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# DYNAMIC MODEL OF 6R ARTICULATED INDUSTRIAL ROBOT USING NEWTON-EULER FORMULATION 

Florin BUGNAR, Claudiu Mihai NEDEZKI, Iuliana Fabiola MOHOLEA


#### Abstract

R\) articulated industrial robot used in welding processes. The paper uses the previously determined geometric and kinematic model. The aim of this analysis is getting the expressions of the generalized driving forces of the robot, representing the equations of the inverse dynamic model. Key words: dynamic model, articulated robot, $6 R$, robotic welding.


## 1. INTRODUCTION

The dynamic model of an industrial robot has a significant importance, due to the complexity of factors that influence the actual performance of the robot. Among them, we can mention the mass distribution parameters (masses, mass centers, position of mass centers, moments of
inertia), the useful forces and moments applied at the end-effecter, factors that are ignored in the process of determining the equations of geometric and kinematic model.

The paper determines the equations of the inverse dynamic model of the 6 R articulated industrial robot, shown in fig. 1 .


Fig. 1. 6R articulated industrial robot

One of the most advantageous methods for dynamic modelling is Newton-Euler formulation, described in [1] and applied in [2], [3], [4], [5]. As a pre-requisite, the geometric [6] and kinematic [7] model of the 6 R robot had to be determined.

An easy way to get the equations of the dynamic model is Robot_Symbolic, a MATLAB toolbox having the following components: Robot_Definition [8], Robot_ Geometry [9], Robot_Kinematics [10] and Robot_Dynamics [3].

## 2. MASS DISTRIBUTION PARAMETERS

Beside the geometric and kinematic parameters, the mass distribution parameters are necessary to be established in order to apply Newton-Euler formulation [11]. Some simplifying hypotheses are useful for setting the mass distribution parameters:
a. The mass centers $\mathrm{C}_{\mathrm{i}}$ will be chosen into the origins $\mathrm{O}_{\mathrm{i}}$ of the frames $\mathrm{O}_{\mathrm{i}} \mathrm{X}_{\mathrm{i}} \mathrm{y}_{\mathrm{i}} \mathrm{z}_{\mathrm{i}}$, $\mathrm{i}=1 \div 6$, such that the position vectors of the mass centers to be null.
b. The mobile frames will be chosen such that their axes to be the main directions of inertia corresponding to the origins of these frames, the centrifugal mechanical moments of inertia being zero.
The mass distribution parameters are: the mass of element $i$, the position vectors of mass centers and the inertia tensors.

The masses are:

$$
\begin{equation*}
\mathrm{M}_{1}, \mathrm{M}_{2}, \mathrm{M}_{3}, \mathrm{M}_{4}, \mathrm{M}_{5}, \mathrm{M}_{6} \tag{1}
\end{equation*}
$$

The position vectors of mass centers, considering the first simplifying hypothesis, are:

$$
\begin{align*}
& \overline{\mathrm{r}}_{\mathrm{c}_{1}}^{1}=\left[\begin{array}{l}
0 \\
0 \\
0
\end{array}\right], \quad \overline{\mathrm{r}}_{\mathrm{c}_{2}}^{2}=\left[\begin{array}{l}
0 \\
0 \\
0
\end{array}\right], \quad \overline{\mathrm{r}}_{\mathrm{c}_{3}}^{3}=\left[\begin{array}{l}
0 \\
0 \\
0
\end{array}\right],  \tag{2}\\
& \overline{\mathrm{r}}_{\mathrm{c}_{4}}^{4}=\left[\begin{array}{l}
0 \\
0 \\
0
\end{array}\right], \quad \overline{\mathrm{r}}_{\mathrm{c}_{5}}^{5}=\left[\begin{array}{l}
0 \\
0 \\
0
\end{array}\right], \quad \overline{\mathrm{r}}_{\mathrm{c}_{6}}^{6}=\left[\begin{array}{l}
0 \\
0 \\
0
\end{array}\right] .  \tag{3}\\
& {\left[\bar{a}_{c}\right]_{3}^{3}=\left[\begin{array}{c}
l_{2}\left(s q_{2} \ddot{q}_{1}+2 \dot{q}_{1} \dot{q}_{2} c q_{2}\right) \\
g s\left(q_{3}+q_{2}\right)-l_{2} c q_{3} \ddot{q}_{2}+\frac{1}{2} l_{2} \dot{q}_{1}^{2} s\left(q_{3}+2 q_{2}\right)-\frac{1}{2} l_{2} \dot{q}_{1}^{2} s q_{3}-s q_{3} l_{2} \dot{q}_{2}^{2} \\
g c\left(q_{3}+q_{2}\right)+l_{2} s q_{3} \ddot{q}_{2}+\frac{1}{2} l_{2} \dot{q}_{1}^{2} c\left(q_{3}+2 q_{2}\right)-\frac{1}{2} l_{2} \dot{q}_{1}^{2} c q_{3}-c q_{3} l_{2} \dot{q}_{2}^{2}
\end{array}\right] .}
\end{align*}
$$

The inertia tensors, considering the second simplifying hypothesis, are the following:

$$
\begin{aligned}
& \mathrm{J}_{1}^{*_{1}}=\left[\begin{array}{ccc}
\mathrm{J}_{\mathrm{x}}^{*_{1}} & 0 & 0 \\
0 & \mathrm{~J}_{\mathrm{y}}^{*_{1}} & 0 \\
0 & 0 & \mathrm{~J}_{\mathrm{z}}^{*_{1}}
\end{array}\right], \mathrm{J}_{2}^{*_{2}}=\left[\begin{array}{ccc}
\mathrm{J}_{\mathrm{x}}^{* 2} & 0 & 0 \\
0 & \mathrm{~J}_{\mathrm{y}}^{*_{2}} & 0 \\
0 & 0 & \mathrm{~J}_{\mathrm{z}}^{* 2}
\end{array}\right] \text {, } \\
& \mathrm{J}_{3}^{*_{3}}=\left[\begin{array}{ccc}
\mathrm{J}_{x}^{* 3} & 0 & 0 \\
0 & \mathrm{~J}_{y}^{* 3} & 0 \\
0 & 0 & \mathrm{~J}_{z}^{* 3}
\end{array}\right], \mathrm{J}_{4}^{* 4}=\left[\begin{array}{ccc}
\mathrm{J}_{x}^{* 4} & 0 & 0 \\
0 & \mathrm{~J}_{y}^{* 4} & 0 \\
0 & 0 & \mathrm{~J}_{z}^{* 4}
\end{array}\right], \\
& \mathbf{J}_{5}^{* 5}=\left[\begin{array}{ccc}
\mathbf{J}_{x}^{* 5} & 0 & 0 \\
0 & \mathrm{~J}_{\mathrm{y}}^{* 5} & 0 \\
0 & 0 & \mathbf{J}_{z}^{* 5}
\end{array}\right], \mathrm{J}_{6}^{* 6}=\left[\begin{array}{ccc}
\mathrm{J}_{\mathrm{x}}^{* 6} & 0 & 0 \\
0 & \mathrm{~J}_{y}^{* 6} & 0 \\
0 & 0 & \mathrm{~J}_{\mathrm{z}}^{*{ }^{*}}
\end{array}\right] \text {. }
\end{aligned}
$$

The non-zero components of the inertia tensor, $J_{x}^{*_{i}}, J_{y}^{*_{i}}, J_{z}^{*_{i}}, i=1 \div 6$ are the axial mechanical moments of inertia with respect to the frame $i$, having the origin in the mass center $\mathrm{C}_{\mathrm{i}}$ and having the same orientation as the frame attached to each of the robot's links.

The accelerations corresponding to the mass centers are determined according to [12]. The following accelerations are established:

$$
\begin{gathered}
\overline{\mathrm{a}}_{\mathrm{c}_{1}}^{1}=\overline{\mathrm{a}}_{1}^{1}+\overline{\mathrm{\varepsilon}}_{1}^{1} \times \overline{\mathrm{r}}_{\mathrm{c}_{1}}^{1}+\bar{\omega}_{1}^{1} \times\left(\bar{\omega}_{1}^{1} \times \overline{\mathrm{r}}_{\mathrm{c}_{1}}^{1}\right), \\
{\left[\bar{a}_{\mathrm{c}}\right]_{1}^{1}=\left[\begin{array}{l}
0 \\
0 \\
\mathrm{~g}
\end{array}\right],} \\
\overline{\mathrm{a}}_{\mathrm{c}_{2}}^{2}=\overline{\mathrm{a}}_{2}^{2}+\bar{\varepsilon}_{2}^{2} \times \overline{\mathrm{r}}_{\mathrm{c}_{2}}^{2}+\bar{\omega}_{2}^{2} \times\left(\bar{\omega}_{2}^{2} \times \overline{\mathrm{r}}_{\mathrm{c}_{2}}^{2}\right),
\end{gathered}
$$

$$
\begin{gathered}
{\left[\overline{\mathrm{a}}_{\mathrm{c}} \mathrm{l}_{2}^{2}=\left[\begin{array}{c}
0 \\
\mathrm{sq}_{2} \mathrm{~g} \\
\mathrm{cq}_{2} \mathrm{~g}
\end{array}\right],\right.} \\
\overline{\mathrm{a}}_{\mathrm{c}_{3}}^{3}=\overline{\mathrm{a}}_{3}^{3}+\bar{\varepsilon}_{3}^{3} \times \times_{\mathrm{c}_{3}}^{3}+\bar{\omega}_{3}^{3} \times\left(\bar{\omega}_{3}^{3} \times \overline{\mathrm{r}}_{\mathrm{c}_{3}^{3}}^{3}\right),
\end{gathered}
$$

Due to the complexity of the equations corresponding to the mass centers 45 and 6 accelerations, they do not fit in this paper. Their computation formulae are the following:

$$
\begin{align*}
& \overline{\mathrm{a}}_{\mathrm{c}_{4}}^{4}=\overline{\mathrm{a}}_{4}^{4}+\bar{\varepsilon}_{4}^{4} \times \overline{\mathrm{r}}_{\mathrm{c}_{4}}^{4}+\bar{\omega}_{4}^{4} \times\left(\bar{\omega}_{4}^{4} \times \overline{\mathrm{r}}_{\mathrm{c}_{4}}^{4}\right),  \tag{13}\\
& \overline{\mathrm{a}}_{\mathrm{c}_{5}}^{5}=\overline{\mathrm{a}}_{5}^{5}+\bar{\varepsilon}_{5}^{5} \times \overline{\mathrm{c}}_{5}^{5}+\bar{\omega}_{5}^{5} \times\left(\bar{\omega}_{5}^{5} \times \overline{\mathrm{r}}_{\mathrm{c}_{5}}^{5}\right),  \tag{14}\\
& \overline{\mathrm{a}}_{\mathrm{c}_{6}}^{6}=\overline{\mathrm{a}}_{6}^{6}+\bar{\varepsilon}_{6}^{6} \times \overline{\mathrm{r}}_{6}^{6}+\bar{\omega}_{6}^{6} \times\left(\bar{\omega}_{6}^{6} \times \overline{\mathrm{r}}_{6}^{6}\right) \tag{15}
\end{align*}
$$

## 3. OUTWARDS ITERATIONS

According to [1], [11] and [12], the mechanical structure of the robot is parsed by outwards iterations, obtaining the system of external forces and their moments.

The external forces, applying the computation relations, are:

$$
[\overline{\mathrm{F}}]_{1}^{1}=\mathrm{M}_{1}\left[\overline{\mathrm{a}}_{\mathrm{c}}\right]_{1}^{1}, \quad \overline{\mathrm{~F}}_{1}^{1}=\left[\begin{array}{c}
0  \tag{16}\\
0 \\
\mathrm{M}_{1} \mathrm{~g}
\end{array}\right]
$$

$$
[\overline{\mathrm{F}}]_{2}^{2}=\mathrm{M}_{2}\left[\overline{\mathrm{a}}_{\mathrm{c}}\right]_{2}^{2}, \quad[\overline{\mathrm{~F}}]_{2}^{2}=\left[\begin{array}{c}
0  \tag{17}\\
\mathrm{M}_{2} \mathrm{sq}_{2} \mathrm{~g} \\
\mathrm{M}_{2} \mathrm{cq}_{2} \mathrm{~g}
\end{array}\right]
$$

$$
\left[\bar{a}_{c}\right]_{3}^{3}=\left[\begin{array}{c}
l_{2}\left(s q_{2} \ddot{q}_{1}+2 \dot{q}_{1} \dot{q}_{2} c q_{2}\right)  \tag{18}\\
g s\left(q_{3}+q_{2}\right)-l_{2} c q_{3} \ddot{q}_{2}+\frac{1}{2} l_{2} \dot{q}_{1}^{2} s\left(q_{3}+2 q_{2}\right)-\frac{1}{2} l_{2} \dot{q}_{1}^{2} s q_{3}-s q_{3} l_{2} \dot{q}_{2}^{2} \\
g c\left(q_{3}+q_{2}\right)+l_{2} s q_{3} \ddot{q}_{2}+\frac{1}{2} l_{2} \dot{q}_{1}^{2} c\left(q_{3}+2 q_{2}\right)-\frac{1}{2} l_{2} \dot{q}_{1}^{2} c q_{3}-c q_{3} l_{2} \dot{q}_{2}^{2}
\end{array}\right] .
$$

The equations of external forces of the other three links are also very complex and they cannot be presented here. Their computation equations are:

$$
\begin{equation*}
[\overline{\mathrm{F}}]_{4}^{4}=\mathrm{M}_{4}\left[\overline{\mathrm{a}}_{\mathrm{c}}\right]_{4}^{4} ;[\overline{\mathrm{F}}]_{5}^{5}=\mathrm{M}_{5}\left[\overline{\mathrm{a}}_{\mathrm{c}}\right]_{5}^{5} \tag{20}
\end{equation*}
$$

$$
\begin{equation*}
[\overline{\mathrm{F}}]_{6}^{6}=\mathrm{M}_{6}\left[\overline{\mathrm{a}}_{\mathrm{c}}\right]_{6}^{6} . \tag{21}
\end{equation*}
$$

According to [12], the moments of the external forces are obtained as:

$$
\begin{equation*}
\overline{\mathrm{M}}_{\mathrm{c}_{1}}^{1}=\mathrm{J}_{1}^{* 1} \bar{\varepsilon}_{1}^{1}+\bar{\omega}_{1}^{1} \times \mathrm{J}_{1}^{* 1} \bar{\omega}_{1}^{1}, \tag{22}
\end{equation*}
$$

$$
\left[\bar{M}_{c}\right]_{1}^{1}=\left[\begin{array}{c}
0  \tag{23}\\
0 \\
J_{z}^{*} \ddot{q}_{1}
\end{array}\right]
$$

$$
\begin{align*}
& \overline{\mathrm{M}}_{\mathrm{c}_{3}}^{3}=\mathrm{J}_{3}^{* 3} \bar{\varepsilon}_{3}^{3}+\bar{\omega}_{3}^{3} \times \mathrm{J}_{3}^{* 3} \bar{\omega}_{3}^{3},  \tag{26}\\
& \overline{\mathrm{M}}_{\mathrm{c}_{4}}^{4}=\mathrm{J}_{4}^{* 4} \bar{\varepsilon}_{4}^{4}+\bar{\omega}_{4}^{4} \times \mathrm{J}_{4}^{* 4} \bar{\omega}_{4}^{4},  \tag{27}\\
& \overline{\mathrm{M}}_{\mathrm{c}_{5}}^{5}=\mathrm{J}_{5}^{* 5} \bar{\varepsilon}_{5}^{5}+\bar{\omega}_{5}^{5} \times \mathrm{J}_{5}^{* 5} \bar{\omega}_{5}^{5},  \tag{28}\\
& \overline{\mathrm{M}}_{\mathrm{c}_{6}}^{6}=\mathrm{J}_{6}^{* 6} \bar{\varepsilon}_{6}^{6}+\bar{\omega}_{6}^{6} \times \mathrm{J}_{6}^{* 6} \bar{\omega}_{6}^{6} \tag{29}
\end{align*}
$$

$$
\begin{equation*}
\overline{\mathrm{F}}_{1_{6}}^{6}=[\mathrm{R}]_{7}^{6} \cdot \overline{\mathrm{~F}}_{1_{7}}^{7}+\overline{\mathrm{F}}_{6}^{6}, \tag{30}
\end{equation*}
$$

## 4. INWARDS ITERATIONS

$$
\begin{equation*}
\overline{\mathrm{F}}_{1_{5}}^{5}=[\mathrm{R}]_{6}^{5} \cdot \overline{\mathrm{~F}}_{\mathrm{l}_{6}}^{6}+\overline{\mathrm{F}}_{5}^{5}, \tag{31}
\end{equation*}
$$

$$
\begin{align*}
& {\overline{F_{14}^{4}}}_{4}=[\mathrm{R}]_{5}^{]^{2}} \cdot \overline{\mathrm{~F}}_{5}^{5}+\overline{\mathrm{F}}_{4}^{4},  \tag{32}\\
& \overline{\mathrm{~F}}_{3}^{3}=[\mathrm{R}]_{4}^{3} \cdot \overline{\mathrm{~F}}_{4}^{4}+\overline{\mathrm{F}}_{3}^{3},  \tag{33}\\
& \overline{\mathrm{~F}}_{2}^{2}=[\mathrm{R}]_{3}^{2} \cdot \overline{\mathrm{~F}}_{3}^{3}+\overline{\mathrm{F}}_{2}^{2},  \tag{34}\\
& \overline{\mathrm{~F}}_{11}^{1}=[\mathrm{R}]_{2}^{1} \cdot \overline{\mathrm{~F}}_{2}^{2}+\overline{\mathrm{F}}_{1}^{1}, \tag{35}
\end{align*}
$$

and the moments of the connection forces have the expressions:

$$
\begin{align*}
& \overline{\mathbf{M}}_{\mathrm{l}_{6}}^{6}=[\mathrm{R}]_{7}^{6} \cdot \overline{\mathbf{M}}_{\mathrm{l}_{7}}^{7}+\overline{\mathrm{r}}_{\mathrm{c}_{6}}^{6} \times \overline{\mathrm{F}}_{6}^{6}+\overline{\mathrm{r}}_{7}^{6} \times[\mathrm{R}]_{7}^{6} \cdot \overline{\mathrm{~F}}_{7}^{7}+\overline{\mathbf{M}}_{\mathrm{c}_{6}}^{6},  \tag{36}\\
& \overline{\mathrm{M}}_{\mathrm{lo}_{5}}^{5}=[\mathrm{R}]_{6}^{5} \overline{\mathrm{M}}_{\mathrm{lo}_{6}}^{6}+\overline{\mathrm{c}}_{\mathrm{cs}}^{5} \times \overline{\mathrm{F}}_{5}^{5}+\overline{\mathrm{r}}_{6}^{5} \times[\mathrm{R}]_{6}^{5} \cdot \overline{\mathrm{~F}}_{16}^{6}+\overline{\mathrm{M}}_{\mathrm{cs}}^{5},  \tag{37}\\
& \overline{\mathrm{M}}_{\mathrm{l}_{4}}^{4}=[\mathrm{R}]_{5}^{4} \cdot \overline{\mathrm{M}}_{\mathrm{lo}_{5}}^{5}+\overline{\mathrm{c}}_{\mathrm{c}_{4}}^{4} \times \overline{\mathrm{F}}_{4}^{4}+\overline{\mathrm{F}}_{5}^{4} \times[\mathrm{R}]_{5}^{4} \cdot \overline{\mathrm{~F}}_{5}^{5}+\overline{\mathrm{M}}_{\mathrm{c}_{4}}^{4},  \tag{38}\\
& \overline{\mathrm{M}}_{\mathrm{l}_{3}}^{3}=[\mathrm{R}]_{4}^{3} \cdot \overline{\mathrm{M}}_{\mathrm{l}_{4}}^{4}+\overline{\mathrm{c}}_{\mathrm{c}_{3}}^{3} \times \overline{\mathrm{F}}_{3}^{3}+\overline{\mathrm{r}}_{4}^{3} \times[\mathrm{R}]_{4}^{3} \cdot \overline{\mathrm{~F}}_{\mathrm{F}_{4}}^{4}+\overline{\mathrm{M}}_{\mathrm{c}_{3}}^{3},  \tag{39}\\
& \overline{\mathrm{M}}_{\mathrm{l}_{2}}^{2}=[\mathrm{R}]_{3}^{2} \cdot \overline{\mathrm{M}}_{\mathrm{l}_{3}}^{3}+\overline{\mathrm{c}}_{\mathrm{c}_{2}}^{2} \times \overline{\mathrm{F}}_{2}^{2}+\overline{\mathrm{r}}_{3}^{2} \times[\mathrm{R}]_{3}^{2} \cdot \overline{\mathrm{~F}}_{3}^{3}+\overline{\mathrm{M}}_{\mathrm{c}_{2}}^{2},  \tag{40}\\
& \overline{\mathrm{M}}_{\mathrm{l}_{1}}^{1}=[\mathrm{R}]_{2}^{1} \cdot \overline{\mathrm{M}}_{\mathrm{l}_{\mathrm{O}_{2}}}^{2}+\overline{\mathrm{r}}_{\mathrm{c}_{1}}^{1} \times \overline{\mathrm{F}}_{1}^{1}+\overline{\mathrm{r}}_{2}^{1} \times[\mathrm{R}]_{2}^{1} \cdot \overline{\mathrm{~F}}_{1}^{2}+\overline{\mathrm{M}}_{\mathrm{c}_{1}}^{1}, \tag{41}
\end{align*}
$$

giving complex equations, available at the reader's request to the email address of the first author.

According to [1] and [13], the generalized driving forces have the following computation formulae:

$$
\begin{align*}
& \mathrm{Q}_{\mathrm{m}}^{1}=\left[\begin{array}{lll}
\overline{\mathrm{M}}_{\mathrm{l}_{1}}^{1}
\end{array}\right] \cdot{ }_{\mathrm{T}} \cdot \overline{\mathrm{k}}_{1}^{1}=\left[\begin{array}{lll}
\mathrm{M}_{\mathrm{l}_{\mathrm{x}}}^{1} & \mathrm{M}_{\mathrm{l}_{\mathrm{y}}}^{1} & \mathrm{M}_{\mathrm{l}_{z}}^{1}
\end{array}\right] \cdot\left[\begin{array}{l}
0 \\
0 \\
1
\end{array}\right]=\mathrm{M}_{\mathrm{l}_{z}}^{1},  \tag{42}\\
& \mathrm{Q}_{\mathrm{m}}^{2}=\left[\bar{M}_{\mathrm{l}_{\mathrm{O}_{2}}}^{2}\right] \mathrm{T}_{\mathrm{T}} \cdot \bar{i}_{2}^{2}=\left[\begin{array}{lll}
\mathrm{M}_{\mathrm{l}_{\mathrm{x}}}^{2} & \mathrm{M}_{\mathrm{l}_{\mathrm{y}}}^{2} & \mathrm{M}_{\mathrm{l}_{\mathrm{z}}}^{2}
\end{array}\right] \cdot\left[\begin{array}{l}
1 \\
0 \\
0
\end{array}\right]=\mathrm{M}_{\mathrm{l}_{\mathrm{x}}}^{2},  \tag{43}\\
& \left.\mathrm{Q}_{\mathrm{m}}^{3}=\left[\begin{array}{c}
\bar{M}_{\mathrm{IO}_{3}}^{3}
\end{array}\right]\right]_{\mathrm{T}} \cdot \bar{i}_{3}^{3}=\left[\begin{array}{lll}
\mathrm{M}_{\mathrm{i}_{\mathrm{x}}}^{3} & \mathrm{M}_{\mathrm{i}_{\mathrm{y}}}^{3} & \mathrm{M}_{\mathrm{i}_{2}}^{3}
\end{array}\right] \cdot\left[\begin{array}{l}
1 \\
0 \\
0
\end{array}\right]=\mathrm{M}_{\mathrm{i}_{\mathrm{x}}}^{3},  \tag{44}\\
& \mathrm{Q}_{\mathrm{m}}^{4}=\left[\begin{array}{l}
\overline{\mathrm{M}}_{\mathrm{l}_{4}}^{4}
\end{array}\right] \mathrm{T} \cdot \overline{\mathrm{k}}_{4}^{4}=\left[\begin{array}{lll}
\mathrm{M}_{\mathrm{l}_{\mathrm{x}}}^{4} & \mathrm{M}_{\mathrm{l}_{\mathrm{y}}}^{4} & \mathrm{M}_{\mathrm{l}_{z}}^{4}
\end{array}\right] \cdot\left[\begin{array}{l}
0 \\
0 \\
1
\end{array}\right]=\mathrm{M}_{\mathrm{l}_{\mathrm{z}}}^{4}, \tag{45}
\end{align*}
$$

$$
\begin{align*}
& \left.\mathrm{Q}_{\mathrm{m}}^{5}=\left[\bar{M}_{\mathrm{lo}_{5}}^{5}\right]\right]_{\mathrm{T}}^{\mathrm{T}} \cdot \bar{i}_{5}^{5}=\left[\begin{array}{lll}
\mathrm{M}_{\mathrm{l}_{\mathrm{x}}}^{5} & \mathrm{M}_{\mathrm{l}_{\mathrm{y}}}^{5} & \mathrm{M}_{\mathrm{l}_{2}}^{5}
\end{array}\right]\left[\begin{array}{l}
1 \\
0 \\
0
\end{array}\right]=\mathrm{M}_{\mathrm{l}_{\mathrm{x}}}^{5}, \tag{46}
\end{align*}
$$

In this case of articulated robot, all the generalized driving forces have the significance of moments.

## 5. CONCLUSION

The dynamic study of articulated robots allows choosing the proper motors for their joints, as well as the optimal arrangement of modules in a modular robotic structure, such that the energy consumption to be minimum for a given task.

The obtaining of the dynamic equations was possible using the component Robot_Dynamics [3] from the toolbox Robot_ Symbolic [1], [8], [9], [10], [3], written in MATLAB.

## 6. FUTURE WORK

There are many possible ways of further development of this work. One of them is to study the errors in the 6 R articulated industrial robot, by means of geometric, kinematic and dynamic operation precision [14].

Another way is to plan the motion of the robot, based on polynomial functions, having a given task in a given environment [15], [16], [17].

The latest research published in [18] - [21] opens a new approach in modeling the dynamics of the sudden motions in robotics and multibody systems, based on the acceleration energy.

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# MODELUL DINAMIC AL ROBOTULUI INDUSTRIAL ARTICULAT 6R, FOLOSIND FORMALISMUL NEWTON-EULER 

> Rezumat: Articolul descrie procesul de obținere a ecuațiilor dinamice ale robotului industrial articulat $6 R$, utilizat in procese de sudură. Lucrarea utilizează ecuațiile modelului geometric și cinematic, determinate în prealabil. Scopul acestei analize este obținerea expresiilor forțelor generalizate ale robotului, reprezentând ecuațiile modelului dinamic invers.

Florin BUGNAR, Ph.D. Eng., Technical University of Cluj-Napoca, B-dul Muncii, no. 103-105, email: bugnarf@hotmail.com, Phone: +40743111693, Cluj-Napoca, Romania.
Claudiu Mihai NEDEZKI, Ph.D. Eng., Associate Professor, Technical University of Cluj Napoca, Department of Engineering and Robotics, e-mail: claudiu_nedezki@yahoo.com, Office Phone +40264401639 , Home Address: Calea Turzii street, no. 67, Home Phone +40264440639 .
Iuliana Fabiola MOHOLEA, Ph.D. Eng., Assistant Professor, Technical University of Cluj Napoca, Department of Mechanical System Engineering, B-dul Muncii, no. 103-105, e-mail: iuliana.moholea@mep.utcluj.ro, Office Phone +40264401781 .

