TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering
Vol. 61, Issue IV, November, 2018

POLYNOMIAL INTERPOLATION FUNCTIONS IN ADVANCED DYNAMICS

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Abstract: *The present paper is devoted to the research of the main author, in what concerns the use of polynomial interpolation functions in generating the motion trajectory of an industrial robot. In this purpose are presented some classical formulations on polynomial interpolation functions of third order with restrictions. Unlike the third order polynomials, the higher order polynomials* (≥ 5) *have the advantage of ensuring the continuity in accelerations of higher order. Are also presented, in explicit form, the expressions for the acceleration energy of first, second and third order, corresponding to the current and sudden motions multi body systems and which are further use to define the motion equations. In the final part of the paper, the formulations are applied to describe the dynamic behavior of a 2TR robot structure. So, are determined the expressions for the acceleration energies of first, second and third order, compulsory in defining the dynamic equations. The time variation laws for generalized coordinates, velocities, accelerations, energy of accelerations and generalized forces of first, second and third order will be also defined.*

Key words: *polynomial interpolation functions, advanced dynamics, acceleration energy, robotics.*

1. INTRODUCTION

A consequence of the fact that the machines and robots used in different technological processes have become very complex and precise, is seen in the improvement of the performed tasks regarding the motions that have to be designed in order to follow accurately a given path. So, it is introduced the concept of motion trajectory which allows engineers to build more powerful tools in order to control the motion of different machines simply by considering the position a function of time.

In most cases, the main approach, consist in defining a certain path and expect the system to be able to follow. But many of the designed paths can't be precisely and effectively followed. Some of the systems already use trajectories, but not in an integrated way. The use of a trajectory to define the motion of a mechanical system gives all the necessary elements to verify that the motion is really feasible. For example, in case of an industrial robot which is interacting with human operators, the safety constraints can be expressed more easily in terms of kinematic constraints.

From mathematical point of view, the trajectory is usually studied between two points belonging to the working space, corresponding to the initial and final moment of the motion. In order to avoid possible collisions with different objects from the working area, a finite set of intermediate points is added to the two points that were initially considered. The motion trajectory of a robot represents the meeting of all polynomial time functions, these being expressed either in the configuration space or in the Cartesian space, depending on the initial conditions of the movement. The degree of interpolation of the polynomial depends on restrictive conditions of the movement, which are generally the points in the work space that the robot is forced to pass at a certain moment and the generalized and operational velocities and accelerations that characterize the motion of these points at the time t_1, according to [5], [6].

According to the research of the main author, example [12] – [15], polynomial interpolation functions can be expressed in a general form as:

$$\left\{ \begin{array}{l} \overset{(m-p)}{q_{ji}}(\tau) = (-1)^p \cdot \dfrac{(\tau_i - \tau)^{p+1}}{t_i \cdot (p+1)!} \cdot \overset{(m)}{q_{ji-1}} + \\ + \dfrac{(\tau - \tau_{i-1})^{p+1}}{t_i \cdot (p+1)!} \cdot \overset{(m)}{q_{ji}} + \delta_p \cdot \displaystyle\sum_{k=1}^{p} \dfrac{\tau^{p-k}}{(p-k)!} \cdot a_{jik} \end{array} \right\} \quad (1)$$

In the expression presented above, m is a parameter defining the deriving order of polynomial ($m \geq 2$, $m = 2,3,4,5,\dots$), $p = 0 \to m$, $j = 1 \to n$ are the degrees of freedom of the analyzed mechanical system. In the same equation, δ_p represents the space travelled during t_i period of time, being defined by the following identity: $\delta_p = \{(0, p = 0);(1; p \geq 1)\}$.
If in case of the real interpolation function, δ_p represents the space travelled during time interval t_i, for the normalized function, it travels a space of unity length in a given time interval. Also, $i = 1 \to s$ defines the intervals of motion trajectories, τ is the actual time variable $\tau \in [\tau_{i-1} \;\; \tau_i]$, and $t_i = \tau_i - \tau_{i-1}$ is the actual time corresponding to each trajectory interval (i). For every trajectory interval $(i = 1 \to s)$, $(m+1)$ represents the number of unknowns, defined as:

$$\left\{ (a_{jik}) \text{ for } k = 1 \to m; \left(\overset{(m)}{q}_{ji-1} \right) \text{ for } i = 2 \to s \right\}; \quad (2)$$

where (a_{jik}) are the integration constants, as well $\left(\overset{(m)}{q}_{ji-1} \right)$ the generalized accelerations of (m) order.
To determine the unknowns defined by the expression (2), is required to apply geometrical and kinematical constraints [11], [15] – [18]:

$$\left\{ \begin{array}{l} (\tau_0) \Rightarrow \overset{(m-p)}{q}_{j0}, \;\; p = 0 \to m; \;\; (\tau_s) \Rightarrow \left\{ \overset{(m)}{q}_{js}, \; q_{js} \right\} \\ (\tau_i) \Rightarrow \left\{ \begin{array}{c} \overset{(2)}{q}_{ji} - \text{generalized accelerations} \\ \left\{ \overset{(m-p)}{q}_{ji}(\tau^+) = \overset{(m-p)}{q}_{ji+1}(\tau^-), \;\; p = 0 \to m \right\} \\ \text{continuity conditions} \\ \text{all conditions are applied to each } (\tau_i) \\ \text{where } \; i = 1 \to s-1 \end{array} \right\} \end{array} \right\} \quad (3)$$

The continuity conditions from (3) are applied to each (τ_i), where $i = 1 \to s-1$. Finally, the results from (1) will be substituted in the kinematic and dynamic equations.

2. CLASSICAL POLYNOMIAL FUNCTIONS

The studies regarding trajectory generation at the joint level are various and win over the interest of many researchers. For example, the authors of the paper [1] were among the first that advanced the idea of using the polynomial functions for robot trajectory generation. Also, according to [2]-[6], a commonly used joint-space trajectory generation method is based on linearly-changing joint velocity using starting and ending parabolic blends.

Although this method is frequently used, its application raises up some problems such as the fact that the first derivative of acceleration (jerk) is characterized by infinite spikes thus requiring three separate functions instead of one. The use of third order polynomial joint-space trajectory generation is also preferred by many authors [4]-[7]. Another approach is that of using the fifth-order polynomial for determining the motion trajectory of a mechanical system.

Some other authors [5]- [7] suggest the use of initial, intermediate and final polynomials of 4-3-4 or 3-5-3 order polynomials, for a single joint motion. The objective of the present paper is to make a review on different methods of applying the polynomials in defining the motion trajectory, by highlighting the advantages and the drawbacks for each of them. Also, it will be presented a generalized algorithm that uses polynomial functions of higher degree for determining the trajectory of motion, based on an original approach.

If the motion of the end effector of a robot characterized by (n) degrees of freedom is well defined, the problem is to determine the interpolation polynomial functions which define the robot trajectory in the configurations space.

The meeting of all time dependent interpolation functions, defined for each segment of motion and for each component of the generalized variable vector represents the motion trajectory. Forwards are presented the most commonly methods based on polynomials functions, used for motion trajectory generation.

2.1 Polynomial functions of third order

For the mathematical modeling of motion trajectories in the configurations space are also used the geometric, kinematic and dynamic constraints of the command and control system, along with the initial conditions imposed by the work process [5], [8], [16]. Depending on the working process, the motion trajectory must go through all $(n+1)$ points corresponding to the moments τ_i $(i=0\rightarrow n)$. According to the initial conditions, the trajectory must provide a restrictive control over the position, velocity and acceleration that characterize the motion at the moments τ_0 and τ_n to ensure the continuity in velocity and accelerations at the moments $\left[\tau_i\left(i=1\rightarrow n-1\right)\right]$. The restrictive conditions for this type of motion trajectory are presented below:

$$\left\{\begin{array}{c}(\tau_0)\Rightarrow\{h_0=q_{j0}\,;\,v_0=\dot{q}_{j0}\,;\,a_0=\ddot{q}_{j0}\}\\(\tau_i)\Rightarrow\left\{\begin{array}{c}h_1=q_{ji}\,;\,i=2\rightarrow n-2\\\left\{\begin{array}{c}v_i(t^+)=v_{i+1}(t^-);\,a_i(t^+)=a_{i+1}(t^-)\\i=1\rightarrow n-1\end{array}\right\}\end{array}\right\}\\(\tau_n)\Rightarrow\{h_n=q_{jn}\,;\,v_n=\dot{q}_{jn}\,;\,a_n=\ddot{q}_{jn}\}\end{array}\right\}\quad(4)$$

The parameters included in (4) result from the geometric and kinematic control modeling. The initial conditions (4) are supplemented by the following kinematic and dynamic constraints:

$$\left\{\begin{array}{ll}|q_{ji}(\tau)|\le q_j^{max}; & |\dot{q}_{ji}(\tau)|\le\dot{q}_j^{max};\\|\ddot{q}_{ji}(\tau)|\le\ddot{q}_j^{max}; & |Q_m^{ji}(\tau)|\le max(Q_j)\end{array}\right\}\quad(5)$$

The interpolation of each segment of the motion trajectory $(i=1\rightarrow n)$ is done by using the cubic spline functions. In order to determine the cubic spline functions is generated a time linear function for generalized accelerations from each robot joint $(j=1\rightarrow N)$, see [8] and [16]:

$$\ddot{q}_{ji}(\tau)=\frac{\tau_i-\tau}{t_i}\cdot\ddot{q}_{ji}(\tau_{i-1})+\frac{\tau-\tau_{i-1}}{t_i}\cdot\ddot{q}_{ji}(\tau_i);\quad(6)$$

where $t_i=\tau_i-\tau_{i-1}$ is the time needed to travel the $i=1\rightarrow n$ segment of trajectory.

According to [16], the unknowns are represented by the generalized acceleration that characterize the motion at moments τ_{i-1} and τ_i:

$$\ddot{q}_{ji}(\tau_{i-1})=\ddot{q}_{ji-1}\,;\qquad\ddot{q}_{ji}(\tau_i)=\ddot{q}_{ji};\quad(7)$$

Integrating the differential equation (6), results:

$$\dot{q}_{ji}(\tau)=-\frac{(\tau_i-\tau)^2}{2\cdot t_i}\cdot\ddot{q}_{ji-1}+\frac{(\tau-\tau_{i-1})^2}{2\cdot t_i}\cdot\ddot{q}_{ji}+a_{ji1};\quad(8)$$

$$\left\{\begin{array}{l}q_{ji}(\tau)=-\dfrac{(\tau_i-\tau)^3}{6\cdot t_i}\cdot\ddot{q}_{ji-1}+\\+\dfrac{(\tau-\tau_{i-1})^3}{6\cdot t_i}\cdot\ddot{q}_{ji}+a_{ji1}\cdot\tau+a_{ji2}\end{array}\right\};\quad(9)$$

By applying the following initial conditions are obtained the integration constants a_{ji1} and a_{ji2}:

$$q_{ji}(\tau_{i-1})=q_{ji-1}\,;\quad q_{ji}(\tau_i)=q_{ji}\,;\quad(10)$$

$$a_{ji1}=\left(\frac{q_{ji}}{t_i}-\frac{t_i}{6}\cdot\ddot{q}_{ji}\right)-\left(\frac{q_{ji-1}}{t_i}-\frac{t_i}{6}\cdot\ddot{q}_{ji-1}\right);\quad(11)$$

$$a_{ji2}=\left(\frac{q_{ji-1}}{t_i}-\frac{t_i}{6}\cdot\ddot{q}_{ji-1}\right)\cdot\tau_i-\left(\frac{q_{ji}}{t_i}-\frac{t_i}{6}\cdot\ddot{q}_{ji}\right)\cdot\tau_{i-1};\quad(12)$$

The integration constants (11) and (12) are substituted in (8) and (9), resulting the polynomial functions of second order, with respect time variable (τ) characterizing the generalized variables and generalized velocities, [8] and [16]:

$$\left\{\begin{array}{l}\dot{q}_{ji}(\tau)=-\dfrac{(\tau_i-\tau)^2}{2\cdot t_i}\cdot\ddot{q}_{ji-1}+\dfrac{(\tau-\tau_{i-1})^2}{2\cdot t_i}\cdot\ddot{q}_{ji}+\\+\left(\dfrac{q_{ji}}{t_i}-\dfrac{t_i}{6}\cdot\ddot{q}_{ji}\right)-\left(\dfrac{q_{ji-1}}{t_i}-\dfrac{t_i}{6}\cdot\ddot{q}_{ji-1}\right)\end{array}\right\};\quad(13)$$

$$\left\{\begin{array}{l}q_{ji}(\tau)=-\dfrac{(\tau_i-\tau)^3}{6\cdot t_i}\cdot\ddot{q}_{ji-1}+\dfrac{(\tau-\tau_{i-1})^3}{6\cdot t_i}\cdot\ddot{q}_{ji}+\\+\left(\dfrac{q_{ji}}{t_i}-\dfrac{t_i}{6}\cdot\ddot{q}_{ji}\right)\cdot(\tau-\tau_{i-1})+\\+\left(\dfrac{q_{ji-1}}{t_i}-\dfrac{t_i}{6}\cdot\ddot{q}_{ji-1}\right)\cdot(\tau-\tau_{i-1})\end{array}\right\};\quad(14)$$

The unknowns are represented by the generalized accelerations and are determined by solving the following matrix equation [8], [16]:

$$\left[\begin{array}{c}\left[\ddot{q}_{jk}\,;\,k=1\rightarrow n-1\right]^T=\\=A^{-1}\cdot\left[\,b_{j1}\;b_{j2}\;\left[b_{jk}\;k=3\rightarrow n-3\right]\;b_{jm-2}\;b_{jm-1}\right]^T\end{array}\right];\quad(15)$$

where A^{-1} represents the inverse of matrix A, the matrix of the coefficients of the unknowns, which is defined by the following expression:

$$\underset{[(m-1)\times(m-1)]}{A}=\left[\,A_1\;\left[A_k\,;k=3\rightarrow m-3\right]^T\;A_{m-1}\right]^T;\quad(16)$$

$$A_{m-1}=\left[\begin{array}{c|ccc}[0]^{m-4} & t_{m-2} & 2\cdot(t_{m-2}+t_{m-1}) & t_{m-1}-\dfrac{t_m^2}{t_{m-1}}\\ & 0 & t_{m-1} & 3\cdot t_m+2t_{m-1}+\dfrac{t_m^2}{t_{m-1}}\end{array}\right]\quad(17)$$

$$A_1=\left[\begin{array}{ccc|c}3\cdot t_1+2\cdot t_2+\dfrac{t_1^2}{2} & t_2 & 0 & [0]^{m-4}\\t_2-\dfrac{t_1^2}{t_2} & 2\cdot(t_2+t_3) & t_3 & \end{array}\right];\quad(18)$$

$$A_k=\left[\,[0]^{k-2}\;t_k\;\;2\cdot(t_k+t_{k+1})\;\;t_{k+1}\;[0]^{m-(k+2)}\right];\quad(19)$$

In the same equation, $\{b_{j1}, b_{j2},...,b_{jm-2}, b_{jm-1}\}$ are defining the components of the column vector of free terms, denoted with B_j, see [8] and [16]:

$$b_{j1} = \left\{ \begin{array}{c} \frac{6\cdot(q_{j2}-q_{j0})}{t_2} - 6\cdot\dot{q}_{j0}\cdot\frac{t_1+t_2}{t_2} - \\ -6\cdot\ddot{q}_{j0}\cdot\left(\frac{t_1}{2}+\frac{1}{3}\cdot\frac{t_1^2}{t_2}\right) \end{array} \right\}; \quad (20)$$

$$b_{j2} = \left\{ \begin{array}{c} \frac{6\cdot(q_{j0}-q_{j2})}{t_2} - \frac{6\cdot(q_{j3}-q_{j2})}{t_3} + \\ 6\cdot\left(\dot{q}_{j0}\cdot\frac{t_1}{t_2}+\frac{1}{3}\cdot\frac{t_1^2}{t_2}\cdot\ddot{q}_{j0}\right) \end{array} \right\}; \quad (21)$$

$$b_{jm-1} = \left\{ \begin{array}{c} \frac{6\cdot(q_{jm-2}-q_{jn})}{t_{m-1}} + 6\cdot\dot{q}_{jm}\cdot\frac{t_{m-1}+t_m}{t_{m-1}} - \\ -6\cdot\ddot{q}_{jm}\cdot\left(\frac{t_m}{2}+\frac{1}{3}\cdot\frac{t_m^2}{t_{m-1}}\right) \end{array} \right\}; \quad (22)$$

$$b_{jm-2} = \left\{ \begin{array}{c} \frac{6\cdot(q_{jm-3}-q_{jm-2})}{t_{n-2}} + \frac{6\cdot(q_{jm}-q_{jm-2})}{t_{n-1}} - \\ -6\cdot\dot{q}_{jm}\cdot\frac{t_m}{t_{m-1}}+2\cdot\frac{t_m^2}{t_{m-1}}\cdot\ddot{q}_{jm} \end{array} \right\} \quad (23)$$

By solving the system of equations from (15) are obtained the generalized accelerations as well as the generalized coordinates. Finally, the polynomial functions defined with (6), (13) and (14) can be written in a final form. Considering the initial input data, for the interpolation of the motion trajectory must be applied the kinematic and dynamic constraints (5) and are defined the maximum values for polynomial functions:

$$\{ q_{ji}(\tau); \dot{q}_{ji}(\tau); \ddot{q}_{ji}(\tau); Q_m^{ji}(\tau)\}.$$

The cubic polynomial functions, because of their interpolation order, exhibits some restrictions in the study of the accelerations of higher order, characteristic to the sudden movements of the multibody systems (MBS).

2. HIGHER ORDER POLYNOMIALS

Polynomial cubic functions usually provide continuity in position and velocity, but accelerations are presenting in some cases discontinuity intervals, which require the use of higher order polynomials.

Depending on the constraints that are imposed by the work process, can be defined different types of trajectories in the configurations space. For example can be mentioned the $(4-3-4)$ respectively $[5-(4-3-4)-5]$ type motion trajectories. In the first case, the end segments of the trajectory are interpolated with polynomials of fourth degree for the position, while on each intermediary segment, $i=2\rightarrow n-1$ of the trajectory are applied cubic polynomial interpolation functions.

For $[5-(4-3-4)-5]$ type motion trajectories developed in [4] and [8], the end segments are interpolated with polynomials of fifth order, first and last intermediary segment with polynomials of fourth order and for the other intermediary segments are used the cubic spline functions.

According to [12], [13] the kinematical constraints, which apply in case of polynomial interpolation functions of fifth order, are:

$$(\tau_0) \Rightarrow \left\{ \begin{array}{l} h_0 = q_{j0};\ v_0 = \dot{q}_{j0}; \\ a_0 = \ddot{q}_{j0};\ \ddot{a}_0 = \ddddot{q}_{j0} \end{array} \right\}; \quad (24)$$

$$(\tau_n) \Rightarrow \{h_n = q_{jn};\ \ddot{a}_n = \ddddot{q}_{jn}\}; \quad (25)$$

$$(\tau_i) \Rightarrow \left\{ \begin{array}{l} a_i = \ddot{q}_{ji};\ i = 1 \rightarrow n-1 \\ h_i(t^+) = h_{i+1}(t^-);\ v_i(t^+) = v_{i+1}(t^-); \\ \dot{a}_i(t^+) = \dot{a}_{i+1}(t^-) \end{array} \right. \quad (26)$$

The time functions for generalized variables of fifth order are defined with the expressions:

$$\ddddot{q}_{ji}(\tau) = \frac{\tau_i-\tau}{t_i}\cdot\ddddot{q}_{ji}(\tau_{i-1}) + \frac{\tau-\tau_{i-1}}{t_i}\cdot\ddddot{q}_{ji}(\tau_i); \quad (27)$$

$$\dddot{q}_{ji}(\tau) = -\frac{(\tau_i-\tau)^2}{2\cdot t_i}\cdot\ddddot{q}_{ji-1} + \frac{(\tau-\tau_{i-1})^2}{2\cdot t_i}\cdot\ddddot{q}_{ji} + a_{ji1}; \quad (28)$$

$$\left\{ \begin{array}{c} \ddot{q}_{ji}(\tau) = \frac{(\tau_i-\tau)^3}{6\cdot t_i}\cdot\ddddot{q}_{ji-1} + \\ +\frac{(\tau-\tau_{i-1})^3}{6\cdot t_i}\cdot\ddddot{q}_{ji} + a_{ji1}\cdot\tau + a_{ji2} \end{array} \right\}; \quad (29)$$

$$\left\{ \begin{array}{c} \dot{q}_{ji}(\tau) = -\frac{(\tau_i-\tau)^4}{24\cdot t_i}\cdot\ddddot{q}_{ji-1} + \\ +\frac{(\tau-\tau_{i-1})^4}{24\cdot t_i}\cdot\ddddot{q}_{ji} + a_{ji1}\cdot\frac{\tau^2}{2} + a_{ji2}\cdot\tau + a_{ji3} \end{array} \right\}; \quad (30)$$

$$\left\{ \begin{array}{c} q_{jik}(\tau) = \frac{(\tau_i-\tau)^5}{120\cdot t_i}\cdot\ddddot{q}_{ji-1k} + \\ +\frac{(\tau-\tau_{i-1})^5}{120\cdot t_i}\cdot\ddddot{q}_{jik} + a_{jik1}\cdot\frac{\tau^3}{6} + \\ + a_{jik2}\cdot\frac{\tau^2}{2} + a_{jik3}\cdot\tau + a_{jik4} \end{array} \right\}; \quad (31)$$

In case of polynomial interpolation functions of fifth order the unknowns are represented by the generalized accelerations of third order [8]. The unknowns are determined as follows:

$$A \cdot X = B \Rightarrow X = A^{-1} \cdot B; \quad (32)$$

In the equation (32) A represents a matrix whose components are represented by the coefficients of the unknowns, B is the column vector of the free terms and X the column vector of the unknowns. Also, A^{-1} is the inverse matrix of the unknown coefficients matrix.
The components of the column vector of the unknowns are presented in the following:

$$\underset{[5n-1\times1]}{X} = \begin{bmatrix} X_1 & [X_i \,;\, i = 2 \to n-1] & X_n \end{bmatrix}^T; \quad (33)$$

$$\underset{[5n-1\times1]}{X} = \left\{ \begin{array}{c} \begin{bmatrix} a_{j1m} \,;\, m = 1 \to 4 \end{bmatrix} \\ \left\{ \begin{array}{c} \begin{bmatrix} \dddot{q}_{ji-1} & a_{jim} \,;\, m = 1 \to 4 \end{bmatrix} \\ i = 2 \to n-1 \end{array} \right\} \\ \begin{bmatrix} \dddot{q}_{j3} & a_{j4m} \,;\, m = 1 \to 4 \end{bmatrix} \end{array} \right\}; \quad (34)$$

The column vector of free terms is defined as:

$$\underset{[5n-1\times1]}{B} = \begin{bmatrix} B_1 & [B_i \,;\, i = 2 \to n-1] & B_n \end{bmatrix}^T; \quad (35)$$

where

$$\underset{(4\times1)}{B_1} = \begin{bmatrix} B_{11} & B_{21} & B_{31} & B_{41} \end{bmatrix}^T, \ \underset{(4\times1)}{B_i} = \underset{(4\times1)}{[0]}; \quad (36)$$

$$\left\{ B_{11} = q_{j0} - \frac{t_1^4}{120} \cdot \dddot{q}_{j0} \,;\, B_{21} = \dot{q}_{j0} + \frac{t_1^3}{24} \cdot \dddot{q}_{j0} \right\}; \quad (37)$$

$$\left\{ B_{31} = \ddot{q}_{j0} - \frac{t_1^2}{6} \cdot \dddot{q}_{j0}; \quad B_{41} = \ddot{q}_{j0} + \frac{t_1}{2} \cdot \dddot{q}_{j0} \right\}; \quad (38)$$

$$\underset{(7\times1)}{B_4} = \begin{bmatrix} \underset{(4\times1)}{[0]} & q_{j4} - \frac{t_4^4}{120} \cdot \dddot{q}_{j4} & \ddot{q}_{j4} - \frac{t_3^2}{6} \cdot \dddot{q}_{j4} & \ddot{q}_{j4} - \frac{t_4}{2} \cdot \dddot{q}_{j4} \end{bmatrix}^T; \quad (39)$$

The matrix of unknown coefficients is written as:

$$\underset{[5n-1\times5n-1]}{A} = \begin{bmatrix} A_1 & [A_i \,;\, i = 2 \to n-1]^T & A_n \end{bmatrix}^T; \quad (40)$$

The components of the A matrix are defined:

$$\underset{[4\times19]}{A_1} = \begin{bmatrix} A_{11} & \underset{[4\times15]}{[0]} \\ A_{12} & \end{bmatrix}; \quad (41)$$

where

$$\underset{[2\times4]}{A_{11}} = \begin{bmatrix} \frac{\tau_0^3}{6} & \frac{\tau_0^2}{2} & \tau_0 & 1 \\ \frac{\tau_0^2}{2} & \tau_0 & 1 & 0 \end{bmatrix}, \ \underset{[2\times4]}{A_{12}} = \begin{bmatrix} \tau_0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

$$\underset{[7\times19]}{A_4} = \left[\begin{array}{c|cc} & A_{411} & A_{412} \\ \underset{[7\times10]}{[0]} & A_{421} & A_{422} \\ & A_{431} & A_{432} \end{array} \right] \quad (42)$$

The components of matrix defined by (42) are:

$$\underset{[2\times5]}{A_{411}} = \begin{bmatrix} \frac{\tau_3^3}{6} & \frac{\tau_3^2}{2} & \tau_3 & 1 & \left(\frac{t_3^4}{120} - \frac{t_4^4}{120} \right) \\ \frac{\tau_3^2}{2} & \tau_3 & 1 & 0 & \left(\frac{t_3^3}{24} + \frac{t_4^3}{24} \right) \end{bmatrix}; \quad (43)$$

$$\underset{[2\times4]}{A_{412}} = \begin{bmatrix} -\frac{\tau_3^3}{6} & -\frac{\tau_3^2}{2} & -\tau_3 & -1 \\ -\frac{\tau_3^2}{2} & -\tau_3 & -1 & 0 \end{bmatrix}; \quad (44)$$

$$\underset{[2\times5]}{A_{421}} = \begin{bmatrix} \tau_3 & 1 & 0 & 0 & \left(\frac{t_3^2}{6} - \frac{t_4^2}{6} \right) \\ 1 & 0 & 0 & 0 & \left(\frac{t_3}{2} + \frac{t_4}{2} \right) \end{bmatrix}; \quad (45)$$

$$\underset{[2\times4]}{A_{422}} = \begin{bmatrix} -\tau_3 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}; \quad (46)$$

$$\underset{(3\times5)}{A_{431}} = \underset{(3\times5)}{[0]}; \ \underset{[4\times3]}{A_{432}} = \begin{bmatrix} \frac{\tau_4^3}{6} & \frac{\tau_4^2}{2} & \tau_4 & 1 \\ \tau_4 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}; \quad (47)$$

$$\underset{[4\times19]}{A_i} = \left[\begin{array}{c|cc|c} \underset{[4\times5(i-2)]}{[0]} & A_{i1} & A_{i2} & \underset{[4\times20-5i]}{[0]} \end{array} \right]; \quad (48)$$

where

$$\underset{[4\times5]}{A_{i1}} = \begin{bmatrix} \frac{\tau_{i-1}^3}{6} & \frac{\tau_{i-1}^2}{2} & \tau_{i-1} & 1 & \left(\frac{t_{i-1}^4}{120} - \frac{t_i^4}{120} \right) \\ \frac{\tau_{i-1}^2}{2} & \tau_{i-1} & 1 & 0 & \left(\frac{t_{i-1}^3}{24} + \frac{t_i^3}{24} \right) \\ \tau_{i-1} & 1 & 0 & 0 & \left(\frac{t_{i-1}^2}{6} - \frac{t_i^2}{6} \right) \\ 1 & 0 & 0 & 0 & \left(\frac{t_{i-1}}{2} + \frac{t_i}{2} \right) \end{bmatrix}; \quad (49)$$

$$\underset{[4\times4]}{A_{i2}} = -\begin{bmatrix} \frac{\tau_{i-1}^3}{6} & \frac{\tau_{i-1}^2}{2} & \tau_{i-1} & 1 \\ \frac{\tau_{i-1}^2}{2} & \tau_{i-1} & 1 & 0 \\ \tau_{i-1} & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}; \quad (50)$$

The matrix A^{-1} from (32) represents the inverse of the matrix of the unknowns coefficients and it exists only if the following condition is met: $\{t_k > 0, \ k = 2 \to n-1\}$.Substituting the expressions (34) - (50)into expression (32), and by performing the necessary transformations, it results a system of linear and nonhomogeneous equations whose solutions are the coefficients that couldn't have been determined by (24)-(26).

3. DYNAMICS EQUATIONS OF HIGHER ORDER FOR 2TR ROBOT STRUCTURE

According to the scientific literature [6], in 1879, Gibbs defines differential equations of nonholonomic motion. These research are developed by Appell who, in 1899, performs a detailed study based on these equations. Today these equations are known as Gibbs-Appell equations and are applied for holonomic and nonholonomic systems, where the kinetic energy was substituted through the acceleration energy, also known as Appell's function. Unlike the studies, above mentioned, in the papers [7] – [11] the author established the higher order acceleration energies in a generalized form, for a multi body system involved in a general motion. According to [12], [15], the generalized driving forces for any mechanical system, including the robots, is determined in a general form [13]:

$$\left\{\begin{array}{l} Q_m^i(t)=\Delta_m^2\cdot\left[\Delta_\theta\cdot Q_{i\mathscr{F}}^i(t)+Q_g^i(t)\right]+ \\ +(-1)^{\Delta_m}\cdot\dfrac{1-\Delta_m}{1+3\cdot\Delta_m}\cdot Q_{SU}^i(t)\end{array}\right\}; \quad (51)$$

$k\ge 1;\ k=\{1;2;3;4;5;\ldots\};\ m\ge(k+1);\quad m=\{2;3;4;5;\ldots\}$

$$\left\{\begin{array}{l}\Delta_m=\{[-1;(SU;\,M_i)];\,(0;SU);\,(1;\,M_i)\} \\ \Delta_\theta=\left\{\left[1;\left\{\dot{\bar\theta};\ddot{\bar\theta}\right\}\ne 0\right];\left[0;\left\{\dot{\bar\theta};\ddot{\bar\theta}\right\}=0\right]\right\}\end{array}\right\}$$

where Δ_m highlight the gravitational load by (M_i) and the manipulating load by the symbol (SU); Δ_θ characterizes the mechanical behavior of the system (0 – statics; 1 – dynamics).

The k order time derivative of the generalized driving force, from every robot joint is defined as:

$$\left\{\begin{array}{l}\overset{(k)}{Q_m^i}(t)=\Delta_m^2\cdot\left[\Delta_\theta\cdot\overset{(k)}{Q_{i\mathscr{F}}^i}(t)+\overset{(k)}{Q_g^i}(t)\right]+ \\ +(-1)^{\Delta_m}\cdot\dfrac{1-\Delta_m}{1+3\cdot\Delta_m}\cdot\overset{(k)}{Q_{SU}^i}(t)\end{array}\right\}; \quad (52)$$

The generalized inertia forces are defined by considering the acceleration energy of first order [12] – [16]. The general expression for higher order time derivatives is determined:

$$\left\{\begin{array}{l}\dfrac{\partial}{\partial\overset{(m)}{q_j}}\left\{\overset{(m-2)}{E_A^{(1)}}\left[\bar\theta(t);\dot{\bar\theta}(t);\ddot{\bar\theta}(t);\cdots;\overset{(m)}{\bar\theta}(t)\right]\right\}= \\ =Q_{i\mathscr{F}}^i\left[\bar\theta(t);\dot{\bar\theta}(t);\overset{(2)}{\bar\theta}(t)\right]\end{array}\right\}; \quad (53)$$

$$\left\{\begin{array}{c}\text{where}\quad \overset{(0)}{E_A^{(1)}}=E_A^{(1)}\quad j=1\to n,\quad k=1 \\ m\ge[(k+1)=2],\ \text{and}\ (k)\ \text{are time deriving orders}\end{array}\right\};$$

The higher order time derivative of generalized inertia forces, from every robot joint is defined:

$$\left\{\begin{array}{l}\overset{(k)}{Q_{i\mathscr{F}}^i}(t)={}^0J_i\left[\bar\theta(t)\right]\cdot{}^0\overset{(k)}{\bar{\mathscr{F}}}{}_{X_i}^*+ \\ +\displaystyle\sum_{m=1}^{k-1}\frac{(k-1)!}{m!(k-m-1)!}\cdot{}^0J_i\left[\overset{(m)}{\bar\theta}(t)\right]\cdot{}^0\overset{[k-m]}{\bar{\mathscr{F}}}{}_{X_i}^*= \\ =\displaystyle\sum_{m=1}^{k}\frac{(k-1)!}{(m-1)!(k-m)!}\cdot{}^0J_i\left[\overset{(m-1)}{\bar\theta}(t)\right]\cdot{}^0\overset{[k-(m-1)]}{\bar{\mathscr{F}}}{}_{X_i}^*\end{array}\right\}; \quad (54)$$

In (52), $\overset{(k)}{Q_g^i}(t)$ defines the k order time derivative of the generalized gravitational force, which according to [16] has the following expression:

$$\left\{\begin{array}{l}\overset{(k)}{Q_g^i}(t)={}^0J_i\left[\bar\theta(t)\right]\cdot{}^0\overset{(k)}{\bar{\mathscr{F}}}_{X_i}+ \\ +\displaystyle\sum_{m=1}^{k-1}\frac{(k-1)!}{m!(k-m-1)!}\cdot{}^0J_i\left[\overset{(m)}{\bar\theta}(t)\right]\cdot{}^0\overset{[k-m]}{\bar{\mathscr{F}}}_{X_i}= \\ =\displaystyle\sum_{m=1}^{k}\frac{(k-1)!}{(m-1)!(k-m)!}\cdot{}^0J_i\left[\overset{(m-1)}{\bar\theta}(t)\right]\cdot{}^0\overset{[k-(m-1)]}{\bar{\mathscr{F}}}_{X_i}\end{array}\right\}; \quad (55)$$

The (k) order time derivative of the generalized manipulating force is defined as follows:

$$\left\{\begin{array}{l}Q_{SU}\left[\overset{(k)}{\bar\theta}(t)\right]={}^0J\left[\bar\theta(t)\right]\cdot{}^0\overset{(k)}{\bar{\mathscr{F}}}_X+ \\ +\displaystyle\sum_{m=1}^{k-1}\frac{(k-1)!}{m!(k-m-1)!}\cdot{}^0J\left[\overset{(m)}{\bar\theta}(t)\right]\cdot{}^0\overset{[k-m]}{\bar{\mathscr{F}}}_X= \\ =\displaystyle\sum_{m=1}^{k}\frac{(k-1)!}{(m-1)!(k-m)!}\cdot{}^0J\left[\overset{(m-1)}{\bar\theta}(t)\right]\cdot{}^0\overset{[k-(m-1)]}{\bar{\mathscr{F}}}_X\end{array}\right\}; \quad (56)$$

In the above equations, (54)-(56), ${}^0J\left[\bar\theta(t)\right]$ is the Jacobian matrix and ${}^0\bar{\mathscr{F}}_{X(i)}$ characterizes the force – moment Cartesian vector, along with their higher order time derivatives [8], [12], [15].

The expressions (52)-(56) are compulsory in establishing of the motion differential equations of various orders. In the following, is presented an application of the theoretical model presented in this paper. For this purpose, is considered a 2TR robot structure which is in fact an (MBS), whose kinematical structure is presented in the Fig1.

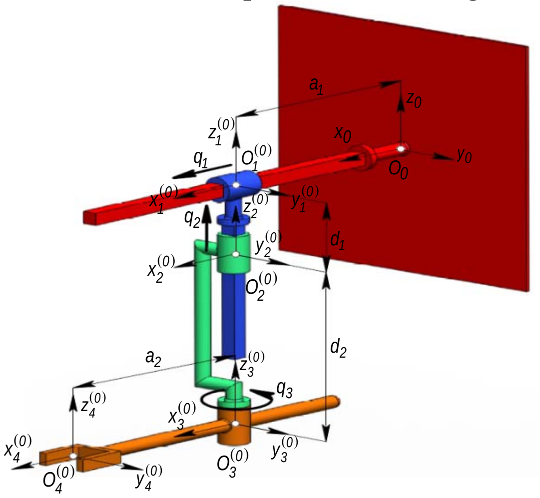

Fig.1 The kinematical structure of 2TR robot

According to [9] – [15], the sudden motion of a MBS (also the robot structure), the transient motion phases, as well as the mechanical systems subjected to the action of a system of external forces, with a time variation law, are characterized by linear and angular accelerations of higher order. The expressions for the acceleration energy of first, second and third order have been determined for the 2TR robot structure. These are further used to write the expressions for the differential equations of motion of first, second and third order.

In this case, the acceleration energy of first order for the 2TR robot structure is expressed as:

$$\left\{\begin{array}{c} E^{(1)}_{Aik}(\tau)=\frac{1}{2}\cdot\Big[\left(M_1+M_2+M_3\right)\cdot\ddot{q}^2_{1ik}(\tau)+ \\ +\left(M_3\cdot a_2^2+{}^3I_z\right)\cdot\left[\ddot{q}^2_{3ik}(\tau)+\dot{q}^4_{3ik}(\tau)\right]\Big]- \\ -\left[M_3\cdot a_2\cdot\dot{q}^2_{3ik}(\tau)\cdot c\left(q_{3ik}(\tau)\right)\right]\cdot\ddot{q}_{1ik}(\tau)- \\ -M_3\cdot a_2\cdot sq_{3ik}(\tau)\cdot\ddot{q}_{1ik}(\tau)\cdot\ddot{q}_{3ik}(\tau)+ \\ +\left(M_2+M_3\right)\cdot\ddot{q}^2_{2ik}(\tau) \end{array}\right\}; \quad (57)$$

Based on aspects regarding the polynomial interpolation functions presented in the previous sections, are determined the differential equations of second order, written in the form:

$$\left\{\begin{array}{c} Q^1_{mik}(\tau)=\left(M_1+M_2+M_3\right)\cdot\ddot{q}_{1ik}(\tau)- \\ -M_3\cdot a_2\cdot sq_{3ik}(\tau)\cdot\ddot{q}_{3ik}(\tau)- \\ -M_3\cdot a_2\cdot\dot{q}^2_{3ik}(\tau)\cdot cq_{3ik}(\tau) \end{array}\right\}; \quad (58)$$

$$\left\{Q^2_{mik}(\tau)=\left(M_2+M_3\right)\cdot\left[\ddot{q}_{2ik}(\tau)+g\right]\right\}; \quad (59)$$

$$\left\{\begin{array}{c} Q^3_{mik}(\tau)=\left(M_3\cdot a_2^2+{}^3I_z\right)\cdot\ddot{q}_{3ik}(\tau)- \\ -M_3\cdot a_2\cdot sq_{3ik}(\tau)\cdot\ddot{q}_{1ik}(\tau) \end{array}\right\}; \quad (60)$$

The equations (58)-(60) define the differential equations of second order, for every robot joint.

The acceleration energy of second order is:

$$\left\{\begin{array}{c} E^{(2)}_{Aik}(\tau)=\frac{1}{2}\cdot\left[\left(M_1+M_2+M_3\right)\cdot\dddot{q}^2_{1ik}(\tau)\right]+ \\ +\frac{1}{2}\cdot\left(M_2+M_3\right)\cdot\dddot{q}^2_{2ik}(\tau)+\frac{1}{2}\cdot\left(M_3\cdot a_2^2+{}^3I_z\right)\cdot\dddot{q}^2_{3ik}(\tau)- \\ -\frac{1}{2}\cdot\left(M_3\cdot a_2^2+{}^3I_z\right)\cdot\left(2\cdot\dot{q}^3_{3ik}(\tau)\cdot\dddot{q}_{3ik}(\tau)\right)+ \\ +\frac{1}{2}\cdot\left[\left(M_3\cdot a_2^2+{}^3I_z\right)\cdot\left(9\cdot\dot{q}^2_{3ik}(\tau)\cdot\ddot{q}^2_{3ik}(\tau)+\dot{q}^6_{3ik}(\tau)\right)\right]- \\ -3\cdot M_3\cdot a_2\cdot\dot{q}_{3ik}(\tau)\cdot cq_{3ik}(\tau)\cdot\dddot{q}_{1ik}(\tau)\cdot\ddot{q}_{3ik}(\tau)- \\ -M_3\cdot a_2\cdot sq_{3ik}(\tau)\cdot\dddot{q}_{1ik}(\tau)\cdot\dddot{q}_{3ik}(\tau)+ \\ +M_3\cdot a_2\cdot sq_{3ik}(\tau)\cdot\dddot{q}_{1ik}(\tau)\cdot\dot{q}^3_{3ik}(\tau) \end{array}\right\}; (61)$$

The differential equations of third order are:

$$\left\{\begin{array}{c} \dot{Q}^1_{mik}(\tau)=\left(M_1+M_2+M_3\right)\cdot\dddot{q}_{1ik}(\tau)- \\ -M_3\cdot a_2\cdot sq_{3ik}(\tau)\cdot\dddot{q}_{3ik}(\tau)- \\ -3\cdot M_3\cdot a_2\cdot\ddot{q}_{3ik}(\tau)\cdot cq_{3ik}(\tau)\cdot\dot{q}_{3ik}(\tau)+ \\ +M_3\cdot a_2\cdot sq_{3ik}(\tau)\cdot\dot{q}^3_{3ik}(\tau) \end{array}\right\}; \quad (62)$$

$$\left\{\dot{Q}^2_{mik}(\tau)=\left(M_2+M_3\right)\cdot\dddot{q}_{2ik}(\tau)\right\}; \quad (63)$$

$$\left\{\begin{array}{c} \dot{Q}^3_{mik}(\tau)=\left(M_3\cdot a_2^2+{}^3I_z\right)\cdot\dddot{q}_{3ik}(\tau)- \\ -M_3\cdot a_2\cdot sq_{3ik}(\tau)\cdot\dddot{q}_{1ik}(\tau)- \\ -M_3\cdot a_2\cdot cq_{3ik}(\tau)\cdot\dot{q}_{3ik}(\tau)\cdot\ddot{q}_{1ik}(\tau) \end{array}\right\}; \quad (64)$$

Considering the kinematical constraints, in general form, (24)-(26), applied for the 2TR robot structure, as well as the time functions for generalized variables of fifth order defined with the expressions (27)-(31) and by applying expressions (33)-(50) are obtained the polynomial time functions for generalized coordinates.

In Table 1 is presented a selection from a sequence of the working process containing the polynomial time functions for the generalized coordinates from the first and second robot joint. By deriving the polynomials for the generalized coordinates are obtained the polynomial functions for velocities, accelerations and accelerations of higher order.

Table 1

Int.	Polynomials for generalized coordinates	
	$q_{1ik}\ \langle m \rangle$	$q_{2ik}\ \langle m \rangle$
1	$0.7681 \cdot \tau^5$	$1.0973 \cdot \tau^5$
2	$-2.3045 \cdot \tau^5 + 2.3045 \cdot \tau^4 - -0.6913 \cdot \tau^3 + 0.1037 \cdot \tau^2 - -0.0077 \cdot \tau + 0.0002$	$-3.2921 \cdot \tau^5 + 3.2921 \cdot \tau^4 - -0.9876 \cdot \tau^3 + 0.1481 \cdot \tau^2 - -0.0111 \cdot \tau + 0.0003$
3	$2.3045 \cdot \tau^5 - 4.609 \cdot \tau^4 + +3.4567 \cdot \tau^3 - 1.1407 \cdot \tau^2 + +0.1788 \cdot \tau - 0.0109$	$3.2921 \cdot \tau^5 - 6.5843 \cdot \tau^4 + +4.9382 \cdot \tau^3 - 1.6296 \cdot \tau^2 + +0.2555 \cdot \tau - 0.0156$
4	$-0.7681 \cdot \tau^5 + 2.3045 \cdot \tau^4 - -2.7654 \cdot \tau^3 + 1.6592 \cdot \tau^2 - -0.4511 \cdot \tau + 0.0457$	$-1.0973 \cdot \tau^5 + 3.2921 \cdot \tau^4 - -3.9506 \cdot \tau^3 + 2.3703 \cdot \tau^2 - -0.6444 \cdot \tau + 0.0653$

In order to determine the time variation laws for the generalized variables, driving forces and acceleration energies of higher order, the time polynomial functions of fifth order are applied.

The study is extended to the energies of higher order. Using the research of the author, have been determined the variation laws for the acceleration energy of first, second and third order in case of the 2TR robot structure.

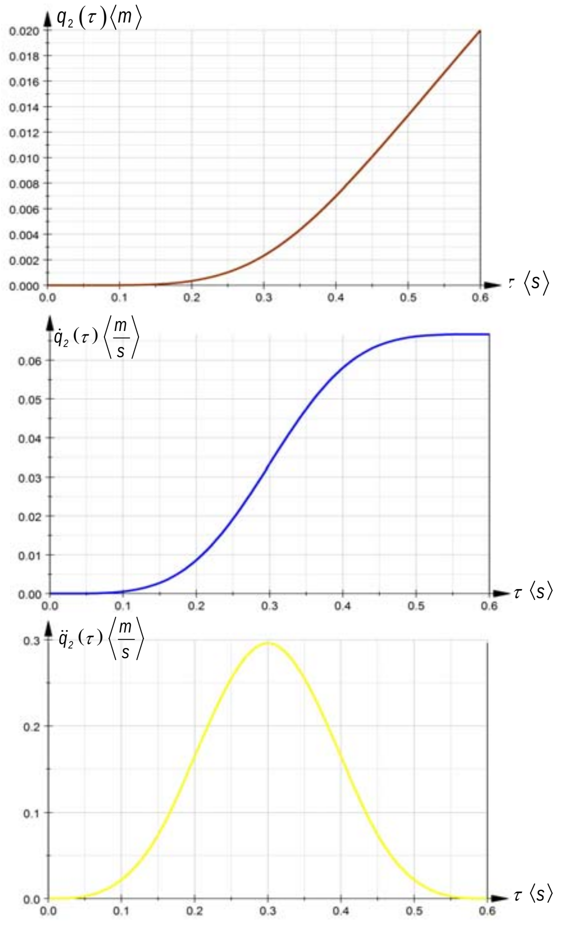

Fig.2 The time variation laws for coordinates, velocities and accelerations for 2TR robot

The polynomial functions are further used to draw the time variation law of the generalized coordinates, velocities and accelerations.

Based on this, in Fig.2 are represented the variation laws for generalized coordinates, velocities, accelerations, accelerations of first, second and third order as well as of acceleration energy of first, second and third order in case of the second joint of the 2TR robot. In order to determine the time variation laws for the generalized variables (Fig.2), higher order accelerations (Fig. 3) and acceleration energies of first, second and third order (Fig. 4), the time polynomial functions of fifth order were applied (Table 1).

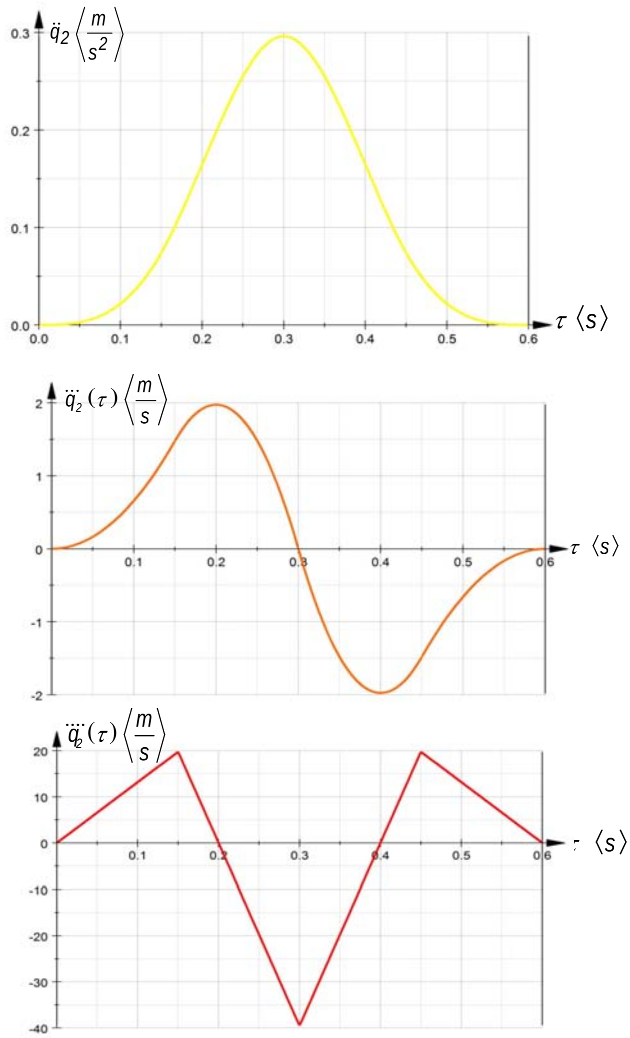

Fig.3 The time variation laws for accelerations of first, second and third order for 2TR robot

Considering the aspects presented in the previous sections, the corresponding time variation laws that characterize the dynamic behavior of a 2TR robot structure have been drawn (Fig. 4).

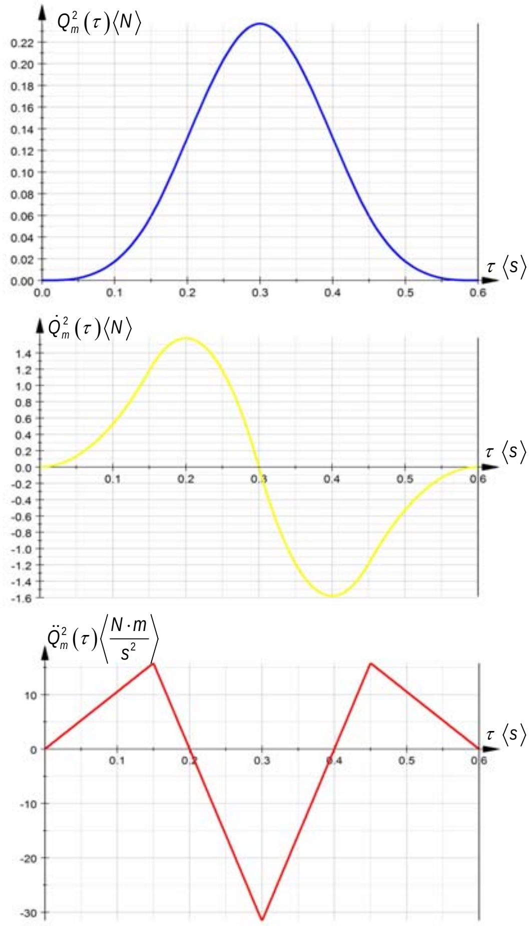

Fig.4 The time variation laws for the generalized forces of first, second and third order

The graphical representation of the variation law for the first order acceleration energy is presented:

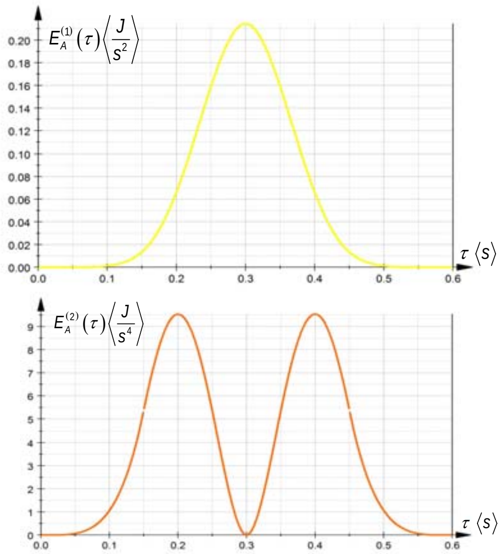

Fig.5 The time variation laws for the energies of accelerations of first and second order

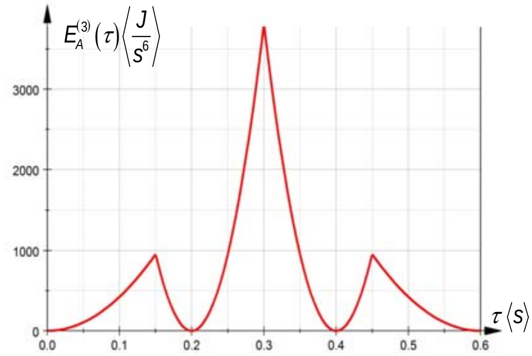

Fig.6 The time variation laws for the energies of accelerations of third order

The graphical representations from Fig.5 and Fig.6 are based on the main author original approaches regarding the acceleration energies of first, second and third order [10], [15].

These were applied for a multibody system (MBS), represented in this case by a robot structure characterized by 3 d.o.f's.

4. CONCLUSIONS

The present paper is devoted to the research of the main author, in what concerns the importance of polynomial interpolation functions in generating the motion trajectory of an industrial robot. Unlike the classical polynomial functions, already mentioned in the scientific works, the first and second sections of the paper is using the previous research of the main author are developed the expressions for the polynomial interpolation functions of higher order, with applications in higher order acceleration energies and dynamic equations of higher order, as it results from the fourth section of the present paper. Unlike the third order polynomials the higher order polynomials have the advantage of ensuring the continuity in accelerations of higher order. In the next section are presented, in explicit form, the expressions of definition for the acceleration energy of first, second and third order. They are corresponding to the current and sudden motions of rigid body and multi body systems. The formulations contain the absolute time derivatives of higher order of the advanced notions, according to differential equations of higher order, characteristic to analytical dynamics. In the final part of the paper, the presented notions are applied to describe the dynamic behavior of a 2TR robot.

5. REFERENCES

[1] R.P. Paul and H. Zhong, 1984*, Robot Motion Trajectory Specification and Generation*, Second International Symposium on Robotics Research, Kyoto, Japan, August.

[2]R.E. Parkin, 1991, *Applied Robotic Analysis*, Prentice Hall, Englewood Cliffs, NJ.

[3]L.Sciavicco and B. Siciliano, 2008, *Robotics: Modeling, Planning, and Control*, McGraw-Hill, New York.

[4]Negrean I., Vuşcan I., Haiduc N., *Robotics. Kinematic and Dynamic Modeling, Editura Didactică şi Pedagogică, R.A. Bucureşti, 1998*

[5]J.J. Craig, 2005, *Introduction to Robotics: Mechanics and Control*, 3rd edition, Pearson Prentice Hall, Upper Saddle River, NJ.

[6]Appell, P., *Sur une forme générale des equations de la dynamique*, Paris, 1899.

[7] Negrean, I., Negrean, D. C., *The Acceleration Energy to Robot Dynamics,* International Conference on Automation, Quality and Testing, Robotics, AQTR 2002, May 23-25, Cluj-Napoca, Tome II, pp. 59-64

[8] Negrean I., *Mecanică avansată în Robotică*, ISBN 978-973-662-420-9, UT Press, Cluj-Napoca, 2008.

[9]Negrean, I., *Energies of Acceleration in Advanced Robotics Dynamics,* Applied Mechanics and Materials, ISSN: 1662-7482, vol 762, pp 67-73 Submitted: 2014-08-05 ©(2015)TransTech Publications Switzerland

[10]Negrean I., *New Formulations on Acceleration Energy in Analytical Dynamics,* Applied Mechanics and Materials, vol. 823 pp 43-48 © (2016) TransTech Publications Switzerland Revised: 2015-09,

[11]Negrean I., *New Formulations on Motion Equations in Analytical Dynamics,* Applied Mechanics and Materials, vol. 823 (2016), pp 49-54 © (2016) TransTech Publications.

[12]Negrean I., *Advanced Notions in Analytical Dynamics of Systems,* Acta Technica Napocensis, Series: Applied Mathematics, Mechanics and Engineering, Vol. 60, Issue IV, November. 2017, pg. 491-502.

[13]Negrean I., *Advanced Equations in Analytical Dynamics of Systems,* Acta Technica Napocensis, Series: Applied Mathematics, Mechanics and Engineering, Vol. 60, Issue IV, November 2017, pg. 503-514.

[14] Negrean I., *New Approaches on Notions from Advanced Mechanics,* Acta Technica Napocensis, Series: Applied Mathematics, Mechanics and Engineering, Vol. 61, Issue II, June 2018, pg. 149-158.

[15]Negrean I., *Generalized Forces in Analytical Dynamic of Systems,* Acta Technica Napocensis, Series: Applied Mathematics, Mechanics and Engineering, Vol. 61, Issue II, June 2018, pg. 357-368.

[16]Fu K., Gonzales R., Lee C., *Control, Sensing, Vision and Intelligence, McGraw-Hill Book Co., International Edition, 1987*

Funcţii polinomiale de interpolare în dinamica avansată

Lucrarea de faţă este dedicată cercetării întreprinse de autorul principal, în domeniul utilizării funcţiilor polinomiale de interpolare în generarea traiectoriei de mişcare a roboţilor industriali. În acest scop sunt prezentate câteva formulări clasice privind funcţiile polinomiale de interpolare de ordinul trei cu restricţii. Spre deosebire de varianta clasică, varianta care utilizează polinoame de ordin superior asigură continuitatea în acceleraţii de ordin superior. Sunt prezentate, în formă explicită, expresiile pentru energia de acceleraţie de ordinul întâi, al doilea şi al treilea, corespunzătoare mişcărilor curente şi bruşte ale sistemelor multicorp, acestea fiind utilizate în continuare pentru a defini ecuaţiile de mişcare. În ultima parte a lucrării, modelele matematice prezentate anterior sunt aplicate pentru a descrie comportamentul dinamic al unei structuri de robot de tipul 2TR. În acest scop, se determină expresiile pentru energiile de acceleraţii de ordinul întâi, al doilea şi al treilea, obligatorii în definirea ecuaţiilor dinamice. Au fost de asemenea definite legile de variaţie în raport cu timpul pentru coordonatele generalizate, viteze, acceleraţii, energii de acceleraţii şi forţe generalizate de ordinul întâi, al doilea şi al treilea.

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