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FORMULATIONS ON INPUT PARAMETERS IN ADVANCED DYNAMICS

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Abstract: The dynamical study of the rigid body and multibody systems, in accordance with fundamental theorems and differential principles typical to analytical dynamics, is based on advanced notions, such as: momentum, angular momentum, kinetic energy, acceleration energies of different orders and their absolute time derivatives of higher order. Advanced notions and theorems are developed in connection with the generalized variables, also named independent parameters of position and orientation (d.o.f.), corresponding to holonomic rigid body and systems. But, mechanically, the expressions of definition of the advanced notions are based on fundamental input parameters. These are referring on the one hand to kinematical parameters and their differential transformations, corresponding to absolute motions, on the other hand to mass properties highlighted by mass and inertial tensor, as well their generalized variation laws. By means of the researches of the author, based on classical transformations and matrix exponentials, in this paper a few essential formulations concerning input expressions and parameters from advanced kinematics and dynamics will be presented. They become input expressions compulsory included in fundamental notions and theorems, in the differential principles typical to analytical mechanics, as well as in the dynamics equations of higher order, corresponding to the current and sudden motions in the case of the rigid body and multibody systems, as example robot structures.

Key words: advanced mechanics, dynamics, advanced notions, dynamics equations, matrix exponentials.

1. POSITION AND ORIENTATION

The solid body consists one of physical form of existence of the matter in material universe. As a result, the solid body is considered material continuum. According to this property, to obtain an exact solution, geometrically, the solid body is decomposed in the infinity of the elementary particles, having elementary mass infinitesimal with continuous distribution entire geometrical shape of the solid body. If the distances between elementary particles are kept constant entire solid body, then it has the rigid character (rigid solid body (S)). When the density property is kept constant inside rigid structure, it obtains homogeneous rigid solid. In this case, when the integration limits on the geometrical outline are well-defined, it obtains homogeneous body with simple or regular geometrical shape. In this last case, geometrical and mass integrals are applied.

Before mechanical study (statics, kinematics and dynamics), it must compulsory established the geometrical state to any rigid solid to each moment in Cartesian space. In the view of this, at beginning, geometrical state of the simplest model, named material point, is studied (Fig.1).

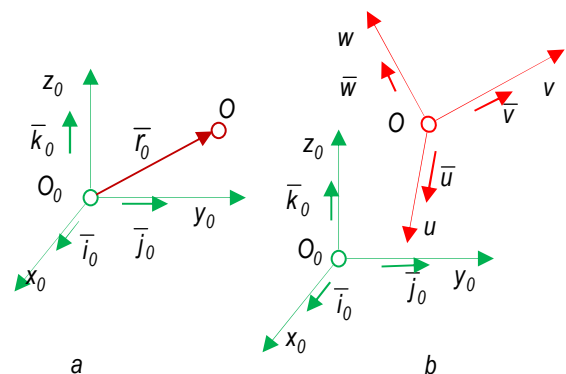


Fig.1 Position and Orientation

Using [1] and [2], a few notations are applied:

$$\left\{ \begin{array}{l} \chi = \{u; v; w\}; \chi_0 = \{u_0; v_0; w_0\} \text{ where} \\ u = \{x; y; z\}; v = \{y; z; x\} \neq u; w = \{z; x; y\} \neq v \end{array} \right\} \quad (1)$$

$$\left\{ \begin{array}{l} \bar{\chi} = \{\bar{u}; \bar{v}; \bar{w}\}; \bar{\chi}_0 = \{\bar{u}_0; \bar{v}_0; \bar{w}_0\} \text{ where} \\ \bar{u} = \{\bar{i}; \bar{j}; \bar{k}\}; \bar{v} = \{\bar{j}; \bar{k}; \bar{i}\} \neq \bar{u}; \bar{w} = \{\bar{k}; \bar{i}; \bar{j}\} \neq \bar{v} \end{array} \right\} \quad (2)$$

$$\delta_x = \{\alpha_x; \beta_x; \gamma_x\}; \cos \delta_x = c\delta_x; \sin \delta_x = s\delta_x \quad (3)$$

$$O_0x_0y_0z_0 \equiv \{0\}; O'_0x'_0y'_0z'_0 \equiv \{0'\}; Oxyz \equiv \{S\}. \quad (4)$$

Notations (1) refer to the Cartesian coordinates or axes, the symbols (2) highlight unit vectors, while (3) express angles and direction cosines.

According to Fig.1a, the geometrical state to any material point (as example O) is named *position*. This is defined by means of *the position vector*:

$$\bar{r}_0 = [x_0 \ y_0 \ z_0]^T, \text{ relative to } \{0\} \text{ frame.} \quad (5)$$

If material point is free in Cartesian frame, the three linear coordinates of (5) are independent. They are also named degrees of freedom (*d.o.f.*).

The study is extended on vector or Cartesian axis (see Fig.1b). In this case, the geometrical state is named *orientation*. This is highlighted by means of *the unit vector*. Using [3] – [13], any unit vector $\bar{\chi} \in \{S\}$ in relation to $\{0\}/\{0'\}$ frame is characterized by the direction cosines, that is:

$$\bar{\chi} = \bar{\chi}^T \cdot \begin{pmatrix} \bar{i}_0 \\ \bar{j}_0 \\ \bar{k}_0 \end{pmatrix} = \begin{pmatrix} c\alpha_\chi \\ c\beta_\chi \\ c\gamma_\chi \end{pmatrix} \equiv \begin{pmatrix} c\alpha \\ c\beta \\ c\gamma \end{pmatrix}_\chi \quad (6)$$

where $\bar{\chi}^T \cdot \bar{\chi} = c^2\alpha_\chi + c^2\beta_\chi + c^2\gamma_\chi = 1. \quad (7)$

Due to (7), *the orientation* to any vector or axis is defined by means of *two independent angles*.

The above geometrical aspects are extended on a reference system orthogonal and right oriented (see Fig.1b, $Ouvw \equiv Oxyz \equiv \{S\}$) relative to $\{0\}$.

Its geometrical state is *position* and *orientation*. Unlike position defined by (5), for orientation, according to [1] – [8], first of all is established:

$${}^0_s[R] = [\bar{i} \ \bar{j} \ \bar{k}] = \begin{bmatrix} \begin{pmatrix} c\alpha \\ c\beta \\ c\gamma \end{pmatrix}_x & \begin{pmatrix} c\alpha \\ c\beta \\ c\gamma \end{pmatrix}_y & \begin{pmatrix} c\alpha \\ c\beta \\ c\gamma \end{pmatrix}_z \end{bmatrix} \quad (8)$$

This is named the resultant rotation (orientation) matrix. It contains the unit vectors belonging to $\{S\}$ in relation to $\{0\}/\{0'\}$. Every unit vector has *two independent angles*. Beside (7), between the unit vectors there are other three relationships:

$$\bar{u}^T \cdot \bar{v} = c\alpha_u \cdot c\alpha_v + c\beta_u \cdot c\beta_v + c\gamma_u \cdot c\gamma_v = 0, \ v \neq u \quad (9)$$

So, in the general case, resultant orientation to any reference frame $\{S\}$ relative to another for example $\{0\}/\{0'\}$ is defined by means of *three orientation angles and independent (three d.o.f.)*. According to [3] – [13], they are symbolized as:

$$\bar{v}(t) = [\alpha_v(t) \ \beta_v(t) \ \gamma_v(t)]^T; \quad (10)$$

Every angle from (10) is, geometrically, dihedral angle between two geometrical plans, that is:

$$\chi_0 = \{u_0; v_0; w_0\} = cst - \text{fixed plan} \in \{0'\}/\{0\},$$

$$\text{and } \chi = \{u; v; w\} = 0 - \text{mobile plan} \in \{S\} \quad (11)$$

Physically, every orientation angle expresses a simple rotation around of the axes: $\chi = \{u; v; w\}$.

Considering the researches from [2] – [13], by combining the three simple rotations, twelve sets of the orientation angles (10) are obtained. The symbol from (10) is named the column matrix of orientation. According to same researches are developed expressions of definition for the three simple rotation matrices, below symbolized as:

$$R(\bar{\chi}; \delta_\chi) = \{R(\bar{x}; \alpha_x); R(\bar{y}; \beta_y); R(\bar{z}; \gamma_z)\} \quad (12)$$

In this paper it proposes the generalized matrix:

$$\left\{ \begin{aligned} R(\bar{\chi}; \delta_\chi) &= \{R(\bar{x}; \alpha_x); R(\bar{y}; \beta_y); R(\bar{z}; \gamma_z)\} \\ &= \begin{bmatrix} c(\delta_\chi \cdot \Delta_{yz}) & -s(\delta_\chi \cdot \Delta_z) & s(\delta_\chi \cdot \Delta_y) \\ s(\delta_\chi \cdot \Delta_z) & c(\delta_\chi \cdot \Delta_{zx}) & -s(\delta_\chi \cdot \Delta_x) \\ -s(\delta_\chi \cdot \Delta_y) & s(\delta_\chi \cdot \Delta_x) & c(\delta_\chi \cdot \Delta_{xy}) \end{bmatrix} \end{aligned} \right\} \quad (13)$$

where $\Delta_{uv} = \{\Delta_{yz}; \Delta_{zx}; \Delta_{xy}\} = \quad (14)$
 $\{\chi = \{u; v\}\}$

$$= \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \text{ if } \delta_\chi = \left\{ \left\{ \begin{matrix} \beta_y \\ \alpha_x \end{matrix} \right\}, \left\{ \begin{matrix} \gamma_z \\ \beta_y \end{matrix} \right\}, \left\{ \begin{matrix} \alpha_x \\ \beta_y \\ \gamma_z \end{matrix} \right\} \right\};$$

and $\Delta_u = \{\Delta_x; \Delta_y; \Delta_z\} = 1 - \Delta_{uv} \quad (15)$
 $\{\chi = u\}$ $\{\chi = \{u; v\}\}$

Successively, substituting (14) and (15) in the generalized form (13) simple rotation matrices (12) are obtained as expressions of definition. In consonance with (13) – (15), new notations are:

$$\overline{\Delta_{uv}} = [c(\delta_\chi \cdot \Delta_{yz}) \ c(\delta_\chi \cdot \Delta_{zx}) \ c(\delta_\chi \cdot \Delta_{xy})]^T; \quad (16)$$

(3×1)

$$\overline{\Delta_u} = [s(\delta_\chi \cdot \Delta_x) \ s(\delta_\chi \cdot \Delta_y) \ s(\delta_\chi \cdot \Delta_z)]^T. \quad (17)$$

(3×1)

The generalized matrix becomes new expression:

$$R(\bar{\chi}; \delta_\chi) = I_3 \cdot \overline{\Delta_{uv}} + [\overline{\Delta_u} \times]; \quad (18)$$

where symbol I_3 is unit matrix, and $[\overline{\Delta_u} \times]$ is skew-symmetric matrix associated to (17). The matrix (18) is obtained with classical formula:

$$R(\bar{\chi}; \delta_\chi) = \bar{\chi} \cdot \bar{\chi}^T \cdot (1 - c\delta_\chi) + I_3 \cdot c\delta_\chi + (\bar{\chi} \times s\delta_\chi) \quad (19)$$

According to researchers from [1] – [6], the three simple rotations from (10) are performed either around of the moving axes or (relatively) fixed axes belonging to $\{S\}$ or $\{0'\}/\{0\}$. Thus, the resultant rotation matrix is determined with:

$${}^0_s[R] = R(\bar{u}; \alpha_u) \cdot R(\bar{v}; \beta_v) \cdot R(\bar{w}; \gamma_w). \quad (20)$$

Using results of the author regarding to matrix exponentials [2] – [13], resultant rotation matrix (20) is below written by means of exponentials:

$$\left\{ \begin{array}{l} \prod_{\{\bar{x}; \delta_x\}} R(\bar{x}; \delta_x) = \exp \left[\sum_{\{\chi=\{u,v,w\}\}} [\bar{x} \times \delta_x] \right] = \\ \prod_{\{\chi=\{u,v,w\}\}} \exp [\bar{x} \times \delta_x] = \prod_{\{u,v\}} \left\{ I_3 \cdot \bar{\Delta}_{uv} + [\bar{\Delta}_u \times] \right\} \end{array} \right\} \quad (21)$$

$$\left\{ \begin{array}{l} {}_s^0[R] = \prod_{\{\bar{x}; \delta_x\}} R(\bar{x}; \delta_x) = e^{\bar{u} \times \alpha_u} \cdot e^{\bar{v} \times \beta_v} \cdot e^{\bar{w} \times \gamma_w} = \\ = \exp[\bar{u} \times \alpha_u] \cdot \exp[\bar{v} \times \beta_v] \cdot \exp[\bar{w} \times \gamma_w] = \\ = \exp[\bar{u} \times \alpha_u + \bar{v} \times \beta_v + \bar{w} \times \gamma_w] \end{array} \right\} \quad (22)$$

Considering the geometrical state of position (5) and the column matrix of orientation (10), the geometrical state corresponding to reference system orthogonal and right oriented is named position and orientation. This is characterized by six independent parameters (see (5) and (10)).

The above mathematical conclusions about position and orientation are generalized in the case of the rigid solid. Considering definitions from the first aligned of this section, any rigid body is composed on the one hand by infinity of material points, one the other hand by infinity of geometrical axes parallel and perpendicular one to another. They have a continuous distributed inside of the geometrical shape of the rigid (S).

The same body is also composed by infinity of assemblies of three geometrical plans orthogonal and continuous distributed in entire rigid solid.

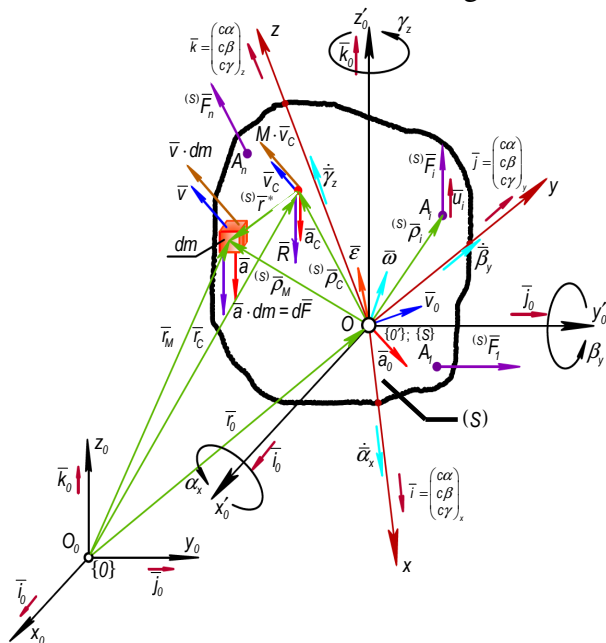


Fig.2 Rigid Body Free in Cartesian Frame

Geometrically, only one ensemble composed of three geometrical plans and orthogonal is enough to choose. It determines a reference system right oriented with the origin in arbitrary point O of the rigid body. According to (4) and Fig. 2, this is symbolized $Oxyz \equiv \{S\}$. Due to rigid character, the system $\{S\}$ is linked of inside body structure.

But, the position and orientation of this frame is defined by the (5) and (10) expressions. Taking two material points from internal structure of the body, as example $M \neq O$ and $C \neq \{O; M\}$ it can write the following position expressions [4]:

$$\bar{r}_M(t) = \bar{r}_O(t) + \bar{\rho}_M(t) = \bar{r}_O(t) + {}_s^0[R](t) \cdot {}^s\bar{\rho}_M; \quad (23)$$

$$\bar{r}_C(t) = \bar{r}_O(t) + \bar{\rho}_C(t) = \bar{r}_O(t) + {}_s^0[R](t) \cdot {}^s\bar{\rho}_C; \quad (24)$$

where $\bar{\rho}_M(t) \neq \bar{\rho}_C(t)$ and $\bar{r}_M(t) \neq \bar{r}_C(t)$. (25)

Using (21) and (22), the position equation is written by matrix exponentials, according to:

$$\left\{ \begin{array}{l} \bar{r}_M(t) = \bar{r}_O(t) + \exp \left[\sum_{\{\chi=\{u,v,w\}\}} [\bar{x} \times \delta_x] \right] \cdot {}^s\bar{\rho}_M = \\ \bar{r}_O(t) + \left\{ \exp[\bar{u} \times \alpha_u + \bar{v} \times \beta_v + \bar{w} \times \gamma_w] \right\} \cdot {}^s\bar{\rho}_M \end{array} \right\} \quad (26)$$

When position $\bar{r}_O(t)$ and orientation ${}_s^0[R](t)$ of the moving frame $Oxyz \equiv \{S\}$ are known, then the position equation for any material point of the body can be determined. At the same time it observes that the orientation is invariant for all points of the rigid solid. So, geometrically and mechanically the body is substituted by means of its moving frame $Oxyz \equiv \{S\}$. These aspects demonstrate the authenticity that the geometrical state of the any rigid solid, free in the Cartesian space, is named *position and orientation*. This is geometrically characterized by means of the six independent parameters (*six d.o.f.*), as follows:

$$\bar{X}_{(6 \times 1)}(t) = \begin{bmatrix} \bar{r}_O(t) \\ \bar{\psi}(t) \end{bmatrix} = \begin{bmatrix} [x_0(t) \ y_0(t) \ z_0(t)]^T \\ [\alpha_u(t) \ \beta_v(t) \ \gamma_w(t)]^T \end{bmatrix} \quad (27)$$

Considering [2] – [9], (*six d.o.f.*) are symbolized:

$$\bar{\theta} \neq \bar{\theta}^{(0)}; \quad \bar{\theta}(t) = [q_j(t); \quad j=1 \rightarrow 6]^T, \quad (28)$$

$$\Delta_j = \left\{ \left(0 \text{ for } q_j - \text{linear} \right); \left(1 \text{ for } q_j - \text{angular} \right) \right\} \quad (29)$$

where $q_j(t)$ is named the generalized coordinate; and (28) is operator that highlights type (*d.o.f.*);

$$\left\{ \begin{array}{l} \left\{ \bar{\theta}(t); \dot{\bar{\theta}}(t); \ddot{\bar{\theta}}(t); \dots; \bar{\theta}^{(m)}(t) \right\} = \\ = \left\{ \begin{array}{l} q_i(t); \dot{q}_i(t); \ddot{q}_i(t); \dots; q_i^{(m)}(t) \\ i=1 \rightarrow 6, m \geq 1 \end{array} \right\} \end{array} \right\}. \quad (30)$$

The symbols, from expressions (30), highlight the generalized variables of higher order in the case of the current and sudden movements. The character (m) represents the time deriving order.

In the advanced mechanics, instead of (10), named column matrix of orientation, the angular vector of orientation is used, according to [4]. Its expression of definition is below written thus:

$$\left\{ \begin{array}{l} {}^0 J_{\psi} [\alpha_u(t) - \beta_v(t) - \gamma_w(t)] = \\ \left[\bar{u} \quad R(\bar{u}; \alpha_u) \cdot \bar{v} \quad R(\bar{u}; \alpha_u) \cdot R(\bar{v}; \beta_v) \cdot \bar{w} \right] \end{array} \right\}; \quad (31)$$

$$\left\{ \begin{array}{l} \bar{\psi}(t) = {}^0 J_{\psi} [\alpha_u(t) - \beta_v(t) - \gamma_w(t)] \cdot \bar{\psi}^{(0)}(t) = \\ = \bar{\psi} [q_j(t) \cdot \Delta_j; \quad j=1 \rightarrow k^*=6, t] \end{array} \right\}; \quad (32)$$

In the matrix study of mechanics the position and orientation for moving frame $Oxyz \equiv \{S\}$ in relation with another, for example fixed frame $\{0\}$ is represented, according to [4] and [5], by matrix of homogeneous transformation, thus:

$${}^0_s [T](t) = \begin{bmatrix} {}^0_s [R](t) & \bar{p}(t) \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad (33)$$

$$\left\{ \begin{array}{l} {}^0_s [T](t) = \begin{bmatrix} {}^0_s [R](t) & \bar{p}(t) \\ 0 & 0 & 0 & 1 \end{bmatrix} = \\ = \left[\begin{array}{l} \exp \left[\sum_{\{\chi=\{u,v,w\}\}} [\bar{\chi} \times \delta_{\chi}(t)] \right] \bar{p}(t) \\ 0 \quad 0 \quad 0 \quad 1 \end{array} \right] \end{array} \right\}; \quad (34)$$

$$\left\{ \begin{array}{l} \bar{p}(t) = \sum_{\{\chi=\{u,v,w\}\}} \left\{ \exp \left\{ \sum_{\chi} [\bar{\chi} \times \delta_{\chi}(t)] \right\} \right\} \cdot \bar{b}_{\chi} \\ + \left\{ \exp \left\{ \sum_{\{\chi=\{u,v,w\}\}} [\bar{\chi} \times \delta_{\chi}(t)] \right\} \right\} \cdot \bar{p}^{(0)} \cdot \Delta_p \end{array} \right\}; \quad (35)$$

$$\left\{ \begin{array}{l} \bar{b}_{\chi} = [\bar{\chi}^{(0)} \cdot \bar{\chi}^{(0)T} \cdot (\delta_{\chi} - s\delta_{\chi}) + I_3 \cdot s\delta_{\chi} + \\ + (\bar{\chi}^{(0)} \times) \cdot (1 - c\delta_{\chi})] \cdot [\bar{p}^{(0)} \times \bar{\chi}^{(0)} \cdot \Delta_{\chi}] \end{array} \right\}; \quad (36)$$

$$\Delta_p = \left\{ \left\{ 0; \bar{p} = \bar{r}_0 \right\}; \left\{ 1; \bar{p} = \bar{r}_M \right\} \right\}. \quad (37)$$

The conclusions and expressions of definition, synthetically disseminated in this introductory section, are compulsory applied in the advanced kinematics and dynamics of mechanical system.

2. INPUT EXPRESSIONS IN DYNAMICS

Based on especially of the author researches, in this paper, a few reformulations and new formulations regarding the advanced notions of dynamics will be presented. In the view of this beside the equations (1) – (37) from first section, the other equations regarding general motion, mass properties and the distribution of the active forces must be synthetically disseminated. For this analysis, it considers the rigid solid (S) in accordance Fig. 2, found in the general motion.

• The parametric equations of motion are (27). The first three (5) express resultant translation motion, while the last three (10) define resultant rotation movement. This is also characterized by (12) – (22), (31) and (32). Taking to study (23), it expresses the absolute position equation. It shows the variable distribution from to another material point of the body. Applying the first time and absolute derivative on (23), it obtains:

$$\left\{ \begin{array}{l} \dot{\bar{r}}_M(t) = \dot{\bar{r}}_0(t) + \dot{\bar{p}}_M(t) = \dot{\bar{r}}_0(t) + {}^0_s [\dot{R}](t) \cdot {}^s \bar{p}_M \\ = \dot{\bar{r}}_0(t) + {}^0_s [\dot{R}](t) \cdot {}^0_s [R]^T(t) \cdot {}^0_s [R](t) \cdot {}^s \bar{p}_M \end{array} \right\}. \quad (38)$$

In accordance to [1] – [7], the skew symmetric matrix associated to angular velocity vector is:

$${}^0_s [\dot{R}](t) \cdot {}^0_s [R]^T(t) = (\bar{\omega} \times). \quad (39)$$

As a result, linear velocity and acceleration are:

$$\bar{v}_M = \bar{v}_0 + \bar{\omega} \times \bar{p}_M; \quad (40)$$

$$\bar{a}_M = \bar{a}_0 + \dot{\bar{\omega}} \times \bar{p}_M + \bar{\omega} \times \bar{\omega} \times \bar{p}_M. \quad (41)$$

The time derivatives of higher order applied on the property (39) are defined with expression:

$$\left\{ \begin{array}{l} {}^0_s [R] \cdot {}^0_s [R]^T = [{}^0_s [\bar{\omega} \times] + {}^0_s [R] \cdot {}^0_s [R]^T \cdot [{}^0_s [\bar{\omega} \times] + \\ + \frac{d^{k-2}}{dt^{k-2}} \{ [{}^0_s [\bar{\omega} \times] \cdot [{}^0_s [\bar{\omega} \times] \} \} \cdot \delta_k + \\ + \sum_{j=1}^{k-3} \frac{d^j}{dt^j} \left\{ [{}^0_s [R] \cdot {}^0_s [R]^T \cdot [{}^0_s [\bar{\omega} \times] \} \cdot \delta_{k-1} + \right. \\ \left. \text{where } k \geq 2, \text{ and } \delta_k = \begin{cases} 0, & k \leq 2 \\ 1, & k > 2 \end{cases} \right\} \end{array} \right\}; \quad (42)$$

In advanced kinematics and dynamics, the time derivatives of higher order for rotation matrices and position vectors must be used as follows:

$${}^0_s[R][q_j(t) \cdot \Delta_j] = \sum_{j=1}^6 \left\{ \frac{\partial}{\partial q_j} \left\{ {}^0_s[R] \right\} \cdot \Delta_j \cdot q_j^{(k)} \right\} \quad (43)$$

$$+ \sum_{j=1}^6 \sum_{r=1}^{k-1} \left\{ \frac{\prod_{p=1}^r (k-p)}{p!} \cdot \left[\frac{p! \cdot m!}{(m+p)!} \cdot \frac{\partial}{\partial q_j} \left\{ {}^0_s[R] \right\} \cdot \Delta_j \cdot q_j^{(k-p)} \right] \right\};$$

$$\left\{ \begin{aligned} \frac{d^k \bar{r}_M(t)}{dt^k} &= \frac{(k-1)}{\bar{v}_M} = \sum_{j=1}^6 \left[\frac{\partial \bar{r}_M^{(m)}(t)}{\partial q_j} \cdot q_j^{(k)} \right] + \\ &\sum_{j=1}^6 \sum_{r=1}^{k-1} \left\{ \frac{\prod_{p=1}^r (k-p)}{p!} \cdot \left[\frac{p! \cdot m!}{(m+p)!} \cdot \frac{\partial \bar{r}_M^{(m)}(t)}{\partial q_j} \cdot q_j^{(k-p)} \right] \right\} \end{aligned} \right\} \quad (44)$$

and $\left\{ \begin{aligned} k &\geq 1; k = \{1; 2; 3; 4; 5; \dots\} \\ m &\geq (k+1); m = \{2; 3; 4; 5; \dots\} \end{aligned} \right\};$

where the symbols: (k) and (m) are the orders of the time derivatives concerning (43) and (44). According to researches of author [2] – [13], expressions of definition for angular velocities, and then angular accelerations of higher order are established on the basis of matrix exponentials:

$$\begin{aligned} \bar{\omega}[\alpha_u(t) - \beta_v(t) - \gamma_w(t)] &= \quad (45) \\ \left\{ \begin{aligned} &\dot{\alpha}_u(t) \cdot \{\exp[0]\} \cdot \bar{u}^{(0)} + \dot{\beta}_v(t) \cdot \{\exp[\bar{u}(t) \times \alpha_u(t)]\} \cdot \bar{v}^{(0)} \\ &+ \dot{\gamma}_w(t) \cdot \{\exp[\bar{u}(t) \times \alpha_u(t)] \cdot \exp[\bar{v}(t) \times \beta_v(t)]\} \cdot \bar{w}^{(0)} \end{aligned} \right\} \end{aligned}$$

$$\begin{aligned} \bar{\omega}[\alpha_u(t) - \beta_v(t) - \gamma_w(t)] &= \quad (46) \\ \left\{ \begin{aligned} &= \frac{d}{dt} \left\{ \dot{\alpha}_u(t) \cdot \{\exp[0]\} \cdot \bar{u}^{(0)} + \right. \\ &\quad \left. + \dot{\beta}_v(t) \cdot \{\exp[\bar{u}(t) \times \alpha_u(t)]\} \cdot \bar{v}^{(0)} + \right. \\ &\quad \left. + \dot{\gamma}_w(t) \cdot \{\exp[\bar{u}(t) \times \alpha_u(t)] \cdot \exp[\bar{v}(t) \times \beta_v(t)]\} \cdot \bar{w}^{(0)} \right\} \end{aligned} \right\} \end{aligned}$$

Remark: The using of matrix exponentials apparently seems to be complicatedly, but these have great advantages of not using reference systems. This is visible in above equations by $\bar{\chi}^{(0)} = \{\bar{u}^{(0)}; \bar{v}^{(0)}; \bar{w}^{(0)}\}$. They are corresponding to initial state of the moving frame $Oxyz \equiv \{S\}$.

• An essential aspect in advanced dynamics is reflected by *mass properties*. First of all, mass and position of mass center is determined in the relation with $O'_0x'_0y'_0z'_0 \equiv \{O'\}$, as follows below:

$$M = \int dm; \text{ where } dm = \{\rho_\sigma \cdot d\sigma\}; \sigma = \{V; A; L\}; \quad (47)$$

$$\bar{\rho}_C(t) = \frac{\int \bar{\rho}_M(t) \cdot dm}{\int dm} = \frac{\int \bar{\rho}_M(t) \cdot dm}{M}; \quad (48)$$

$$\int \bar{\rho}_M(t) \cdot dm = M \cdot \bar{\rho}_C(t); \quad (49)$$

where (48) is the static moment relative to $\{O'\}$.

The position of the mass center, in the relation to $O_0x_0y_0z_0 \equiv \{O\}$, is expressed in classical form, and then on the basis of matrix exponentials as:

$$\left\{ \begin{aligned} \bar{r}_C(t) &= \frac{\int \bar{r}_M(t) \cdot dm}{\int dm} = \frac{\int \bar{r}_M(t) \cdot dm}{M} = \\ &= \bar{r}_0(t) + \bar{\rho}_C(t) = \bar{r}_0(t) + {}^0_s[R](t) \cdot {}^s\bar{\rho}_C \end{aligned} \right\}; \quad (50)$$

$$\left\{ \begin{aligned} \bar{r}_C(t) &= \bar{r}_0(t) + \exp \left[\sum_{\chi=\{u,v,w\}} [\bar{\chi} \times \delta_\chi] \right] \cdot {}^s\bar{\rho}_C = \\ &\bar{r}_0(t) + \left\{ \exp[\bar{u} \times \alpha_u + \bar{v} \times \beta_v + \bar{w} \times \gamma_w] \right\} \cdot {}^s\bar{\rho}_C \end{aligned} \right\} \quad (51)$$

Applying the time derivative on (50), the linear velocity and acceleration of the mass center are:

$$\left\{ \begin{aligned} \dot{\bar{r}}_C(t) &= \dot{\bar{r}}_0(t) + \dot{\bar{\rho}}_C(t) = \\ &\dot{\bar{r}}_0(t) + {}^0_s[\dot{R}](t) \cdot {}^0_s[R]^T(t) \cdot {}^0_s[R](t) \cdot {}^s\bar{\rho}_C \end{aligned} \right\}; \quad (52)$$

$$\bar{v}_C(t) = \bar{v}_0(t) + \bar{\omega}(t) \times \bar{\rho}_C(t); \quad (53)$$

$$\left\{ \begin{aligned} \dot{\bar{v}}_C(t) &= \frac{d}{dt} \left\{ \bar{r}_0(t) + \exp \left[\sum_{\chi=\{u,v,w\}} [\bar{\chi} \times \delta_\chi] \right] \cdot {}^s\bar{\rho}_C \right\} = \\ &\frac{d}{dt} \left\{ \bar{r}_0(t) + \left\{ \exp[\bar{u} \times \alpha_u + \bar{v} \times \beta_v + \bar{w} \times \gamma_w] \right\} \cdot {}^s\bar{\rho}_C \right\} \end{aligned} \right\}$$

Using classical expression (52) / (53) it obtains:

$$\dot{\bar{v}}_C(t) = \dot{\bar{v}}_0(t) + \frac{d}{dt} [\bar{\omega}(t) \times \bar{\rho}_C(t)], \quad (54)$$

$$\bar{a}_C = \bar{a}_0 + \bar{\varepsilon} \times \bar{\rho}_C + \bar{\omega} \times \bar{\omega} \times \bar{\rho}_C \quad (55)$$

In dynamics next expression is necessary:

$$\bar{\rho}_M(t) = \bar{\rho}_C(t) + \bar{r}^*(t) = {}^0_s[R](t) \cdot ({}^s\bar{\rho}_C + {}^s\bar{r}^*);$$

$$\bar{\omega}(t) \times \bar{\rho}_M(t) = \bar{\omega}(t) \times \bar{\rho}_C(t) + \bar{\omega}(t) \times \bar{r}^*(t). \quad (56)$$

• Essential aspect they have inertia properties in the cases of the rotation motions. These are named mechanical moments of inertia [4] – [5]. According to Fig. 2, the elementary mass (dm) relative to mass center is defined by means of the position vector as: $\bar{r}^* = \bar{r}^*(t)$. Considering (50), the next mass property is obtained thus:

$$\frac{\int \bar{r}^*(t) \cdot dm}{\int dm} = \frac{\int \bar{r}^*(t) \cdot dm}{M} = 0; \quad (57)$$

$$\text{wence } \int \bar{r}^*(t) \cdot dm = {}^0_s[R](t) \cdot \int {}^s\bar{r}^* \cdot dm = 0. \quad (58)$$

Using the researches of the author [1] – [4], the inertial tensor and its variation law relative to concurrent frames is established, as follows:

$$\left\{ \begin{array}{l} I_s^* = \int (\bar{r}^* \times) \cdot (\bar{r}^* \times)^T \cdot dm = \\ = {}^0_s [R] \cdot \left[\int ({}^s \bar{r}^* \times) \cdot ({}^s \bar{r}^* \times)^T \cdot dm \right] \cdot {}^0_s [R]^T \end{array} \right\}; \quad (59)$$

where the mass integral is squared matrix, thus:

$$\int ({}^s \bar{r}^* \times) \cdot ({}^s \bar{r}^* \times)^T \cdot dm = {}^s I_s^*. \quad (60)$$

This is inertial tensor axial and centrifugal of the body (S) in with relation $\{S^*\}$ applied in the mass center (C), having property: $\{S^*\}_{OR} \equiv \{S\}_{OR}$.

From (59) the variation law of the inertial tensor relative to concurrent frames in the mass center: $\{S^*\}$ and $\{O^*\}_{OR} \equiv \{O\}_{OR}$ is established, as follows

$$I_s^* = \int (\bar{r}^* \times) \cdot (\bar{r}^* \times)^T \cdot dm = {}^0_s [R] \cdot {}^s I_s^* \cdot {}^0_s [R]^T. \quad (61)$$

In the following steps the inertial tensor axial and centrifugal in relation with $\{O^*\}$ is defined:

$$\left\{ \begin{array}{l} I'_s = \int (\bar{\rho}_M \times) \cdot (\bar{\rho}_M \times)^T \cdot dm = \\ = {}^0_s [R] \cdot \left[\int ({}^s \bar{\rho}_M \times) \cdot ({}^s \bar{\rho}_M \times)^T \cdot dm \right] \cdot {}^0_s [R]^T \end{array} \right\}; \quad (62)$$

$$\text{where } \int ({}^s \bar{\rho}_M \times) \cdot ({}^s \bar{\rho}_M \times)^T \cdot dm = {}^s I_s; \quad (63)$$

$$\text{and } I'_s = \int (\bar{\rho}_M \times) \cdot (\bar{\rho}_M \times)^T \cdot dm = {}^0_s [R] \cdot {}^s I_s \cdot {}^0_s [R]^T.$$

The position equation (56) is changed in a skew symmetric matrix, and this is substituted in (62):

$$(\bar{\rho}_M \times) = (\bar{\rho}_C \times) + (\bar{r}^* \times);$$

$$I'_s = (\bar{\rho}_C \times) \cdot (\bar{\rho}_C \times)^T \cdot \int dm + \int (\bar{r}^* \times) \cdot (\bar{r}^* \times)^T \cdot dm; \quad (64)$$

$$(\bar{\rho}_C \times) \cdot (\bar{\rho}_C \times)^T \cdot \int dm = M \cdot (\bar{\rho}_C \times) \cdot (\bar{\rho}_C \times)^T = I'_{SC} \quad (65)$$

$$I'_s = {}^0_s [R] \cdot {}^s I_s \cdot {}^0_s [R]^T = I'_{SC} + I_s^*. \quad (66)$$

According to [4] – [5], the matrix expression (66) characterizes the generalized variation law of the inertial tensor axial and centrifugal in relation with frame $\{O^*\}$. The expression (65) is named the inertia matrix axial and centrifugal of the mass center relative to $\{O^*\}$. Sometimes, the inertial tensor axial and centrifugal is defined in relation with absolute frame $\{O\}_{OR} = \{O^*\}_{OR}$, thus:

$$\left\{ \begin{array}{l} I_s = \int (\bar{r}_M \times) \cdot (\bar{r}_M \times)^T \cdot dm = \\ M \cdot (\bar{r}_0 \times) \cdot (\bar{r}_0 \times)^T + I'_s = I_{S0} + I'_s = I_{S0} + I'_{SC} + I_s^* \end{array} \right\}. \quad (67)$$

• Another essential aspect for any dynamical study it consists in the distribution of the active forces, that determine the general motion of the rigid solid. Its distribution is shown below as:

$$\left\{ ({}^s) \bar{F}_i = ({}^s) \bar{u}_i \cdot F_i; A_i; ({}^s) \bar{\rho}_i; i = 1 \rightarrow n \right\}; \quad (68)$$

$$\text{where } \bar{F}_i = {}^0_s [R] \cdot {}^s \bar{F}_i = {}^0_s [R] \cdot {}^s \bar{u}_i \cdot F_i; \quad (69)$$

$$\text{and } \bar{r}_i(t) = \bar{r}_0(t) + \bar{\rho}_i(t) = \bar{r}_0(t) + {}^0_s [R](t) \cdot {}^s \bar{\rho}_i. \quad (64)$$

As a result, the reduction torsor relative $\{O^*\}$ is:

$$\left\{ \begin{array}{l} \bar{R}^* = \sum_{i=1}^n \bar{F}_i = {}^0_s [R] \cdot \sum_{i=1}^n {}^s \bar{F}_i = \\ = {}^0_s [R] \cdot \sum_{i=1}^n {}^s \bar{u}_i \cdot F_i = \int d\bar{F} = \int \bar{a}_M \cdot dm \end{array} \right\}; \quad (70)$$

$$\left\{ \begin{array}{l} \bar{M}'_O = \sum_{i=1}^n \bar{\rho}_i \times \bar{F}_i = {}^0_s [R] \cdot \sum_{i=1}^n ({}^s \bar{\rho}_i \times {}^s \bar{F}_i) = \\ = {}^0_s [R] \cdot \sum_{i=1}^n F_i \cdot ({}^s \bar{\rho}_i \times {}^s \bar{u}_i) = \int \bar{\rho}_M \times d\bar{F} \end{array} \right\}. \quad (71)$$

Since the resultant vector (70) is invariant with any reduction pole, this means that changing the pole from (O) in $(O_0) \in \{O\}$ resultant moment is highlighted by the variation law, as follows:

$$\bar{M}_O = \bar{r}_0 \times \bar{R} + \bar{M}'_O = \int \bar{r}_M \times d\bar{F} = \int \bar{r}_M \times \bar{a}_M \cdot dm. \quad (72)$$

In the above equations \bar{a}_M is substituted by (41).

3. PARAMETERS OF HIGHER ORDER

Velocities, as well as the accelerations of higher order (45), (46), (53) – (55) can be also established by means of the following vectors:

$$\left\{ \begin{array}{l} \bar{r}_0(t) + \exp \left[\sum_{\{\chi=\{u,v,w\}\}} [\bar{\chi}(t) \times \delta_\chi] \right] \cdot {}^s \bar{\rho}_C = \\ = \bar{r}_C [q_j(t); j = 1 \rightarrow k^* = 6, t] = \bar{r}_C(t) \end{array} \right\}; \quad (73)$$

$$\bar{\psi}(t) = \bar{\psi} [q_j(t) \cdot \Delta_j; j = 1 \rightarrow k^* = 6, t] = \quad (74)$$

$$\left\{ \begin{array}{l} = \alpha_u(t) \cdot \{ \exp[0] \} \cdot \bar{u}^{(0)} + \beta_v(t) \cdot \{ \exp[\bar{u}(t) \times \alpha_u(t)] \} \cdot \bar{v}^{(0)} \\ + \gamma_w(t) \cdot \{ \exp[\bar{u}(t) \times \alpha_u(t)] \cdot \exp[\bar{v}(t) \times \beta_v(t)] \} \cdot \bar{w}^{(0)} \end{array} \right\}$$

where (73) is identical with (50) / (51), and (74) named the orientation vector is written by means of expressions: (10), (22), (29), (31) and (32).

An essential component (31) included in (74) is known as angular transfer matrix defined as function of set of orientation angles. Considering (73) and (74) it observes that they are functions of generalized variables (28) – (30). Actually,

the six generalized variables are the independent parameters of position and orientation from (27).

• Using researches of author from [9] – [13], on the time vector functions of position (73) and orientation (74), the differentials properties compulsory applied in advanced kinematics and dynamics have been developed as below follows:

$$\left\{ \frac{d^{k-1}}{dt^{k-1}} \left(\frac{\partial \bar{r}_c}{\partial q_j} \right) = \frac{(k-1)! \cdot m!}{(m+k-1)!} \cdot \frac{\partial \bar{r}_c^{(m+k-1)}}{\partial q_j^{(m)}} \right\}, \quad (75)$$

$$\left\{ \begin{aligned} \frac{d^{k-1}}{dt^{k-1}} \left(\frac{\partial \bar{\psi}_i}{\partial q_j} \cdot \Delta_j \right) &= \frac{(k-1)! \cdot m!}{(m+k-1)!} \cdot \frac{\partial \bar{\psi}_i^{(m+k-3)}}{\partial q_j^{(m)}} \cdot \Delta_j = \\ &= \frac{(k-1)! \cdot m!}{(m+k-1)!} \cdot \frac{\partial \bar{\psi}_i^{(m+k-1)}}{\partial q_j^{(m)}} \cdot \Delta_j \end{aligned} \right\} \quad (76)$$

$$\{k \geq 1; m \geq (k+1); m = \{2; 3; 4; 5; \dots\}\}. \quad (77)$$

The symbols (77) highlight time deriving orders. Using (74) – (77), the next expressions become:

$$\bar{\omega}_i(t) = {}^0 J_\psi [\alpha_u(t) - \beta_v(t) - \gamma_{Cw}(t)] \cdot \frac{\partial \bar{\psi}_i(t)}{\partial t} = \quad (78)$$

$$\left\{ = \sum_{j=1}^{k^*=n} \frac{\partial \bar{\psi}_i(t)}{\partial q_j} \cdot \Delta_j \cdot \dot{q}_j(t) = \sum_{j=1}^{k^*=n} \frac{\partial \bar{\psi}_i^{(m)}(t)}{\partial q_j^{(m)}} \cdot \Delta_j \cdot \dot{q}_j(t) \right\};$$

$$\bar{v}_c(t) = \sum_{j=1}^{k^*=6} \frac{\partial \bar{r}_c(t)}{\partial q_j} \cdot \dot{q}_j(t) = \sum_{j=1}^{k^*=6} \frac{\partial \bar{r}_c^{(m)}(t)}{\partial q_j^{(m)}} \cdot \dot{q}_j(t); \quad (79)$$

$$\left\{ \begin{aligned} \bar{\varepsilon}_i^{(k-1)}(t) &= \bar{\omega}_i^{(k)}(t) = \sum_{j=1}^{k^*=n} \frac{d^{k-1}}{dt^{k-1}} \left[\frac{\partial \bar{\psi}_i^{(m)}(t)}{\partial q_j^{(m)}} \cdot \Delta_j \cdot \ddot{q}_j(t) \right] \\ &+ \frac{1}{m+1} \cdot \sum_{j=1}^{k^*=n} \frac{d^{k-1}}{dt^{k-1}} \left[\frac{\partial \bar{\psi}_i^{(m+1)}(t)}{\partial q_j^{(m)}} \cdot \Delta_j \cdot \dot{q}_j(t) \right] = \bar{\psi}_i^{(k+1)}(t) \end{aligned} \right\} \quad (80)$$

$$\left\{ \begin{aligned} \bar{a}_{c_i}^{(k-1)}(t) &= \bar{v}_{c_i}^{(k)}(t) = \sum_{j=1}^{k^*=n} \frac{d^{k-1}}{dt^{k-1}} \left[\frac{\partial \bar{r}_{c_i}^{(m)}(t)}{\partial q_j^{(m)}} \cdot \ddot{q}_j(t) \right] \\ &+ \sum_{j=1}^{k^*=n} \frac{d^{k-1}}{dt^{k-1}} \left[\frac{1}{m+1} \cdot \frac{\partial \bar{r}_{c_i}^{(m+1)}(t)}{\partial q_j^{(m)}} \cdot \dot{q}_j(t) \right] = \bar{r}_{c_i}^{(k+1)}(t) \end{aligned} \right\} \quad (81)$$

The expressions (79) and (81) are identical with (53) – (55), and they are referring to the linear velocity and linear accelerations of higher order, corresponding to mass center. The others: (78)

and (80) identical with (45) and (46) they are the angular velocities and angular accelerations of higher order for rigid solid in general motion.

Remarks: The input expressions and parameters of higher order form the three sections of this paper are compulsory applied in the definition of the dynamic notions of higher order, such as: momentum, angular momentum, kinetic energy, acceleration energy of higher order. They will be included in the dynamics theorems of the current and sudden mechanical motion of the bodies.

4. CONCLUSIONS

The currently paper was devoted especially to presentation essential formulations concerning some expressions and parameters from advanced kinematics and dynamics of the rigid bodies and systems. They become the input expressions compulsory included in dynamics equations of higher order, corresponding to the current and sudden motions in the case of the rigid body. These are extended on the multibody systems.

So, unlike the classical models the author has presented in first section of paper reformulations and new formulations regarding the independent parameters of position and orientation, for any rigid body found free in the Cartesian space. In the same section is proposes a new general expression for the simple and resultant rotation matrices, as well homogeneous transformation matrix. In the second section of the paper, they have been presented the input expressions that define the general motion of the body. In the view of this matrix exponentials and the time derivatives of higher order have been applied, concerning the rotation matrices, position vectors, as well linear and angular accelerations of higher order. In the same section of this paper were presented the mass and inertia properties, represented by: mass, position of mass center, inertial tensor and its generalized variation law. In the final part of the second section of the paper the author presented the distribution of active forces corresponding to general motion. In the third section, author developed important differential properties concerning position of the mass center and orientation vector. They are also used for determine the linear and angular accelerations of higher order above mentioned.

5. REFERENCES

- [1] Appell, P., *Sur une forme générale des equations de la dynamique*, Paris, 1899.
- [2] Negrean, I., Negrean, D. C., *Matrix Exponentials to Robot Kinematics*, 17th International Conference on CAD/CAM, Robotics and Factories of the Future, CARS&FOF 2001, Durban South Africa, 2001, Vol.2, pp. 1250-1257, 32 rel., 4 ref.
- [3] Negrean, I., Negrean, D. C., *The Acceleration Energy to Robot Dynamics*, International Conference on Automation, Quality and Testing, Robotics, AQTR 2002, May 23-25, Cluj-Napoca, Tome II, pp. 59-64
- [4] Negrean I., *Mecanică avansată în Robotică*, ISBN 978-973-662-420-9, UT Press, 2008.
- [5] Negrean I., *Mechanics, Theory and Applications*, Editura UT PRESS, Cluj – Napoca, ISBN 978-606-737-061-4, 2015.
- [6] Negrean, I., *Formulations in Advanced Dynamics of Mechanical Systems*, The 11 th IFTOMM International Symposium on Science of Mechanisms and Machines, Mechanisms and Machine Science 18, DOI:10.1007/978-3-319-01845-4-19 Springer International Publis. Switzerland, pp. 185-195.
- [7] Negrean, I., *Energies of Acceleration in Advanced Robotics Dynamics*, Applied Mechanics and Materials, ISSN: 1662-7482, vol 762, pp 67-73 Submitted: 2014-08-05 ©(2015)TransTech Publications Switzerland
- DOI:10.4028/www.scientific.net/AMM.762.67, Revised:2014-11-16.
- [8] Negrean I., *New Formulations on Acceleration Energy in Analytical Dynamics*, Applied Mechanics and Materials, vol. 823 pp 43-48 © (2016) TransTech Publications Switzerland Revised: 2015-09, Doi: 10.4028 /www.scientific.net/AMM.823.43.
- [9] Negrean I., *Formulations on the Advanced Notions in analytical Dynamics of Mechanical Systems*, published in IJERM, ISSN: 2349-2058, Volume-03, Issue-04, pp 123-130, 2016.
- [10] Negrean I., *Advanced Notions and Differential Principles of Motion in Analytical Dynamics*, Journal of Engineering Sciences and Innovation, Technical Sciences Academy of Romania, Vol. 1/2016, Issue 1, pp. 49-72, ISSN 2537-320X.
- [11] Negrean I., *Advanced Notions in Analytical Dynamics of Systems*, Acta Technica Napocensis, Series: Applied Mathematics, Mechanics, and Engineering, Vol. 60, Issue IV, 2017, ISSN 1221-5872, pp. 491-502
- [12] Negrean I., *Advanced Equations in Analytical Dynamics of Systems* Acta Technica Napocensis, Series: Applied Mathematics, Mechanics, and Engineering, Vol. 60, Issue IV, 2017, ISSN 1221-5872, pp. 503-514
- [13] [12] Negrean I., *New Approaches on Notions from Advanced Dynamics*, Acta Technica Napocensis, Series: Applied Mathematics, Mechanics, and Engineering, Vol.61, Issue II, June 2018, pp. 149 – 158.

Formulări asupra parametrilor de intrare în dinamica avansată

Studiul dinamic al corpului rigid și sistemelor multicorp, în conformitate cu teoremele fundamentale și principiile diferențiale specifice dinamicii analitice, se bazează pe noțiunile avansate, cum sunt: impulsul, momentul cinetic, energia cinetică, energiile de accelerații de diferite ordine și derivatele absolute în raport cu timpul a acestora de ordin superior. Noțiunile și teoremele avansate sunt dezvoltate în consonanță cu variabilele generalizate, cunoscute și ca parametri independenți de poziție și orientare (g.d.l.), corespunzători corpului rigid și sistemelor olonome. Dar, sub aspect mecanic, expresiile de definiție ale noțiunilor avansate se bazează pe parametri de intrare fundamentali. Aceștia se referă pe de o parte la parametri cinematici și transformările lor diferențiale, corespunzătoare mișcării absolute, pe de altă parte parametri de intrare se referă la proprietățile maselor evidențiate prin masă și tensorii inerțiali, de asemenea prin legile de variație generalizată. Cu ajutorul cercetărilor autorului, bazate pe transformări clasice și exponențiale de matrice, în această lucrare vor fi prezentate formulări esențiale privind expresiile și parametri de intrare pentru cinematica și dinamica avansată. Aceștia devin expresii de intrare obligatorii pentru noțiunile și teoremele fundamentale, pentru principiile diferențiale ale mecanicii analitice, de asemenea pentru ecuațiile dinamicii de ordin superior, corespunzătoare mișcărilor curente și rapide ale corpului rigid și sistemelor multicorp, ca exemplu structurile de roboți.

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