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DYNAMIC MODEL OF 6R ARTICULATED INDUSTRIAL ROBOT USING NEWTON-EULER FORMULATION

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Abstract: the article describes the process of obtaining the dynamic equations of the 6R articulated industrial robot used in welding processes. The paper uses the previously determined geometric and kinematic model. The aim of this analysis is getting the expressions of the generalized driving forces of the robot, representing the equations of the inverse dynamic model. **Key words:** dynamic model, articulated robot, 6R, robotic welding.

1. INTRODUCTION

The dynamic model of an industrial robot has a significant importance, due to the complexity of factors that influence the actual performance of the robot. Among them, we can mention the mass distribution parameters (masses, mass centers, position of mass centers, moments of inertia), the useful forces and moments applied at the end-effecter, factors that are ignored in the process of determining the equations of geometric and kinematic model.

The paper determines the equations of the inverse dynamic model of the 6R articulated industrial robot, shown in fig. 1.



Fig. 1. 6R articulated industrial robot

One of the most advantageous methods for modelling dynamic is Newton-Euler formulation, described in [1] and applied in [2], [3], [4], [5]. As a pre-requisite, the geometric [6] and kinematic [7] model of the 6R robot had to be determined.

An easy way to get the equations of the dynamic model is Robot Symbolic, a MATLAB toolbox having the following components: Robot_Definition [8], Robot_ Geometry [9], Robot Kinematics [10] and Robot Dynamics [3].

2. MASS DISTRIBUTION PARAMETERS

Beside the geometric and kinematic parameters, the mass distribution parameters are necessary to be established in order to apply Newton-Euler formulation [11]. Some simplifying hypotheses are useful for setting the mass distribution parameters:

- a. The mass centers C_i will be chosen into the origins O_i of the frames $O_i x_i y_i z_i$, $i = 1 \div 6$, such that the position vectors of the mass centers to be null.
- b. The mobile frames will be chosen such that their axes to be the main directions of inertia corresponding to the origins of these frames, the centrifugal mechanical moments of inertia being zero.

The mass distribution parameters are: the mass of element i, the position vectors of mass centers and the inertia tensors.

The masses are:

$$M_1$$
, M_2 , M_3 , M_4 , M_5 , M_6 . (1)
The position vectors of mass centers,

considering the first simplifying hypothesis, are: [0]

$$\vec{r}_{c_{1}}^{1} = \begin{bmatrix} 0\\0\\0 \end{bmatrix}, \quad \vec{r}_{c_{2}}^{2} = \begin{bmatrix} 0\\0\\0 \end{bmatrix}, \quad \vec{r}_{c_{3}}^{3} = \begin{bmatrix} 0\\0\\0 \end{bmatrix}, \quad (2) \qquad \qquad \lfloor cq_{2}g \rfloor$$

$$\vec{a}_{c_{3}}^{3} = \vec{a}_{3}^{3} + \vec{e}_{3}^{3} \times \vec{r}_{c_{3}}^{3} + \vec{\omega}_{3}^{3} \times (\vec{\omega}_{3}^{3} \times \vec{r}_{c_{3}}^{3}), \quad (11)$$

$$\vec{r}_{c_{4}}^{4} = \begin{bmatrix} 0\\0\\0\\0 \end{bmatrix}, \quad \vec{r}_{c_{5}}^{5} = \begin{bmatrix} 0\\0\\0\\0 \end{bmatrix}, \quad \vec{r}_{c_{6}}^{6} = \begin{bmatrix} 0\\0\\0\\0 \end{bmatrix}, \quad (3)$$

$$[\vec{a}_{c}]_{3}^{3} = \begin{bmatrix} s(q_{3} + q_{2}) - l_{2}cq_{3}\ddot{q}_{2} + \frac{1}{2}l_{2}\dot{q}_{1}^{2}s(q_{3} + 2q_{2}) - \frac{1}{2}l_{2}\dot{q}_{1}^{2}sq_{3} - sq_{3}l_{2}\dot{q}_{2}^{2} \\ gc(q_{3} + q_{2}) + l_{2}sq_{3}\ddot{q}_{2} + \frac{1}{2}l_{2}\dot{q}_{1}^{2}c(q_{3} + 2q_{2}) - \frac{1}{2}l_{2}\dot{q}_{1}^{2}cq_{3} - cq_{3}l_{2}\dot{q}_{2}^{2} \end{bmatrix}. \quad (12)$$

The inertia tensors, considering the second simplifying hypothesis, are the following:

$$J_{1}^{*1} = \begin{bmatrix} J_{x}^{*1} & 0 & 0 \\ 0 & J_{y}^{*1} & 0 \\ 0 & 0 & J_{z}^{*1} \end{bmatrix}, \quad J_{2}^{*2} = \begin{bmatrix} J_{x}^{*2} & 0 & 0 \\ 0 & J_{y}^{*2} & 0 \\ 0 & 0 & J_{z}^{*2} \end{bmatrix}, \quad (4)$$
$$J_{3}^{*3} = \begin{bmatrix} J_{x}^{*3} & 0 & 0 \\ 0 & J_{y}^{*3} & 0 \\ 0 & 0 & J_{z}^{*3} \end{bmatrix}, \quad J_{4}^{*4} = \begin{bmatrix} J_{x}^{*4} & 0 & 0 \\ 0 & J_{y}^{*4} & 0 \\ 0 & 0 & J_{z}^{*4} \end{bmatrix}, \quad (5)$$
$$J_{5}^{*5} = \begin{bmatrix} J_{x}^{*5} & 0 & 0 \\ 0 & J_{y}^{*5} & 0 \\ 0 & 0 & J_{z}^{*5} \end{bmatrix}, \quad J_{6}^{*6} = \begin{bmatrix} J_{x}^{*6} & 0 & 0 \\ 0 & J_{y}^{*6} & 0 \\ 0 & 0 & J_{z}^{*6} \end{bmatrix}. \quad (6)$$

The non-zero components of the inertia tensor, J_x^{*i} , J_y^{*i} , J_z^{*i} , $i = 1 \div 6$ are the axial mechanical moments of inertia with respect to the frame i, having the origin in the mass center C_i and having the same orientation as the frame attached to each of the robot's links.

The accelerations corresponding to the mass centers are determined according to [12]. The following accelerations are established:

$$\overline{a}_{c_1}^1 = \overline{a}_1^1 + \overline{c}_1^1 \times \overline{r}_{c_1}^1 + \overline{\omega}_1^1 \times \left(\overline{\omega}_1^1 \times \overline{r}_{c_1}^1\right), \tag{7}$$

$$\begin{bmatrix} \bar{\mathbf{a}}_c \end{bmatrix}_1^1 = \begin{bmatrix} 0\\ 0\\ g \end{bmatrix}, \tag{8}$$

$$\overline{a}_{c_2}^2 = \overline{a}_2^2 + \overline{c}_2^2 \times \overline{r}_{c_2}^2 + \overline{\omega}_2^2 \times \left(\overline{\omega}_2^2 \times \overline{r}_{c_2}^2\right), \qquad (9)$$

$$[\bar{\mathbf{a}}_{c}]_{2}^{2} = \begin{bmatrix} 0\\ sq_{2}g\\ cq_{2}g \end{bmatrix}, \qquad (10)$$

$$\overline{a}_{c_3}^3 = \overline{a}_3^3 + \overline{e}_3^3 \times \overline{r}_{c_3}^3 + \overline{\omega}_3^3 \times \left(\overline{\omega}_3^3 \times \overline{r}_{c_3}^3\right), \qquad (11)$$

Due to the complexity of the equations corresponding to the mass centers 4 5 and 6 accelerations, they do not fit in this paper. Their computation formulae are the following:

$$\overline{a}_{c_4}^{_4} = \overline{a}_4^{_4} + \overline{\epsilon}_4^{_4} \times \overline{r}_{c_4}^{_4} + \overline{\omega}_4^{_4} \times \left(\overline{\omega}_4^{_4} \times \overline{r}_{c_4}^{_4}\right), \qquad (13)$$

$$\overline{a}_{c_5}^5 = \overline{a}_5^5 + \overline{c}_5^5 \times \overline{r}_{c_5}^5 + \overline{\omega}_5^5 \times \left(\overline{\omega}_5^5 \times \overline{r}_{c_5}^5\right), \qquad (14)$$

$$\bar{a}_{c_6}^6 = \bar{a}_6^6 + \bar{\epsilon}_6^6 \times \bar{r}_{c_6}^6 + \overline{\omega}_6^6 \times \left(\overline{\omega}_6^6 \times \bar{r}_{c_6}^6\right).$$
(15)

3. OUTWARDS ITERATIONS

According to [1], [11] and [12], the mechanical structure of the robot is parsed by outwards iterations, obtaining the system of external forces and their moments.

The external forces, applying the computation relations, are:

$$\left[\overline{\mathbf{F}}\right]_{i}^{l} = \mathbf{M}_{1}\left[\overline{\mathbf{a}}_{c}\right]_{1}^{l}, \qquad \overline{\mathbf{F}}_{1}^{l} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}_{1}\mathbf{g} \end{bmatrix}; \qquad (16)$$

$$[\overline{\mathbf{F}}]_{2}^{2} = \mathbf{M}_{2}[\overline{\mathbf{a}}_{c}]_{2}^{2}, \qquad [\overline{\mathbf{F}}]_{2}^{2} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}_{2}\mathbf{s}\mathbf{q}_{2}\mathbf{g} \\ \mathbf{M}_{2}\mathbf{c}\mathbf{q}_{2}\mathbf{g} \end{bmatrix}; \quad (17)$$

$$[\overline{\mathbf{F}}]_{3}^{3} = \mathbf{M}_{3}[\overline{\mathbf{a}}_{c}]_{3}^{3}; \qquad (18)$$

$$\begin{bmatrix} \bar{a}_{c} \end{bmatrix}_{3}^{3} = \begin{bmatrix} l_{2}(sq_{2}\ddot{q}_{1} + 2\dot{q}_{1}\dot{q}_{2}cq_{2}) \\ gs(q_{3} + q_{2}) - l_{2}cq_{3}\ddot{q}_{2} + \frac{1}{2}l_{2}\dot{q}_{1}^{2}s(q_{3} + 2q_{2}) - \frac{1}{2}l_{2}\dot{q}_{1}^{2}sq_{3} - sq_{3}l_{2}\dot{q}_{2}^{2} \\ gc(q_{3} + q_{2}) + l_{2}sq_{3}\ddot{q}_{2} + \frac{1}{2}l_{2}\dot{q}_{1}^{2}c(q_{3} + 2q_{2}) - \frac{1}{2}l_{2}\dot{q}_{1}^{2}cq_{3} - cq_{3}l_{2}\dot{q}_{2}^{2} \end{bmatrix}.$$
(19)

(20)

(21)

(29)

The equations of external forces of the other three links are also very complex and they cannot be presented here. Their computation equations are:

 $[\overline{\mathbf{F}}]_4^4 = \mathbf{M}_4[\overline{\mathbf{a}}_a]_4^4; [\overline{\mathbf{F}}]_5^5 = \mathbf{M}_5[\overline{\mathbf{a}}_a]_5^5$

 $[\overline{F}]_6^6 = M_6 [\overline{a}_c]_6^6 \cdot$

According to [12], the moments of the external forces are obtained as:

$$\overline{\mathbf{M}}_{c_1}^{\scriptscriptstyle 1} = \mathbf{J}_1^{*_1} \overline{\boldsymbol{\epsilon}}_1^{\scriptscriptstyle 1} + \overline{\boldsymbol{\omega}}_1^{\scriptscriptstyle 1} \times \mathbf{J}_1^{*_1} \overline{\boldsymbol{\omega}}_1^{\scriptscriptstyle 1}, \qquad (22)$$

$$[\bar{M}_c]_1^1 = \begin{bmatrix} 0\\0\\J_z^{*1}\ddot{q}_1 \end{bmatrix};$$
 (23)

$$\overline{\mathbf{M}}_{c_2}^2 = \mathbf{J}_2^{*2} \overline{\mathbf{e}}_2^2 + \overline{\mathbf{\omega}}_2^2 \times \mathbf{J}_2^{*2} \overline{\mathbf{\omega}}_2^2, \qquad (24)$$

$$[\bar{M}_{c}]_{2}^{2} = \begin{bmatrix} J_{x}^{*2}\ddot{q}_{2} - J_{y}^{*2}\dot{q}_{1}^{2}cq_{2}sq_{2} + J_{z}^{*2}\dot{q}_{1}^{2}cq_{2}sq_{2} \\ J_{y}^{*2}\ddot{q}_{1}sq_{2} + J_{y}^{*2}\dot{q}_{1}\dot{q}_{2}cq_{2} + J_{x}^{*2}\dot{q}_{1}\dot{q}_{2}cq_{2} - J_{z}^{*2}\dot{q}_{1}\dot{q}_{2}cq_{2} \\ J_{z}^{*2}\ddot{q}_{1}cq_{2} - J_{z}^{*2}\dot{q}_{1}\dot{q}_{2}sq_{2} - J_{x}^{*2}\dot{q}_{1}\dot{q}_{2}sq_{2} + J_{y}^{*2}\dot{q}_{1}\dot{q}_{2}sq_{2} \end{bmatrix}.$$
(25)

-

$$\overline{\mathbf{M}}_{c_3}^3 = \mathbf{J}_3^{*3}\overline{\mathbf{\varepsilon}}_3^3 + \overline{\mathbf{\omega}}_3^3 \times \mathbf{J}_3^{*3}\overline{\mathbf{\omega}}_3^3, \qquad (26)$$

$$\overline{\mathbf{M}}_{c_4}^{\, 4} = \mathbf{J}_{_4}^{*4} \overline{\boldsymbol{\epsilon}}_{_4}^{\, 4} + \overline{\boldsymbol{\omega}}_{_4}^{\, 4} \times \mathbf{J}_{_4}^{*4} \overline{\boldsymbol{\omega}}_{_4}^{\, 4}, \qquad (27)$$

$$\overline{\mathbf{M}}_{c_{5}}^{5} = \mathbf{J}_{5}^{*5}\overline{\mathbf{e}}_{5}^{5} + \overline{\mathbf{\omega}}_{5}^{5} \times \mathbf{J}_{5}^{*5}\overline{\mathbf{\omega}}_{5}^{5} , \qquad (28)$$

 $\overline{M}_{c_6}^6 = J_6^{*6} \overline{\epsilon}_6^6 + \overline{\omega}_6^6 \times J_6^{*6} \overline{\omega}_6^6 \cdot$

$$\overline{F}_{l_6}^6 = [R]_7^6 \cdot \overline{F}_{l_7}^7 + \overline{F}_6^6, \qquad (30)$$

$$\overline{F}_{l_5}^5 = [R]_6^5 \cdot \overline{F}_{l_6}^6 + \overline{F}_5^5, \qquad (31)$$

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$$\overline{F}_{l_4}^4 = [R]_5^4 \cdot \overline{F}_{l_5}^5 + \overline{F}_4^4, \qquad (32)$$

$$\overline{F}_{l_3}^3 = [R]_4^3 \cdot \overline{F}_{l_4}^4 + \overline{F}_3^3, \qquad (33)$$

$$\overline{F}_{l_2}^2 = [R]_3^2 \cdot \overline{F}_{l_3}^3 + \overline{F}_2^2, \qquad (34)$$

$$\overline{F}_{l_1}^1 = \left[R \right]_2^1 \cdot \overline{F}_{l_2}^2 + \overline{F}_{l}^1, \qquad (35)$$

and the moments of the connection forces have the expressions:

$$\begin{split} \overline{\mathbf{M}}_{l_{0_{6}}}^{6} &= [\mathbf{R}]_{7}^{6} \cdot \overline{\mathbf{M}}_{l_{0_{7}}}^{7} + \overline{\mathbf{r}}_{c_{6}}^{6} \times \overline{\mathbf{F}}_{6}^{6} + \overline{\mathbf{r}}_{7}^{6} \times [\mathbf{R}]_{7}^{6} \cdot \overline{\mathbf{F}}_{l_{7}}^{7} + \overline{\mathbf{M}}_{c_{6}}^{6}, (36) \\ \\ \overline{\mathbf{M}}_{l_{0_{5}}}^{5} &= [\mathbf{R}]_{6}^{5} \cdot \overline{\mathbf{M}}_{l_{0_{6}}}^{6} + \overline{\mathbf{r}}_{c_{5}}^{5} \times \overline{\mathbf{F}}_{5}^{5} + \overline{\mathbf{r}}_{6}^{5} \times [\mathbf{R}]_{6}^{5} \cdot \overline{\mathbf{F}}_{l_{6}}^{6} + \overline{\mathbf{M}}_{c_{5}}^{5}, (37) \end{split}$$

$$\overline{\mathbf{M}}_{\mathbf{l}_{O_4}}^4 = [\mathbf{R}]_5^4 \cdot \overline{\mathbf{M}}_{\mathbf{l}_{O_5}}^5 + \overline{\mathbf{r}}_{c_4}^4 \times \overline{\mathbf{F}}_4^4 + \overline{\mathbf{r}}_5^4 \times [\mathbf{R}]_5^4 \cdot \overline{\mathbf{F}}_{\mathbf{l}_5}^5 + \overline{\mathbf{M}}_{c_4}^4,$$
(38)

$$\overline{\mathbf{M}}_{\mathbf{l}_{O_3}}^3 = [\mathbf{R}]_4^3 \cdot \overline{\mathbf{M}}_{\mathbf{l}_{O_4}}^4 + \overline{\mathbf{r}}_{c_3}^3 \times \overline{\mathbf{F}}_3^3 + \overline{\mathbf{r}}_4^3 \times [\mathbf{R}]_4^3 \cdot \overline{\mathbf{F}}_{\mathbf{l}_4}^4 + \overline{\mathbf{M}}_{c_3}^3, \quad (39)$$

$$\overline{\mathbf{M}}_{\mathbf{l}_{O_2}}^2 = [\mathbf{R}]_3^2 \cdot \overline{\mathbf{M}}_{\mathbf{l}_{O_3}}^3 + \overline{\mathbf{r}}_{\mathbf{c}_2}^2 \times \overline{\mathbf{F}}_2^2 + \overline{\mathbf{r}}_3^2 \times [\mathbf{R}]_3^2 \cdot \overline{\mathbf{F}}_{\mathbf{l}_3}^3 + \overline{\mathbf{M}}_{\mathbf{c}_2}^2, \quad (40)$$

$$\overline{\mathbf{M}}_{\mathbf{l}_{O_1}}^{1} = [\mathbf{R}]_2^1 \cdot \overline{\mathbf{M}}_{\mathbf{l}_{O_2}}^2 + \overline{\mathbf{r}}_{c_1}^1 \times \overline{\mathbf{F}}_1^1 + \overline{\mathbf{r}}_2^1 \times [\mathbf{R}]_2^1 \cdot \overline{\mathbf{F}}_{l_2}^2 + \overline{\mathbf{M}}_{c_1}^1, \quad (41)$$

giving complex equations, available at the reader's request to the email address of the first author.

According to [1] and [13], the generalized driving forces have the following computation formulae:

$$Q_{m}^{1} = \left[\overline{M}_{l_{0_{1}}}^{1}\right]^{T} \cdot \overline{k}_{1}^{1} = \left[M_{l_{x}}^{1} \quad M_{l_{y}}^{1} \quad M_{l_{z}}^{1}\right] \cdot \begin{bmatrix}0\\0\\1\end{bmatrix} = M_{l_{z}}^{1}, \quad (42)$$

$$Q_{m}^{2} = \left[\overline{M}_{l_{0_{2}}}^{2}\right]^{T} \cdot \overline{i}_{2}^{2} = \left[M_{l_{x}}^{2} \quad M_{l_{y}}^{2} \quad M_{l_{z}}^{2}\right] \cdot \begin{bmatrix}1\\0\\0\end{bmatrix} = M_{l_{x}}^{2}, \quad (43)$$

$$Q_{m}^{3} = \left[\overline{M}_{l_{0_{3}}}^{3}\right]^{T} \cdot \overline{i}_{3}^{3} = \left[M_{l_{x}}^{3} \quad M_{l_{y}}^{3} \quad M_{l_{z}}^{3}\right] \cdot \begin{bmatrix}1\\0\\0\end{bmatrix} = M_{l_{x}}^{3}, \quad (44)$$

$$Q_{m}^{4} = \left[\overline{M}_{l_{0_{4}}}^{4}\right]^{T} \cdot \overline{k}_{4}^{4} = \left[M_{l_{x}}^{4} \quad M_{l_{y}}^{4} \quad M_{l_{z}}^{4}\right] \cdot \begin{bmatrix}0\\0\\1\end{bmatrix} = M_{l_{z}}^{4}, \quad (45)$$

$$Q_{m}^{5} = \left[\overline{M}_{l_{0_{5}}}^{5}\right]^{T} \cdot \overline{i}_{5}^{5} = \left[M_{l_{x}}^{5} \quad M_{l_{y}}^{5} \quad M_{l_{z}}^{5}\right] \cdot \begin{bmatrix}1\\0\\0\end{bmatrix} = M_{l_{x}}^{5}, \quad (46)$$

$$Q_{m}^{6} = \left[\overline{M}_{l_{0_{6}}}^{6}\right]^{T} \cdot \overline{k}_{6}^{6} = \left[M_{l_{x}}^{6} \quad M_{l_{y}}^{6} \quad M_{l_{z}}^{6}\right] \cdot \begin{bmatrix}0\\0\\1\end{bmatrix} = M_{l_{z}}^{6} \cdot (47)$$

In this case of articulated robot, all the generalized driving forces have the significance of moments.

5. CONCLUSION

The dynamic study of articulated robots allows choosing the proper motors for their joints, as well as the optimal arrangement of modules in a modular robotic structure, such that the energy consumption to be minimum for a given task.

The obtaining of the dynamic equations was possible using the component *Robot_Dynamics* [3] from the toolbox *Robot_ Symbolic* [1], [8], [9], [10], [3], written in MATLAB.

6. FUTURE WORK

There are many possible ways of further development of this work. One of them is to study the errors in the 6R articulated industrial robot, by means of geometric, kinematic and dynamic operation precision [14].

Another way is to plan the motion of the robot, based on polynomial functions, having a given task in a given environment [15], [16], [17].

The latest research published in [18] - [21] opens a new approach in modeling the dynamics of the sudden motions in robotics and multibody systems, based on the acceleration energy.

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MODELUL DINAMIC AL ROBOTULUI INDUSTRIAL ARTICULAT 6R, FOLOSIND FORMALISMUL NEWTON-EULER

Rezumat: Articolul descrie procesul de obținere a ecuațiilor dinamice ale robotului industrial articulat 6R, utilizat în procese de sudură. Lucrarea utilizează ecuațiile modelului geometric și cinematic, determinate în prealabil. Scopul acestei analize este obținerea expresiilor forțelor generalizate ale robotului, reprezentând ecuațiile modelului dinamic invers.

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