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# IDEAL WING APPROXIMATED BY STRAIGHT LINES SEEN FROM ABOVE 

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#### Abstract

The ideal wing shape is ellipsoidal seen from above. It can be approximated by straight lines as rectangular or trapezoidal wing. The trapezoidal wing, having $60 \%$ depth at the end of depth of the central of wing, the area is close to the ellipsoidal one. The trapezoidal wing has leading and trailing edges as straight lines, area and weight close, lift identical to, and drag is close to the ellipsoidal one. It is easily to construct, edges as straight lines, and ribbons decreasing linearly from central to the end of the wing. Rectangular wing has increased area, weight, and drag comparative to the ellipsoidal one, at the same lift. A reasonable approach can be rectangular - trapezoidal Uoptimum wing.


Key words: Ideal wing, rectangular wing, ellipsoidal wing, trapezoidal wing, rectangular - trapezoidal Uopt. wing.

## 1. INTRODUCTION

The wing is the main element of lift for free flight airplane models (glider, rubber, and engine).

Forces of flight on a wing are thrust, drag, lift, and weight, Figure 1. Drag and lift are generated by the wing during the flight.


Fig. 1. Forces of flight on airfoil.
The wing can be rectangular in shape (constant chord), Figure 1, having wingspan (s)


Fig. 1. Rectangular wing (constant chord): s - wingspan, $\mathrm{d}-$ wing depth (chord).
and wing depth (d). The area of rectangular wing, Arec, is
Arec = s.d.

The ideal shape of wing is parabolic seen from front [ $1-5$ ] and elliptic seen from above, Figure 2, having wingspan (s) and wing depth (d). The area of ellipsoidal wing, Ael, is

$$
\begin{equation*}
\mathrm{Ael}=\pi . \mathrm{s} . \mathrm{d} / 4 \tag{2}
\end{equation*}
$$



Fig. 2. Ellipsoidal wing: s - wingspan, and d - wing depth.

Ellipsoidal wing is the most convenient for use. The aerodynamic forces are convenient and uses minimum amount of material to be constructed. It is the tendency to use this shape in almost all constructions. Difficulties are in constructions; leading edge and trailing edge are curves, difficult to make; aero foils are not linearly modified. So, a necessity appears to transform this ellipse into a geometric shape,
having leading and trailing edges made of straight lines. The preferred shape is a trapeze.

## 2. TRANSFORMATION OF THE ELLIPSE IN THE TRAPEZE

It is considered the wing made of two trapezes having the same common base, along d , Figure 3, having wingspan (s), central wing depth (d), and end wing depth (d1). It is called the trapezoidal wing.

The area of trapezoidal wing, Atr, is given by

$$
\begin{equation*}
\operatorname{Atr}=(d+\mathrm{d} 1) \mathrm{s} / 2 . \tag{3}
\end{equation*}
$$



Fig. 3. Trapezoidal wing: s - wingspan,
$\mathrm{d}-$ central wing depth, and $\mathrm{d} 1-$ end depth wing.
To transform ellipsoidal wing into trapezoidal wing, considering the same area, equations (2) and (3), should be equal

$$
\begin{equation*}
\mathrm{Ael}=\mathrm{Atr}, \tag{4}
\end{equation*}
$$

resulting

$$
\begin{equation*}
\mathrm{d} 1=0.57 \mathrm{~d} . \tag{5}
\end{equation*}
$$

It is accepted

$$
\begin{equation*}
\mathrm{d} 1=0.60 \mathrm{~d} \tag{6}
\end{equation*}
$$

for constructive and aerodynamics (Reynolds number) reasons.

Generally, the weight, lift, and drag are proportional to the area for a certain wing. This is a roughly approximation. The air speed around the wing airfoil is considered subsonic. It influences lift and drag in the same manner. The wing weight is a result of wing building; it is proportional to the area too.
The rectangular wing area/weight is 1.273 reported to the ellipsoidal wing of the same area/weight. Lift and drag are the same.
Considering for trapezoidal wing d 1 is 0.6 d , equation (4), the area of trapezoidal wing is 1.8 $\%$ more than of elliptic one.
Area, weight, lifts, and drags for rectangular, elliptic, and trapezoidal wings are presented in Table 1. The elliptic wing is considered $100 \%$ for area, weight, lift, and drag.

Table 1
Area, weight, lift, and drag of different wings

| Wing | Rectangular <br> $\mathbf{\%}$ | Ellipsoidal <br> $\mathbf{\%}$ | Trapezoidal <br> $\mathbf{\%}$ |
| :---: | :---: | :---: | :---: |
| Area | 127.3 | 100 | 101.8 |
| Weight | 127.3 | 100 | 101.8 |
| Lift | 100 | 100 | 100 |
| Drag | 127.3 | 100 | 101.8 |

If ellipsoidal wing is a base for area, weight, lift, and drag ( $100 \%$ ), Table1, the rectangular wing with identical lift has an increased area, weight and drag with $27.3 \%$. Trapezoidal wing (having $\mathrm{d} 1=0.6 \mathrm{~d}$ ) for the same lift has an increased area, weight, and drag with $1.8 \%$.

Considering approximation of the ideal wing by straight lines for Uoptimum shape, Figure 4 [3] arrives at the solution of Figure 4., having wingspan (s), wingspan of rectangular part (s1), central wing depth (d), and end wing depth (d1). There are

$$
\mathrm{d} 1=0.6 \mathrm{~d}
$$

of present considerations, equation (6), and

$$
\begin{equation*}
\mathrm{s} 1=0.3 \mathrm{~s} \tag{7}
\end{equation*}
$$

solution of [3].


Fig. 4. Rectangular - trapezoidal Uoptimum wing: s - wingspan, s1 - wingspan of rectangular part, d - central wing depth, d 1 - end wing depth.

Uoptimum wing, Figure 4 [3], Figure 5, is a wing seen from the front, made by straight lines. The ideal parabola wing is approximated by three straight lines: one tangent in the central part and two passing thru ends, as close as parabola (see consideration in the mentioned paper [3]). The wing has wingspan (s), wingspan of rectangular part (s1), and height of end to base wing (h).

The fact s1, Figures 4 and 5, is close to the wing central part (wing root) makes the wing stronger, static moment of inertia of the
structural strength being bigger (as a solid of equal strength under bending).


Fig. 5. Uoptimum wing seen from the front: s - wingspan, s1 - wingspan of rectangular part, $h-$ height of end wing to the base wing.

Rectangular - trapezoidal Uoptimum wing, Figures 4 and 5, having

$$
\mathrm{s} 1=0.3 \mathrm{~s}, \text { equation (7), }
$$

and

$$
\mathrm{d} 1=0.6 \mathrm{~d}, \text { equation }(6),
$$

has an area increased with $10.2 \%$ over the ellipsoidal wing of the same sizes s and d. The other characteristics as lift and drag are included in Table 2.

Table 2
Area, weight, lift, and drag of ellipsoidal and rectangular - trapezoidal Uoptimum wings

| rectangular - trapezoidal Uoptimum win |  |  |
| :---: | :---: | :---: |
| Wing, \% | Ellipsoidal, \% | Rectangular - <br> trapezoidal <br> $\%$ |
| Area | 100 | 110.2 |
| Weight | 100 | 110.2 |
| Lift | 100 | 100 |
| Drag | 100 | 110.2 |

It is seen from Table 2 the rectangular trapezoidal Uoptimum wing comparative to the ellipsoidal wing, at the same lift the area, lift, and drag are increased to $10.2 \%$, which means some loss of aerodynamics properties, it is easy to build and easy to repair after damages.

When someone wants to make a rectangular trapezoidal Uoptimum wing with exact area as ellipsoidal one has, the value of wing end is
d1 = 0.3857d.

The wing with such d 1 has the area, weight, lift, and drag as ellipsoidal wing. Maybe the small d1 is not good in building, flight, and shocks, definitely not according to Reynolds number.
As an example of using rectangular trapezoidal Uoptimum wing from Figures 4 and 5 is presented a design of a glider, STRONG CLIMBER, Figure 6.The glider with automatic
steering is equipped with a pilot having a magnet as the driving part.

Strong Climber is an excellent air model for automatic steering flight, flying with a speed of $7 \mathrm{~m} / \mathrm{s}$. It is stable at the winds up to $9 \mathrm{~m} / \mathrm{s}$, loaded with $100(8 \mathrm{~m} / \mathrm{s})$ to $250(9 \mathrm{~m} / \mathrm{s}) \mathrm{g}$ lead.

## 3. CONCLUSION

The ideal wing shape is ellipsoidal seen from above. It can be approximated by straight lines as rectangular wing or trapezoidal wing. In the case of trapezoidal wing, having $60 \%$ depth at the end wing of depth of the central of wing, the area is close to the ellipsoidal one. The trapezoidal wing has the leading and trailing edges as straight lines, area and weight close, lift identical, and drag is close to the ellipsoidal one. It is easily to construct, edges are straight lines, and ribbons are decreasing linearly from central part to the end of the wing. Rectangular wing has increased area, weight, and drag comparative to the ellipsoidal one, at the same lift. It is easy to construct, but heavier and higher drag comparative to the trapezoidal wing. A reasonable approach can be rectangular trapezoidal Uoptimum wing.

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Fig. 6. Air model STRONG CLIMBER with rectangular - trapezoidal Uoptimum wing [4 ].
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## ARIPA IDEALĂ APROXIMATĂ PRIN LINII DREPTE VĂZUTĂ DE SUS

Aripa ideală are forma de elipsă văzută de sus. Poate fi aproximată prin linii drepte ca dreptunghiulară sau trapezoidală. Aripa trapezoidală având la capete ptrofunzimea de $60 \%$ din cea de la centru are suprafaţa apropiată de cea eliptică. Aripa trapezoidală are bordul de atac și fugă linii drepte; suprafaţa, greutatea și forţa ascensională identică aripii eliptice, iar rezistenţa la înaintare apropiată de aripa eliptică. Este ușor de constuit bordurile prin linii drepte și nervurile descrescând linear de la centrul la capetele aripii. Aripa dreptunghiulară are crescută suprafaţa, greutatea și rezistanţa la înaintare comparative celei eliptice, la aceași foră ascensională. O nouă abordare poate fi aripa dreptunghiulară trapezoidală Uoptim.

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