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THE JACOBIAN MATRIX BASED ON THE TRANSFER MATRICES

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Abstract: *The main purpose of this paper is to present a mathematical model which can be applied to determine one of the most important differential matrix from robot kinematics also known as the Jacobian matrix or the velocities transfer matrix. The velocity of a robot link with respect to the previous link usually depends on the type of joint that connects them. The velocity of the end effector is a result of the contribution made by local velocities from each joint of the robot. The translation joint is characterized only by a linear velocity which is transferred to the end effector. In case of a rotation driving joint, both, angular velocity and linear velocity will be transferred at the end effector. In this paper is defined a* $(m \times n)$ *matrix called Jacobian matrix which establishes the mathematical relation between the velocities from each robot joint and the corresponding linear and angular velocities at a given point on the end-effector. Also, are presented the linear and angular transfer matrices based on which the Jacobian matrix is defined.*

Key words: *kinematics, mechanics, Jacobian matrix, transfer matrix, robotics.*

1. INTRODUCTION

Kinematics deals with the study of rigid bodies' motion without taking into account the effect of the force system that caused this motion. From kinematics point of view, any industrial robot can be defined as a mechanical system consisting of a series of rigid links connected by means of driving joints which can perform automatically various tasks involving motion [9]–[13]. The main feature of a robot is its capability to move in the tridimensional space characterized by six independent parameters which define the position and orientation of each driving joint. In case of an industrial robot, which is considered a system of rigid bodies, determining the appropriate kinematic models for the mechanical structure is essential in the analysis of its behavior. In robotics, the kinematical modeling of robot structures can be achieved by using either Cartesian coordinates or quaternions. The transformation between two Cartesian coordinate systems can be reduced to a rotation and a translation (around and along one of the axis of the coordinate system). The resultant rotation can be mathematically defined by means of Euler angles, Gibbs vector, orthonormal matrices, Pauli spin matrices or Hamilton's quaternions. Of all these possible representations, the orthonormal matrices or homogenous transformations based on (4×4) matrices are mostly used. The forward kinematical modeling consists in defining the position of the end-effector as a function of the generalized coordinates from each joint, while the inverse kinematical model refers to finding the generalized coordinates from the robot joint that define a certain configuration of the robot. An industrial robot is considered a complex system which from mechanical point of view can be modeled as system of rigid bodies. In this case we often have to transfer the velocities from a fixed reference system represented by $\{0\}$ to a mobile reference system $\{S\}$. The linear and angular velocity at the end-effector is computed by transferring velocities through the robot links. By computing and transferring linear and angular velocities from the fixed base to the end-effector, is established a relation between joint velocities and end effector velocities. This is an iterative method to compute the Jacobian matrix also known as the velocity transfer matrix.

This matrix is essential in the dynamic modeling of industrial robots.

2. THE TRANSFER MATRICES

2.1 The linear transfer matrices

In this section are analyzed the linear and angular velocities associated with multi-body systems (robot mechanical structures) and also the transfer matrices that characterize this linear velocities and accelerations. The linear velocity of the origin O_n of the mobile system $\{n\}$with respect to the fixed system $\{0\}$ is defined by:

$$^{0}\bar{v}_n=\left[\bar{V}_i\left[\bar{\theta}(t)\right]\cdot\dot{q}_i(t);\ i=1\rightarrow n\right]^T=V\left[\bar{\theta}(t)\right]\cdot\dot{\bar{\theta}}(t);\quad (1)$$

In the equation presented above, $V\left[\bar{\theta}(t)\right]$ is the transfer matrix of linear velocities, defined as a $(3\times n)$ matrix, where $V_i\left[\bar{\theta}(t)\right]$represents each column of the matrix, $i=1\rightarrow n$ and n defines the degrees of freedom of the analyzed structure:

$$\underset{(3\times n)}{V\left[\bar{\theta}(t)\right]}=\underset{(3\times n)}{Matrix}\left\{\underset{(i=1\rightarrow n)}{V_i\left[\bar{\theta}(t)\right]}=\frac{\partial\bar{p}_n\left[\bar{\theta}(t)\right]}{\partial q_i(t)}\right\}.\quad (2)$$

The motion from each driving joint is independent, so, the resultant motion of the end-effector can be obtained by applying the effects superposing principle. So, for each driving joint, $i=1\rightarrow n$, are applied the column vectors of the generalized variables, defines as following:

$$\underset{(n\times 1)}{\bar{\theta}(t)}=\left[q_1(t)\quad q_2(t)\quad \dots\quad q_i(t)\right]^T;\qquad (3)$$

$$\underset{(n\times 1)}{\dot{\bar{\theta}}(t)}=\left[\dot{q}_1(t)\quad \dot{q}_2(t)\quad \dots\quad \dot{q}_i(t)\right]^T;\qquad (4)$$

At the end effector is transferred a velocity which is due exclusively to the motion from the robot driving joint(i). Depending on the type of the driving joint, this velocity can be either a rotational velocity or a linear velocity:

$$\left\{\begin{array}{c}\bar{V}_i[\bar{\theta}(t)]\cdot\dot{q}_i(t)=(1-\Delta_i)\cdot\ ^{\bar{0}}k_i(t)\cdot\dot{q}_i(t)+\\ +\Delta_i\cdot\ ^{\bar{0}}k_i(t)\cdot\dot{q}_i(t)\times[\bar{p}_n(t)-\bar{p}_i(t)]\end{array}\right\};\quad (5)$$

where $$^{0}\bar{k}_i(t)={}^{0}_{i}[R](t)\cdot{}^{i}\bar{k}_i;\qquad (6)$$

$$\bar{p}_n(t)-\bar{p}_i(t)=\sum_{j=i}^{n-1}{}^{0}_{j}[R](t)\cdot{}^{j}p_{j+1j};\qquad (7)$$

By identifying the terms from (5) which are multiplied by the generalized velocities,$\dot{q}_i(t)$, can be obtained a new general expression that can be used to define each vector $\bar{V}_i$ from the componence of the linear transfer matrix:

$$\left\{\begin{array}{c}V_i\left[\bar{\theta}(t)\right]=\dfrac{\partial\bar{p}_n\left[\bar{\theta}(t)\right]}{\partial q_i(t)}=\left(1-\Delta_i\right)\cdot{}^{0}\bar{k}_i(t)+\\ +\Delta_i\cdot{}^{0}\bar{k}_i(t)\times\left[\bar{p}_n(t)-\bar{p}_i(t)\right]\end{array}\right\};(8)$$

The linear acceleration of the origin O_n of the mobile system$\{n\}$relative to the fixed system $\{0\}$, is determined by applying the second order time derivative on the position vector $\bar{p}_n(t)$:

$$\left\{\begin{array}{c}{}^{0}\dot{\bar{v}}_n=\sum_{i=1}^{n}\dfrac{\partial\bar{p}_n[\bar{\theta}(t)]}{\partial q_i(t)}\cdot\ddot{q}_i(t)+\\ +\sum_{i=1}^{n}\sum_{j=1}^{n}\dfrac{\partial^2\bar{p}_n[\bar{\theta}(t)]}{\partial q_i(t)\cdot\partial q_j(t)}\cdot\dot{q}_i(t)\cdot\dot{q}_j(t)\end{array}\right\};\quad (9)$$

The expression (9) can be reformulated based on (8), in the following form:

$$\left\{\begin{array}{c}{}^{0}\dot{\bar{v}}_n=\sum_{i=1}^{n}\bar{V}_i[\bar{\theta}(t)]\cdot\ddot{q}_i(t)+\\ +\sum_{i=1}^{n}\sum_{j=1}^{n}\dfrac{\partial\bar{V}_i[\bar{\theta}(t)]}{\partial q_j(t)}\cdot\dot{q}_i(t)\cdot\dot{q}_j(t)\end{array}\right\};(10)$$

$${}^{0}\dot{\bar{v}}_n=V[\bar{\theta}(t)]\cdot\ddot{\bar{\theta}}(t)+\dot{V}\left[\bar{\theta}(t);\dot{\bar{\theta}}(t)\right]\cdot\dot{\bar{\theta}}(t).\quad (11)$$

$$\underset{(3\times n)}{\dot{V}\left[\bar{\theta}(t);\dot{\bar{\theta}}(t)\right]}=\left[\underset{(i=1\rightarrow n)}{\dot{V}_i[\bar{\theta}(t)]}=\sum_{j=1}^{n}\frac{\partial V_i[\bar{\theta}(t)]}{\partial q_j(t)}\cdot\dot{q}_j(t)\right](12)$$

The expression(11), is rewritten by introducing a new matrix, symbolized $A(\bar{\theta})$, also known as the *transfer matrix of linear accelerations*.

$${}^{0}\dot{\bar{v}}_n=A\left[\bar{\theta}(t);\dot{\bar{\theta}}(t)\right]\cdot\begin{bmatrix}\ddot{\bar{\theta}}(t)\\ \dot{\bar{\theta}}(t)\end{bmatrix};\qquad (13)$$

The expression for this matrix is:

$$\underset{(3\times 2\cdot n)}{A\left[\bar{\theta}(t);\dot{\bar{\theta}}(t)\right]}=\left[\underset{(3\times n)}{V[\bar{\theta}(t)]}\quad \underset{(3\times n)}{\dot{V}\left[\bar{\theta}(t);\dot{\bar{\theta}}(t)\right]}\right];\quad (14)$$

where $V\left[\bar{\theta}(t)\right]$ and its derivative $\dot{V}\left[\bar{\theta}(t);\dot{\bar{\theta}}(t)\right]$ are $(3\times n)$ matrices, whose elements V_i and $\dot{V}_i$ are defined according to the expressions (8) and (12) respectively.

2.2 The angular transfer matrices

In this section are presented, according to [8] the angular transfer matrices corresponding to angular velocities and accelerations. First, the antisymmetric matrix associated to the absolute angular velocity of system $\{n\}$ is written as:

$$\left\{{}^{0}\bar{\omega}_n(t)\times\right\}={}^{0}_{n}\left[\dot{R}\right](t)\cdot{}^{0}_{n}[R]^T(t); \qquad (15)$$

In order to determine the angular velocity, the vector operator $vect\{\bar{\omega}\times\}$ is implemented:

$${}^{0}\bar{\omega}_n\left[\bar{\theta}(t);\dot{\bar{\theta}}_\Delta(t)\right]=\sum_{i=1}^{n}\bar{\Omega}_i(t)\cdot\dot{q}_i(t)\cdot\Delta_i; \qquad (16)$$

Within the expression was implemented a new matrix, known as the *transfer matrix of the angular velocities*, defined as a $(3\times n)$ matrix, and which can be written in the following form [7]:

$$\underset{(3\times n)}{\Omega}\left[\bar{\theta}_\Delta(t)\right]=\left\{\underset{(3\times 1)}{\Omega_i}\left[\bar{\theta}_\Delta(t)\right],\quad i=1\to n\right\}; \qquad (17)$$

where each component of the transfer matrix is:

$$\underset{(i=1\to n)}{\bar{\Omega}_i(t)}=vect\left\{{}^{0}_{n}[\dot{R}]\cdot{}^{0}_{n}[R]^T\right\}\cdot\Delta_i. \qquad (18)$$

Considering the principle of the superposing effects and by applying (17) and (18), on the end effector is transferred an angular velocity which appears as a consequence of the motion from the driving joint (i). Therefore, the expression for $\bar{\Omega}_i$ can be rewritten as follows:

$$\bar{\Omega}_i[\bar{\theta}_\Delta(t)]\cdot\dot{q}_i(t)-\Delta_i\cdot\dot{q}_i(t)\cdot{}^{0}\bar{k}_i(t); \qquad (19)$$

$$\bar{\Omega}_i\left[\bar{\theta}_\Delta(t)\right]\equiv\Delta_i\cdot{}^{0}\bar{k}_i(t)\equiv\Delta_i\cdot{}^{0}_{i}[R](t)\cdot{}^{i}\bar{k}_i. \qquad (20)$$

By applying the absolute time derivative on the expressions (15) or (16) is established the absolute angular acceleration which characterizes the motion of the $\{n\}$ system.

$$\left\{\begin{array}{c}{}^{0}\dot{\bar{\omega}}_n\left[\bar{\theta}_\Delta(t);\dot{\bar{\theta}}_\Delta(t);\ddot{\bar{\theta}}_\Delta(t)\right]=\\=\sum_{i=1}^{n}\bar{\Omega}_i(t)\cdot\ddot{q}_i(t)\cdot\Delta_i+\sum_{i=1}^{n}\dot{\bar{\Omega}}_i(t)\cdot\dot{q}_i(t)\cdot\Delta_i\end{array}\right\}; \qquad (21)$$

In the expression (21) is implemented a $(3\times 2\cdot n)$ matrix, denoted E and which is known as *the transfer matrix of angular accelerations*.

$$\left\{\begin{array}{c}{}^{0}\dot{\bar{\omega}}_n\left[\bar{\theta}_\Delta(t);\dot{\bar{\theta}}_\Delta(t);\ddot{\bar{\theta}}_\Delta(t)\right]=\\=E\left[\bar{\theta}_\Delta(t);\dot{\bar{\theta}}_\Delta(t)\right]\cdot\left[\ddot{\bar{\theta}}_\Delta^T(t)\quad\dot{\bar{\theta}}_\Delta^T(t)\right]^T\end{array}\right\}. \qquad (22)$$

This transfer matrix can be expressed as follows:

$$\left\{\begin{array}{c}\underset{(3\times 2\cdot n)}{E}\left[\bar{\theta}_\Delta(t);\dot{\bar{\theta}}_\Delta(t)\right]=\\\underset{(3\times 2\cdot n)}{Matrix}\left\{\underset{(i=1\to n)}{\Omega_i[\bar{\theta}_\Delta(t)]}\quad\underset{(i=1\to n)}{\dot{\Omega}_i\left[\bar{\theta}_\Delta(t);\dot{\bar{\theta}}_\Delta(t)\right]}\right\}\end{array}\right\}. \qquad (23)$$

The time derivative of the transfer matrix of angular accelerations from (23) is the following:

$$\left\{\begin{array}{c}\dot{\Omega}\left[\bar{\theta}_\Delta(t);\dot{\bar{\theta}}_\Delta(t)\right]={}^{0}\dot{\bar{k}}_i(t)\cdot\Delta_i=\\\underset{(3\times n)}{Matrix}\left\{\underset{(i=1\to n)}{\dot{\Omega}_i}=\sum_{j=1}^{n}\frac{\partial\bar{\Omega}_i(t)}{\partial q_j(t)}\cdot\dot{q}_j(t)\cdot\Delta_i\cdot\Delta_j\right\}\end{array}\right\}; \qquad (24)$$

where,

$$\left\{\begin{array}{c}\dot{\Omega}_i\left[\bar{\theta}_\Delta(t);\dot{\bar{\theta}}_\Delta(t)\right]=\\=\left\{{}^{0}\omega_n+\sum_{j=1}^{i}\Delta_j\cdot\dot{q}_j(t)\cdot{}^{0}\bar{k}_j(t)\right\}\times{}^{0}\bar{k}_i(t)\cdot\Delta_i\end{array}\right\}. \qquad (25)$$

The angular transfer matrices presented in this section are further applied in order to determine the absolute angular velocities and accelerations which characterize the equations of direct kinematical modeling (DKM).

3. THE JACOBIAN MATRIX

In this section is determined the expression of the Jacobian matrix and its time derivative by using the linear and angular transfer matrices. According to [4] and [7], the generalized velocities which defines the motion of the end effector, with projections on the fixed system are:

$$\underset{(6\times 1)}{{}^{0}\dot{X}} = \begin{bmatrix} {}^{0}\bar{v}_n \\ {}^{0}\bar{\omega}_n \end{bmatrix} = \begin{bmatrix} V(\bar{\theta}) \\ \Omega(\bar{\theta}) \end{bmatrix} \cdot \dot{\bar{\theta}}, i = 1 \to n; \quad (26)$$

Where the linear and angular transfer matrices are defined with expressions (2), (17) and (18) which are rewritten in another form, such as:

$$\left\{ \begin{array}{c} \underset{(3\times n)}{V[\bar{\theta}(t)]} = \\ \underset{(3\times n)}{Matrix} \left\{ \underset{(i=1\to n)}{V_i[\bar{\theta}(t)]} = \left\{ \frac{\partial}{\partial q_i(t)} \left[\begin{array}{c} f_j(q_k(t); k=1\to n) \\ j=1\to 3 \end{array} \right] \right\}^T \right\} \end{array} \right\}; (27)$$

$$\left\{ \begin{array}{l} \underset{(i=1\to n)}{V_i[\bar{\theta}(t)]} = \Delta_i \cdot {}^{0}\bar{k}_i(t) \times [\bar{p}_n(t) - \bar{p}_i(t)] + \\ \quad + \Delta_i \cdot {}^{0}\bar{k}_i(t) \times \bar{p}_{ni}(t) + (1-\Delta_i) \cdot {}^{0}\bar{k}_i(t) \end{array} \right\}; (28)$$

The angular transfer matrices are defined as:

$$\left\{ \begin{array}{l} \underset{(3\times n)}{\Omega[\bar{\theta}_\Delta(t)]} = Matrix\{\Omega_i[\bar{\theta}_\Delta(t)], i = 1 \to n\} = \\ = \left\{ vect \left\{ \frac{\partial}{\partial q_i(t)} \{{}^{0}_{n}[R](t)\} \cdot {}^{0}_{n}[R]^T(t) \right\} \cdot \Delta_i \right\} \end{array} \right\}; (29)$$

In the equation (26) is implemented the transfer matrix of linear velocities, thus resulting:

$$\underset{(6\times 1)}{{}^{0}\dot{X}} = \begin{bmatrix} {}^{0}J_v(\bar{\theta}) \\ {}^{0}J_\Omega(\bar{\theta}) \end{bmatrix} \cdot \dot{\bar{\theta}}; \quad (30)$$

Based on (30) is defined the Jacobian ${}^{0}J(\bar{\theta})$, whose components are: the linear motion Jacobian denoted with $J_v(\bar{\theta})$ and the angular motion Jacobian denoted ${}^{0}J_\Omega(\bar{\theta})$.

$$\left\{ {}^{0}J(\bar{\theta}) = \left[\frac{V(\bar{\theta})}{\Omega(\bar{\theta})} \right] = \left[\frac{{}^{0}J_v(\bar{\theta})}{{}^{0}J_\Omega(\bar{\theta})} \right] \right\}; \quad (31)$$

The matrices ${}^{0}J_v(\bar{\theta})$ and ${}^{0}J_\Omega(\bar{\theta})$ are defined as $(3\times n)$ matrices, each column (n) representing the linear and angular Jacobian corresponding to each robot joint. In order to define each column of the Jacobian, the following expression is used:

$$\left\{ {}^{0}J_i = \left[\frac{{}^{0}J_{iv}}{{}^{0}J_{i\Omega} \cdot \Delta_i} \right] = \left[\frac{\bar{V}_i}{\bar{\Omega}_i \cdot \Delta_i} \right] \right\}; \quad (32)$$

In the expression (32), ${}^{0}J_i$ represents the Jacobian matrix associated to each robot joint.

According to [8], the connection between the column vectors of the generalized and operational velocities respectively, can be achieved by means of the Jacobian matrix also known as the transfer matrix of linear velocities or the matrix of partial derivatives. The operational accelerations of the end effector with respect to the fixed system is defined as follows:

$$\underset{(6\times 1)}{{}^{0}\ddot{X}} = \begin{bmatrix} {}^{0}\dot{\bar{v}}_n \\ {}^{0}\dot{\bar{\omega}}_n \end{bmatrix} = \left\{ \begin{bmatrix} V(\bar{\theta}) \\ \Omega(\bar{\theta}) \end{bmatrix} \quad \begin{bmatrix} \dot{V}(\bar{\theta}) \\ \dot{\Omega}(\bar{\theta}) \end{bmatrix} \right\} \cdot \begin{bmatrix} \ddot{\bar{\theta}} \\ \dot{\bar{\theta}} \end{bmatrix} =$$

$$= \left\{ \begin{matrix} [V(\bar{\theta}) & \dot{V}(\bar{\theta})] \\ [\Omega(\bar{\theta}) & \dot{\Omega}(\bar{\theta})] \end{matrix} \right\} \cdot \begin{bmatrix} \ddot{\bar{\theta}} \\ \dot{\bar{\theta}} \end{bmatrix} = \begin{bmatrix} A(\bar{\theta}) \\ E(\bar{\theta}) \end{bmatrix} \cdot \begin{bmatrix} \ddot{\bar{\theta}} \\ \dot{\bar{\theta}} \end{bmatrix}; (33)$$

In the previous expression, is implemented the Jacobian matrix and its first time derivative, according to the following expression:

$$\left\{ \begin{bmatrix} A(\bar{\theta}) \\ E(\bar{\theta}) \end{bmatrix} = [{}^{0}J(\theta) \quad {}^{0}\dot{J}(\theta)] \right\}; \quad (34)$$

$${}^{0}J(\bar{\theta}) = \begin{bmatrix} {}^{0}J_v(\bar{\theta}) \\ {}^{0}J_\Omega(\bar{\theta}) \end{bmatrix}; \quad {}^{0}\dot{J}(\bar{\theta}) = \begin{bmatrix} {}^{0}\dot{J}_v(\bar{\theta}) \\ {}^{0}\dot{J}_\Omega(\bar{\theta}) \end{bmatrix}; \quad (35)$$

The expression (34) highlight the transfer matrices corresponding to linear and angular accelerations. The absolute time derivative of the Jacobian matrix, based on transfer matrices being defined as follows:

$$\left\{ \begin{array}{l} \underset{(6\times n)}{{}^{0}\dot{J}(\bar{\theta})} = \left[\frac{{}^{0}\dot{J}_v(\bar{\theta})}{{}^{0}\dot{J}_\Omega(\bar{\theta})} \right] = \left[\frac{\dot{V}(\bar{\theta})}{\dot{\Omega}(\bar{\theta})} \right] = \\ = \left\{ {}^{0}\dot{J}_i = \left[\frac{{}^{0}\dot{J}_{iv}}{{}^{0}\dot{J}_{i\Omega} \Delta_i} \right] = \left[\frac{\dot{\bar{V}}_i}{\dot{\bar{\Omega}}_i \Delta_i} \right] \quad i = 1 \to n \right\} \end{array} \right\}; \quad (36)$$

According to [5], the operational kinematic parameters which are expressing the absolute motion of the end effector can be projected on the $\{n\}$ mobile reference system.

In this situation, from the kinematical modeling, according to [6] and [9] the equations are used:

$${}^{n}\dot{X} = \begin{bmatrix} {}^{n}\bar{v}_n \\ {}^{n}\bar{\omega}_n \end{bmatrix} = {}^{n}R \cdot \begin{bmatrix} {}^{0}\bar{v}_n \\ {}^{0}\bar{\omega}_n \end{bmatrix} = {}^{n}R \cdot {}^{0}J(\bar{\theta}) \cdot \dot{\bar{\theta}}; \quad (37)$$

$$ {}^{n}\ddot{X} = \begin{bmatrix} {}^{n}\dot{v}_n \\ {}^{n}\dot{\omega}_n \end{bmatrix} = {}^{n}R \cdot \begin{bmatrix} {}^{0}\dot{v}_n \\ {}^{0}\dot{\omega}_n \end{bmatrix} = {}^{n}R \cdot \begin{bmatrix} {}^{0}J(\bar{\theta}) & {}^{0}\dot{J}(\bar{\theta}) \end{bmatrix} \cdot \begin{bmatrix} \ddot{\bar{\theta}} \\ \dot{\bar{\theta}} \end{bmatrix} \quad (38) $$

where, $$ {}^{n}R = \left[\begin{array}{c|c} {}^{0}_{n}[R]^{-1} & [0] \\ \hline [0] & {}^{0}_{n}[R]^{-1} \end{array} \right]; \quad (39) $$

$$ {}^{n}J(\bar{\theta}) = {}^{n}R \cdot {}^{0}J(\bar{\theta}); \quad (40) $$

$$ {}^{n}\dot{J}(\bar{\theta}) = {}^{n}R \cdot {}^{0}\dot{J}(\bar{\theta}); \quad (41) $$

In the expression (39), ${}^{n}R$ represents the matrix transfer operator which is used to achieve the transfer of the Jacobian matrix from the fixed system $\{0\}$ to the $\{S\}$ mobile system.

According to the main aspects presented in this section, the expressions of the Jacobian matrix based on transfer matrices are used for establishing the forward kinematics equations. These equations are essential in modeling the dynamics of any mechanical system.

4. APPLICATION

In the following is considered a RTR type mechanical robot structure on which is applied the previously presented mathematical model.

The RTR structure is defined in the initial configuration by the following positions and orientations of the kinematic joints included in the matrix of nominal geometry $M_{vn}^{(0)}$:

$M_{vn}^{(0)} \in RRT$

Table 1

Link $i=1 \to 4$	Joint type	${}^{i-1}\bar{p}_{ii-1}^{(0)T}$			$\bar{k}_i^{(0)T}$		
1	R	0	0	l_0	0	0	1
2	R	0	l_1	0	0	1	0
3	T	0	0	l_2	0	0	1
4	-	0	0	l_3	0	1	0

The homogenous transformations between the mobile systems $\{i\}$ and $\{i-1\}$, in a certain configuration (different from the initial configuration), are defined by:

$$ {}^{0}_{1}[T] = \left[\begin{array}{ccc|c} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & l_0 \\ \hline 0 & 0 & 0 & 1 \end{array} \right]; \quad {}^{1}_{2}[T] = \left[\begin{array}{ccc|c} cq_2 & 0 & sq_2 & 0 \\ 0 & 1 & 0 & l_1 \\ -sq_2 & 0 & cq_2 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array} \right]; $$

$$ {}^{2}_{3}[T] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_2+q_3 \\ \hline 0 & 0 & 0 & 1 \end{array} \right]; \quad {}^{3}_{4}[T] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ \hline 0 & 0 & 0 & 1 \end{array} \right]. $$

The homogenous transformations between the mobile systems $\{i\}$ and fixed system $\{0\}$, are:

$$ \left\{ \begin{array}{l} {}^{0}_{2}[T] = {}^{0}_{1}[T] \cdot {}^{1}_{2}[T] = \\ = \left[\begin{array}{ccc|c} cq_1 \cdot cq_2 & -sq_1 & cq_1 \cdot sq_2 & -l_1 \cdot sq_1 \\ sq_1 \cdot cq_2 & cq_1 & sq_1 \cdot sq_2 & l_1 \cdot cq_1 \\ -sq_2 & 0 & cq_2 & l_0 \\ \hline 0 & 0 & 0 & 1 \end{array} \right] \end{array} \right\}; $$

$$ \left\{ \begin{array}{l} {}^{0}_{3}[T] = {}^{0}_{2}[T] \cdot {}^{2}_{3}[T] = \\ = \left[\begin{array}{ccc|c} cq_1 \cdot cq_2 & -sq_1 & cq_1 \cdot sq_2 & cq_1 \cdot sq_2 \cdot (l_2+q_3) - l_1 \cdot sq_1 \\ sq_1 \cdot cq_2 & cq_1 & sq_1 \cdot sq_2 & sq_1 \cdot sq_2 \cdot (l_2+q_3) + l_1 \cdot cq_1 \\ -sq_2 & 0 & cq_2 & l_0 + cq_2 \cdot (l_2+q_3) \\ \hline 0 & 0 & 0 & 1 \end{array} \right] \end{array} \right\}. $$

According to [6] and [8], and expressions (28), (29), (31) and (32), presented in the previous section, the components of the angular and linear Jacobian, based on transfer matrices in case of the RTR robot structure are computed as follows:

$$ {}^{0}J_{\Omega}(\bar{\theta}) = [\Omega_1 \;\; \Omega_2 \;\; \Omega_3]; \quad (42) $$

where, $$ {}^{0}J_{\Omega}(\bar{\theta}) = \begin{bmatrix} 0 & sq_1 & 0 \\ 0 & cq_1 & 0 \\ 1 & 0 & 0 \end{bmatrix}; \quad (43) $$

$$ {}^{0}J_{v}(\bar{\theta}) = [V_1 \;\; V_2 \;\; V_3]; \quad (44) $$

$$ V_1 = \begin{bmatrix} -l_1 \cdot cq_1 - (l_2+q_3) \cdot sq_1 \cdot sq_2 \\ -l_1 \cdot sq_1 - (l_2+q_3) \cdot cq_1 \cdot sq_2 \\ 0 \end{bmatrix}; \quad (45) $$

$$ V_2 = \begin{bmatrix} (l_2+q_3) \cdot cq_1 \cdot cq_2 \\ (l_2+q_3) \cdot sq_1 \cdot cq_2 \\ -(l_2+q_3) \cdot sq_2 \end{bmatrix}; \; V_3 = \begin{bmatrix} cq_1 \cdot sq_2 \\ sq_1 \cdot sq_2 \\ cq_2 \end{bmatrix}. \quad (46) $$

$$ {}^{0}J(\bar{\theta}) = \left[\begin{array}{c} V(\bar{\theta}) \\ \hline \Omega(\bar{\theta}) \end{array} \right] = \left[\begin{array}{c} {}^{0}J_{v}(\bar{\theta}) \\ \hline {}^{0}J_{\Omega}(\bar{\theta}) \end{array} \right] $$

Substituting (45) and (46) in (44) and the result and (43) into the expression (31), it is obtained the Jacobian matrix with respect to fixed system ${}^{0}J(\bar{\theta})$, which in case of a RTR robot is defined

by a (6×3) matrix whose components are represented by the angular and linear transfer matrices, $\Omega(\bar{\theta})$ and $V(\bar{\theta})$, respectively.
According to the expression (30), defined in the previous section, in the following is determined the column vector of operational velocities with respect to the fixed system $\{0\}$ representing the absolute velocities at the end - effector.
For the robot structure defined according to *Table 1*, this vector is defined for $i=1\rightarrow 3$ as:

$$\underset{(6\times 1)}{{}^{0}\dot{X}} = \begin{bmatrix} \bar{v}_3 \\ \bar{\omega}_3 \end{bmatrix} = \begin{bmatrix} {}^{0}J_v(\bar{\theta}) \\ {}^{0}J_\Omega(\bar{\theta}) \end{bmatrix} \cdot \dot{\bar{\theta}}.$$

The absolute linear velocity at the end effector:

where $\quad \bar{v}_3 = \begin{bmatrix} v_{3x} & v_{3y} & v_{3z} \end{bmatrix}^T$; (47)

$$\begin{Bmatrix} v_{3x} - \dot{q}_3 \cdot cq_1 \cdot sq_2 - \dot{q}_1 \cdot l_1 \cdot cq_1 + \\ +\dot{q}_1 \cdot (l_2 + q_3) \cdot sq_1 \cdot sq_2 + \dot{q}_2 \cdot (l_2 + q_3) \cdot cq_1 \cdot cq_2 \end{Bmatrix}; \quad (48)$$

$$\begin{Bmatrix} v_{3y} = \dot{q}_3 \cdot sq_1 \cdot sq_2 - \dot{q}_1 \cdot l_1 \cdot sq_1 + \\ +\dot{q}_1 \cdot (l_2 + q_3) \cdot cq_1 \cdot sq_2 + \dot{q}_2 \cdot (l_2 + q_3) \cdot sq_1 \cdot cq_2 \end{Bmatrix}; \quad (49)$$

$$\{v_{3z} = \dot{q}_3 \cdot cq_2 - \dot{q}_2 \cdot (l_2 + q_3) \cdot sq_2\}. \quad (50)$$

The angular velocity of the end effector, with projections on the $\{0\}$ fixed system are defined:

$$\bar{\omega}_3 = \begin{bmatrix} \omega_{3x} \\ \omega_{3y} \\ \omega_{3z} \end{bmatrix} = \begin{bmatrix} -\dot{q}_2 \cdot sq_1 \\ \dot{q}_2 \cdot cq_1 \\ \dot{q}_1 \end{bmatrix}; \quad (51)$$

Based on the expressions (31), (37) respectively (43) - (46) we can also define the operational kinematic parameters which are expressing the relative motion of the end effector. In case of the RTR robot and for $i=1\rightarrow 3$, the column vector of the operational velocities, projected on $\{n\}$ mobile reference system, ${}^{n}\dot{X}$ is computed as:

$${}^{n}\dot{X} = {}^{n}R \cdot {}^{0}J(\bar{\theta}) \cdot \dot{\bar{\theta}} = [3\bar{v}_3 \quad 3\omega_3]^T.$$

where, $\quad \dot{\bar{\theta}} = [\dot{q}_1 \quad \dot{q}_2 \quad \dot{q}_3]^T$; (52)

and ${}^{n}R$ is defined according to (39), where:

$${}^{0}_{n}[R]^{-1} = {}^{0}_{3}[R]^{-1} = \begin{bmatrix} cq_1 \cdot cq_2 & -sq_1 & cq_1 \cdot sq_2 \\ sq_1 \cdot cq_2 & cq_1 & sq_1 \cdot sq_2 \\ -sq_2 & 0 & cq_2 \end{bmatrix}; \quad (53)$$

In case of the analyzed RTR robot structure, the components of the column vector of operational velocities with respect to the mobile system are:

where, $\dot{\bar{\theta}} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}$; and $\bar{\omega}_3 = \begin{bmatrix} \dot{q}_1 \cdot sq_2 \\ \dot{q}_2 \\ \dot{q}_1 \cdot cq_2 \end{bmatrix}$; (54)

$$\bar{v}_3 = \begin{bmatrix} \dot{q}_2 \cdot l_2 + q_3 \cdot \dot{q}_2 - l_1 \cdot \dot{q}_1 \cdot cq_2 \\ \dot{q}_1 \cdot (l_2 + q_3) \cdot sq_2 \\ \dot{q}_3 - l_1 \cdot \dot{q}_1 \cdot sq_2 \end{bmatrix}. \quad (55)$$

In the following are determined the expressions which define the transfer matrices corresponding to linear and angular accelerations. For this reason, in case of the RTR robot and according to (36) is determined the absolute time derivative of the Jacobian matrix, based on transfer matrices. The components of this matrix are computed with:

$$\underset{(6\times n)}{{}^{0}\dot{J}(\bar{\theta})} = \begin{bmatrix} {}^{0}\dot{J}_v(\bar{\theta}) \\ \hline {}^{0}\dot{J}_\Omega(\bar{\theta}) \end{bmatrix} = \begin{bmatrix} \dot{V}(\bar{\theta}) \\ \hline \dot{\Omega}(\bar{\theta}) \end{bmatrix};$$

For $i=1\rightarrow 3$, the components of the absolute time derivative of the linear Jacobian are:

$${}^{0}\dot{J}_v(\bar{\theta}) = [\dot{V}_1 \quad \dot{V}_2 \quad \dot{V}_3]; \quad (56)$$

where, for the considered RTR robot structure and for $i=1\rightarrow 3$, the linear transfer matrices are:

$$\dot{V}_1 = \begin{bmatrix} \dot{q}_1 \cdot l_1 \cdot sq_1 - \dot{q}_1 \cdot (l_2 + q_3) \cdot cq_1 \cdot sq_2 - \\ -\dot{q}_2 \cdot (l_2 + q_3) \cdot sq_1 \cdot cq_2 - \dot{q}_3 \cdot sq_1 \cdot sq_2 \\ -\dot{q}_1 \cdot l_1 \cdot cq_1 - \dot{q}_1 \cdot (l_2 + q_3) \cdot sq_1 \cdot sq_2 + \\ +\dot{q}_2 \cdot (l_2 + q_3) \cdot cq_1 \cdot cq_2 + \dot{q}_3 \cdot cq_1 \cdot sq_2 \\ 0 \end{bmatrix}; \quad (57)$$

$$\dot{V}_2 = \begin{bmatrix} -\dot{q}_1 \cdot (l_2 + q_3) \cdot sq_1 \cdot cq_2 - \\ -\dot{q}_2 \cdot (l_2 + q_3) \cdot cq_1 \cdot sq_2 + \dot{q}_3 \cdot cq_1 \cdot cq_2 \\ \dot{q}_1 \cdot (l_2 + q_3) \cdot cq_1 \cdot cq_2 - \\ \dot{q}_2 \cdot (l_2 + q_3) \cdot sq_1 \cdot sq_2 + \dot{q}_3 \cdot sq_1 \cdot cq_2 \\ -\dot{q}_3 \cdot sq_2 - \dot{q}_2 \cdot (l_2 + q_3) \cdot cq_2 \end{bmatrix}; \quad (58)$$

$$\dot{V}_3 = \begin{bmatrix} -\dot{q}_1 \cdot sq_1 \cdot sq_2 + \dot{q}_2 \cdot cq_1 \cdot cq_2 \\ \dot{q}_1 \cdot cq_1 \cdot sq_2 + \dot{q}_2 \cdot sq_1 \cdot cq_2 \\ -\dot{q}_2 \cdot sq_2 \end{bmatrix}. \quad (59)$$

According to expression (33) can be also computed the operational accelerations of the end effector with respect to the fixed system:

$$\underset{(6\times 1)}{\ddot{\delta}_X} = \begin{bmatrix} A(\bar{\theta}) \\ E(\bar{\theta}) \end{bmatrix} \cdot \begin{bmatrix} \ddot{\bar{\theta}} \\ \dot{\bar{\theta}} \end{bmatrix} = \begin{bmatrix} \dot{v}_n \\ \dot{\omega}_n \end{bmatrix};$$

where, $A(\bar{\theta}) = [V(\bar{\theta}) \quad \dot{V}(\bar{\theta})]$;

and $E(\bar{\theta}) = [\Omega(\bar{\theta}) \quad \dot{\Omega}(\bar{\theta})]$;

So, for $n=3$ and for the considered robot structure, the angular and linear accelerations at the end effector, based on transfer matrices are:

$$\dot{v}_3 = [\dot{v}_{3x} \quad \dot{v}_{3y} \quad \dot{v}_{3z}]^T; \tag{60}$$

where,

$$\left\{\begin{matrix} \dot{v}_{3x} = \dot{q}_3 \cdot (\dot{q}_2 \cdot cq_1 \cdot cq_2 - \dot{q}_1 \cdot sq_1 \cdot sq_2) - \\ -\dot{q}_1 \cdot \dot{q}_3 \cdot sq_1 \cdot sq_2 + \dot{q}_2 \cdot \dot{q}_3 (l_2 + q_3) \cdot sq_1 \cdot cq_2 - \\ -\dot{q}_1^2 \cdot (l_1 \cdot sq_1 - (l_2+q_3) \cdot cq_1 \cdot sq_2) - \ddot{q}_1 \cdot l_1 \cdot cq_1 - \\ -\ddot{q}_1 (l_2+q_3) \cdot sq_1 \cdot sq_2 - \dot{q}_2 \cdot \dot{q}_1 \cdot (l_2+q_3) \cdot sq_1 \cdot cq_2 + \\ +\dot{q}_2^2 \cdot (l_2+q_3) \cdot cq_1 \cdot sq_2 - \dot{q}_1 \cdot \dot{q}_2 \cdot (l_2+q_3) \cdot cq_1 \cdot cq_2 + \\ +\ddot{q}_2 (l_2+q_3) \cdot cq_1 \cdot cq_2 + \ddot{q}_3 \cdot cq_1 \cdot sq_2 \end{matrix}\right\}; \tag{61}$$

$$\left\{\begin{matrix} \dot{v}_{3y} = \dot{q}_3 \cdot (\dot{q}_2 \cdot sq_1 \cdot cq_2 + \dot{q}_1 \cdot cq_1 \cdot sq_2) + \\ +\dot{q}_1 \cdot \dot{q}_3 \cdot cq_1 \cdot sq_2 - \dot{q}_2 \cdot \dot{q}_3 \cdot (l_2+q_3) \cdot cq_1 \cdot cq_2 - \\ -\dot{q}_1^2 \cdot (l_1 \cdot cq_1 + (l_2+q_3) \cdot sq_1 \cdot sq_2) - \ddot{q}_1 \cdot l_1 \cdot sq_1 - \\ -\ddot{q}_1 (l_2+q_3) \cdot cq_1 \cdot sq_2 + \dot{q}_2 \cdot \dot{q}_1 \cdot (l_2+q_3) \cdot cq_1 \cdot cq_2 - \\ -\dot{q}_2^2 \cdot (l_2+q_3) \cdot sq_1 \cdot sq_2 + \dot{q}_1 \cdot \dot{q}_2 \cdot sq_1 \cdot cq_2 \cdot (l_2+q_3) + \\ +\ddot{q}_2 (l_2+q_3) \cdot sq_1 \cdot cq_2 + \ddot{q}_3 \cdot sq_1 \cdot sq_2 \end{matrix}\right\}; \tag{62}$$

$$\left\{\begin{matrix} \dot{v}_{3z} = \ddot{q}_2 \cdot cq_2 - \dot{q}_2 \cdot \dot{q}_3 \cdot sq_2 - \\ -\dot{q}_2^2 \cdot (l_2 + q_3) \cdot cq_2 - \dot{q}_2 \cdot \dot{q}_3 \cdot sq_2 - \\ -\ddot{q}_2 \cdot (l_2 + q_3) \cdot sq_2 \end{matrix}\right\}; \tag{63}$$

The angular accelerations at the end-effector are defined for the same robot structure according to the following matrix relations:

$$\dot{\omega}_3 = [\dot{\omega}_{3x} \quad \dot{\omega}_{3y} \quad \dot{\omega}_{3z}]^T; \tag{64}$$

$$\left\{\begin{matrix} \dot{\omega}_{3x} = -\dot{q}_1 \cdot \dot{q}_3 \cdot sq_1 \cdot sq_2 + \dot{q}_2 \cdot \dot{q}_3 \cdot cq_1 \cdot cq_2 - \\ -\ddot{q}_2 \cdot sq_1 - \dot{q}_1 \cdot \dot{q}_2 \cdot cq_1 \end{matrix}\right\}; \tag{65}$$

$$\left\{\begin{matrix} \dot{\omega}_{3y} = -\dot{q}_1 \cdot \dot{q}_3 \cdot cq_1 \cdot sq_2 + \dot{q}_2 \cdot \dot{q}_3 \cdot sq_1 \cdot cq_2 + \\ +\ddot{q}_2 \cdot cq_1 - \dot{q}_1 \cdot \dot{q}_2 \cdot sq_1 \end{matrix}\right\}; \tag{66}$$

$$\{\dot{\omega}_{3y} = \ddot{q}_1 - \dot{q}_2 \cdot \dot{q}_3 \cdot sq_2\}. \tag{66}$$

The linear and angular accelerations at the end-effector, with respect to the mobile system $\{n\}$, are determined based on expressions (38), which applied for the above presented RTR robot structure, lead to the following results:

$${}^{3}\dot{v}_3 = \begin{bmatrix} \ddot{q}_2 \cdot (l_2 + q_3) - 2 \cdot \dot{q}_2 \cdot \dot{q}_3 - \\ -(l_2 \cdot \dot{q}_1^2 \cdot sq_2 + l_1 \cdot \ddot{q}_1) \cdot cq_2 \\ \ddot{q}_1 \cdot (l_2 + q_3) \cdot sq_2 - l_1 \cdot \dot{q}_1^2 + \\ +2 \cdot \dot{q}_1 \cdot \dot{q}_3 \cdot sq_2 + 2 \cdot \dot{q}_1 \cdot \dot{q}_3 \cdot (l_2 + q_3) \cdot cq_2 \\ \ddot{q}_3 - (l_2 + q_3) \cdot \dot{q}_2^2 - l_1 \cdot \ddot{q}_1 \cdot sq_2 - \\ -\dot{q}_1^2 \cdot (l_2 + q_3) \cdot s^2(q_2) \end{bmatrix}; \tag{67}$$

$${}^{3}\dot{\omega}_3 = \begin{bmatrix} \ddot{q}_1 \cdot sq_2 - \dot{q}_1 \cdot \dot{q}_2 \cdot cq_2 + \dot{q}_2 \cdot \dot{q}_3 \\ \ddot{q}_2 + \dot{q}_1 \cdot \dot{q}_3 \cdot sq_2 \\ \ddot{q}_1 \cdot cq_2 - \dot{q}_1 \cdot \dot{q}_2 \cdot sq_2 \end{bmatrix}; \tag{68}$$

5. CONCLUSIONS

This paper is devoted to present a mathematical model which can be applied to determine one of the most important differential matrix from robot kinematics known as Jacobian matrix or the velocities transfer matrix. The velocity of a robot link with respect to the previous link usually depends on the type of joint that connects them. The velocity of the end effector is a result of the contribution made by local velocities from each joint of the robot. The motion of a prismatic joint is defined only by a linear velocity which is transferred to the end effector. For a rotation joint, both, angular velocity and linear velocity will be transferred at the end effector. In this paper is defined a $(m \times n)$ matrix called Jacobian matrix which establishes the mathematical relation between the velocities from each robot joint and the corresponding linear and angular velocities at a given point on the end-effector. Also, are presented the linear and angular transfer matrices based on which the Jacobian matrix is defined.

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MATRICEA JACOBIANĂ BAZATĂ PE MATRICELE DE TRANSFER

Rezumat: *Obiectivul acestei lucrări este de a prezenta un model matematic care poate fi aplicat pentru determinarea uneia dintre cele mai importante matrice diferențiale din cinematica și dinamica roboților, cunoscută sub numele de matricea Jacobiană sau matricea de transfer a vitezelor. Viteza unui element cinetic față de elementul cinetic anterior, depinde în principal de tipul cuplei motoare care realizează legătura dintre cele două elemente cinetice. În cazul cuplelor prismatice, viteza liniară a unui element față de elementul anterior este orientată de-a lungul axei de translație în timp ce pentru cuplele de rotație, viteza unghiulară se realizează în jurul axei de rotație. Viteza efectorului final depinde de vitezele din fiecare cuplă a robotului. Cuplele de translație sunt caracterizate doar de viteză liniară care este transferată efectorului final. Pentru cupelele de rotație, la efectorul final vor fi transferate atât viteza de rotație cât și viteza liniară din fiecare cuplă a robotului. În cadrul acestei lucrări a fost definirea unei matrice, de dimensiuni* $(m\times n)$ *cunoscută în literatura de specialitate sub numele de matricea Jacobiană al cărei rol constă în stabilirea unei relații matematice între vitezele din fiecare cuplă a robotului și vitezele liniare și unghiulare corespunzătoare unei poziții ocupate de efectorul final, în spațiul de lucru, la un moment dat în timpul mișcării. De asemenea au fost prezentate și matricele de transfer a vitezelor și accelerațiilor liniare și unghiulare precum și legătura care există între acestea și matricea Jacobiană. În final modelul teoretic prezentat a fost aplicat pentru o structură de robot de tip RTR.*
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