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THE JACOBIAN MATRIX BASED ON DIFFERENTIAL MATRICES

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Abstract: *The present paper aims to present a mathematical model of determining the Jacobian matrix by using the differential matrices of the homogenous transformations. The expressions for the Jacobian matrix were developed using the differential matrices of first and second order and are obtained by differentiating the expressions for the locating matrices that define the transformations between the systems. Based on these, are determined the expressions for the Jacobian matrix with projections on the fixed and mobile system. The role of Jacobian matrix is to establish the connection between the generalized velocities and operational velocities, both defining the forward kinematics equations for any robot mechanical structure.*
Key words: *kinematics, Jacobian matrix, robotics, kinematics, differential matrix.*

1. INTRODUCTION

In the kinematical modeling the Jacobian matrix plays an essential role in the process of robot control and analysis. The computation of this matrix is required when planning smooth trajectories, determining the singularities, transforming the control space, deriving the dynamics equations of motion or in designing the control schemes. The Jacobian-based algorithms are used to solve various kinematic control problems that emerge in case of serial or parallel robots, cooperative multi-arm systems, or in case of different robot structures used in special applications (underwater robots, spacecraft systems or flexible manipulators).

The kinematic control of any robot structure consists in solving the motion control problem. There are two approaches in solving this problem. The first approach refers to applying the inverse kinematical model for transforming the desired trajectory of the end-effector into the corresponding joint trajectories, which will represent the reference inputs for some joint space control scheme [3]. The second approach consists in handling the manipulator outside the control loop, by this allowing that the singularities and redundancies corresponding to the robot structure to be solved apart from the motion control problem [3], [4]. The key point of kinematic control is the solution to the inverse kinematics problem.

In case of a serial robot structure, the differential kinematic equations are established using geometric Jacobian. This matrix is usually obtained by following a geometric method which consists in computing the intake of each joint velocity to the linear and angular velocities at the end-effector [5]. In the scientific literature [2] - [22] is made a clear distinction between the geometric Jacobian and the analytical Jacobian. The geometric Jacobian is commonly used when physical quantities are of concern, while the analytical Jacobian is adopted when task space quantities are involved. In the kinematic modeling is possible to switch between one Jacobian to the other, excepting the case when a singularity case is reported. The parametric errors lead to Jacobian error which further causes velocity error in the Cartesian space. This is a result of the fact that the Jacobian matrix is a function of joint variables and it comprises the kinematical parameters.

The present paper aims to present a mathematical model of determining the Jacobian matrix by using the differential matrices of the homogenous transformations. The expressions for the Jacobian, based on the differential matrices of first order, are determined with respect to fixed and mobile systems.

2. MATRIX DIFFERENTIAL OPERATOR

This section aims to establishing a matrix operator which is meant to replace the partial derivative ($\partial q / \partial q_i;\ i = 1 \rightarrow n$) in case of any homogenous transformation matrix. The use of this matrix operator in the kinematic and dynamic modeling of robot structures, leads to the generalization of the matrix and differential calculus. According to [10] - [15], the locating (position and orientation) of two systems $\{i\}$ and $\{i-1\}$ attached in the geometrical center of two adjoined robot joints, can be defined by means of the homogenous transformation matrices (also known as locating matrices):

$$ {}^{i-1}_{i}[T]\left[\tau_i \cdot q_i(t)\right] = \begin{bmatrix} {}^{i-1}_{i}[R] & {}^{i-1}\overline{p}_{ii-1} \\ 0\ 0\ 0 & 1 \end{bmatrix}; \qquad (1) $$

where,

$$ {}^{i-1}_{i}[R] = R_{ii-1} \cdot R\left[\overline{k}_i; \tau_i \cdot q_i(t) \cdot \Delta_i\right]; \qquad (2) $$

$$ {}^{i-1}\overline{p}_{ii-1} = {}^{i-1}\overline{p}^{(0)}_{ii-1} + \left(1 - \Delta_i\right) \cdot \tau_i \cdot q_i(t) \cdot {}^{i-1}\overline{k}_i; \qquad (3) $$

In the expressions (2) and (3) the matrix R_{ii-1} and ${}^{i-1}\overline{p}^{(0)}_{ii-1}$ characterize the initial configuration of the robot, while $\tau_i = \pm 1$ expresses the sign of the generalized coordinate with respect to the versor of the driving axis.

According to [10] - [16], the differential of the homogenous transformation is defined with the following expression:

$$ \left\{ \begin{array}{l} dT_{ii-1}(t) = \dfrac{\partial\left\{ {}^{i-1}_{i}[T]\left[q_i(t)\right]\right\}}{\partial q_i(t)} \cdot dq_i(t) = \\ = \left\{ \begin{array}{l} U_i \cdot {}^{i-1}_{i}[T](t) \cdot dq_i(t) \text{L} \ \ for\ {}^{i-1}\overline{k}_i \\ {}^{i-1}_{i}[T](t) \cdot U_i \cdot dq_i(t)\ \text{L} \ \ for\ {}^{i}\overline{k}_i \end{array} \right\} \end{array} \right\}; \quad (4) $$

In the expression above presented, the partial derivative of the homogenous transformation between $\{i\}$ and $\{i-1\}$ reference systems is:

$$ \left\{ \begin{array}{l} \dfrac{\partial}{\partial q_i(t)}\left\{ {}^{i-1}_{i}[T]\left[q_i(t)\right]\right\} = \\ \left\{ \begin{array}{l} {}^{i-1}_{i}[T] \cdot \left\{ {}^{i-1}_{i}[T]^{-1} \cdot U_i \cdot {}^{i-1}_{i}[T]\right\} \ldots for\ q_i(t) \cdot {}^{i-1}\overline{k}_i \\ \left\{ {}^{i-1}_{i}[T] \cdot U_i \cdot {}^{i-1}_{i}[T]^{-1}\right\} \cdot {}^{i-1}_{i}[T] \ldots for\ q_i(t) \cdot {}^{i}\overline{k}_i \end{array} \right\} \end{array} \right\}; (5) $$

$$ \left\{ \begin{array}{l} {}^{i-1}_{i}[T](t) \cdot U_i \cdot {}^{i-1}_{i}[T]^{-1}(t) = \text{const.} \\ = T_{ii-1}(t) \cdot U_i \cdot T^{-1}_{ii-1}(t) \equiv T^{(0)}_{ii-1} \cdot U_i \cdot \left\{T^{(0)}_{ii-1}\right\}^{-1} \end{array} \right\}; \quad (6) $$

$$ \left\{ \begin{array}{c} U_{i\Delta}(\overline{z}_i) \\ \left\{U_{iT}(\overline{z}_i); U_{iR}(\overline{z}_i)\right\} \end{array} \right\} = \left\{ \begin{array}{l} \tau_i \cdot \left\{ \begin{array}{cc} \{\overline{z}_i \times\} \cdot \Delta_i & \overline{z}_i \cdot (1 - \Delta_i) \\ 0\ \ 0\ \ 0 & 0 \end{array} \right\} \\ \Delta_i = \left\{ \{1; i = R\}\ \ \{0; i = T\} \right\} \end{array} \right\}; (7) $$

$$ \left\{ \left\{ \begin{array}{c} U_{i\Delta}(\overline{z}_i) \\ \left\{U_{iT}(\overline{z}_i); U_{iR}(\overline{z}_i)\right\} \end{array} \right\} = \tau_i \cdot \begin{bmatrix} 0 & -\Delta_i & 0 & 0 \\ \Delta_i & 0 & 0 & 0 \\ 0 & 0 & 0 & (1 - \Delta_i) \\ 0 & 0 & 0 & 0 \end{bmatrix} \right\}; (8) $$

According to (4) and (6), the matrix U_i which is defined with (7) and (8) substitute the classical partial derivative $\partial q / \partial q_i;\ i = 1 \rightarrow n$ belonging to homogenous transformation. This matrix is also known as the matrix differential operator (Uicker). The expression of the Uicker operator changes due to the unit vector $\overline{k}_i = \{\overline{x}_i;\ \overline{y}_i;\ \overline{z}_i\}$ which characterizes the driving axis. The application of this operator to the right side or to the left side of the homogenous transformation matrix depends on the physical state of the driving axis (either fixed ${}^{i-1}\overline{k}_i$ or mobile ${}^{i}\overline{k}_i$).

3. THE DIFFERENTIAL MATRCES OF THE HOMOGENOUS TRANSFORMATIONS

The differential matrices of homogenous transformations play an essential role in the kinematic and dynamic modeling of the mechanical systems mainly because of their computational advantages. Based on these matrices can be determined the Jacobian matrix and the dynamics equations in a matrix form.

In order to determine the differential matrices, the expressions for the locating matrices that define the position and orientation of the systems $\{i\} \rightarrow \{0\}$ are applied:

$$ \left\{ \begin{array}{c} {}^{0}_{i}[T]\left[\overline{\theta}_i(t)\right] = \\ = \begin{bmatrix} {}^{0}_{i}[R]\left[q_j(t) \cdot \Delta_j\right] & \overline{p}_i\left[q_j(t) \cdot \delta_j\right] \\ 0\quad 0\quad 0 & 1 \end{bmatrix} \end{array} \right\}; \quad (9) $$

$$\left\{\begin{matrix} {}^{0}_{n}[T]\left[\bar{\theta}(t)\right]= \\ \begin{bmatrix} {}^{0}_{n}[R]\left[q_i(t)\cdot\Delta_i\right] & \bar{p}_n\left[q_i(t)\cdot\delta_i\right] \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}\end{matrix}\right\}; \quad (10)$$

where, $i=1\rightarrow n$ and $j=1\rightarrow i$

The operational velocities are determined as the absolute derivative of the locating matrices, as:

$$\left\{\begin{matrix} {}^{0}_{i}[\dot{T}]\begin{pmatrix} q_j(t)\cdot\delta_i \\ j=1\rightarrow i\end{pmatrix}=\sum_{j=1}^{i}A_{ij}\cdot\dot{q}_j \\ =\dfrac{d}{dt}\left\{\prod_{j=1}^{i}{}^{j-1}_{j}[T]\right\}=\sum_{j=1}^{i}\dfrac{\partial}{\partial q_j}\left\{{}^{0}_{i}[T]\begin{pmatrix} q_j(t)\cdot\delta_i \\ j=1\rightarrow i\end{pmatrix}\right\}\cdot\dot{q}_j\end{matrix}\right\}; \quad (11)$$

$$\left\{\begin{matrix} {}^{0}_{i}[\dot{T}]\left[\bar{\theta}_i(t);\dot{\bar{\theta}}_i(t)\right]= \\ \sum_{j=1}^{i}\left\{{}^{0}_{j-1}[T]\left[\bar{\theta}_{j-1}(t)\right]\cdot\dfrac{\partial}{\partial q_j}\left\{{}^{j-1}_{j}[T]\left[q_j(t)\right]\right\}\cdot{}^{j}_{i}[T]\left[A_{ij}\left[\bar{\theta}_i(t)\right]\right]\right\}\cdot\dot{q}_j\end{matrix}\right\}; \quad (12)$$

In the expression (12) was introduced the notation A_{ij} which defines the first order differential matrix of the locating matrices, which can be written according to:

$$\left\{\begin{matrix} A_{ij}\left[\bar{\theta}_i(t)\right]=\dfrac{\partial}{\partial q_j}\left\{{}^{0}_{i}[T]\left[\bar{\theta}_i(t)\right]\right\}= \\ =\begin{bmatrix} \dfrac{\partial}{\partial q_j}\left\{{}^{0}_{i}[R]\left[\bar{\theta}_{i\Delta}(t)\right]\right\} & \dfrac{\partial\bar{p}_i\left[\bar{\theta}_i(t)\right]}{\partial q_j} \\ 0 \quad 0 \quad 0 & 0\end{bmatrix}\end{matrix}\right\}; \quad (13)$$

$$A_{ij}\left[\bar{\theta}_i(t)\right]=\left\{{}^{0}_{j-1}[T]\left[\bar{\theta}_{j-1}(t)\right]\cdot\dfrac{\partial}{\partial q_j}\left\{{}^{j-1}_{j}[T]\left[q_j(t)\right]\right\}\cdot{}^{j}_{i}[T]\left[\bar{\theta}_{ij}(t)\right]\right\}; \quad (14)$$

The components of the differential matrix of first order are symbolized as follows:

$$\left\{\begin{matrix} A_{ij}\left[\bar{\theta}_i(t)\right]=\dfrac{\partial}{\partial q_j}\left\{{}^{0}_{i}[T]\left[\bar{\theta}_i(t)\right]\right\}= \\ =\begin{bmatrix} A_{ij}(R)\left[\bar{\theta}_{i\Delta}(t)\right] & A_{ij}(\bar{p})\left[\bar{\theta}_i(t)\right] \\ 0 \quad 0 \quad 0 & 0\end{bmatrix}\end{matrix}\right\}; \quad (15)$$

The partial derivative of the matrix function from (14) is substituted by the following property:

$$\left\{\begin{matrix} \dfrac{\partial}{\partial q_j(t)}\left\{{}^{j-1}_{j}[T]\left[q_j(t)\right]\right\}= \\ =\left\{\begin{matrix} U_j\cdot{}^{j-1}_{j}[T](t) \ \ldots \ for\ {}^{j-1}\bar{k}_j \\ {}^{j-1}_{j}[T](t)\cdot U_j \ \ldots \ for\ {}^{j}\bar{k}_j\end{matrix}\right\}\end{matrix}\right\}; \quad (16)$$

The first order differential matrices of the locating matrices for transformations around fixed or mobile axes are defined according to:

$$\left\{\begin{matrix} A_{ij}\left[\bar{\theta}_i(t)\right]\equiv A_{ji}\left[\bar{\theta}_i(t)\right]= \\ \left\{\begin{matrix} {}^{0}_{j-1}[T]\left[\bar{\theta}_{j-1}(t)\right]\cdot U_j\cdot{}^{j-1}_{i}[T]\left[\bar{\theta}_{ij-1}(t)\right] \\ {}^{0}_{j}[T]\left[\bar{\theta}_j(t)\right]\cdot U_j\cdot{}^{j}_{i}[T]\left[\bar{\theta}_{ij}(t)\right]\end{matrix}\right\}\end{matrix}\right\}; \quad (17)$$

The first expression from (17) is applied when $q_j(t)\cdot{}^{j-1}\bar{k}_j$ and the second one for $q_j(t)\cdot{}^{j}\bar{k}_j$.

For the particular case when $i=n$ and $j=i$, results the following expression for the differential matrices:

$$\left\{\begin{matrix} A_{ni}\left[\bar{\theta}(t)\right]= \\ =\left\{\begin{matrix} {}^{0}_{i-1}[T]\left[\bar{\theta}_{i-1}(t)\right]\cdot U_i\cdot{}^{i-1}_{n}[T]\left[\bar{\theta}_{ni-1}(t)\right] \\ {}^{0}_{i}[T]\left[\bar{\theta}_i(t)\right]\cdot U_i\cdot{}^{i}_{n}[T]\left[\bar{\theta}_{ni}(t)\right]\end{matrix}\right\}\end{matrix}\right\}; \quad (18)$$

where the first expression is applied for $q_i(t)\cdot{}^{i-1}\bar{k}_i$ and the second for $q_i(t)\cdot{}^{i}\bar{k}_i$.

In the same way, according to [….] are determined the defining expressions for the second order differential matrix of the homogenous transformations:

$$\left\{\begin{matrix} A_{ijk}\left[\bar{\theta}_i(t)\right]=A_{ikj}\left[\bar{\theta}_i(t)\right]= \\ =\left\{\begin{matrix} {}^{0}_{k-1}[T]\left[\bar{\theta}_{k-1}(t)\right]\cdot U_k\cdot{}^{k-1}_{j-1}[T]\left[\bar{\theta}_{j-1k-1}(t)\right]\cdot U_j\cdot{}^{j-1}_{i}[T]\left[\bar{\theta}_{ij-1}(t)\right] \\ for\ \left\{q_k(t)\cdot{}^{k-1}\bar{k}_k;\ q_j(t)\cdot{}^{j-1}\bar{k}_j\right\}\end{matrix}\right\} \\ =\left\{\begin{matrix} {}^{0}_{k}[T]\left[\bar{\theta}_k(t)\right]\cdot U_k\cdot{}^{k}_{j}[T]\left[\bar{\theta}_{jk}(t)\right]\cdot U_j\cdot{}^{j}_{i}[T]\left[\bar{\theta}_{ij}(t)\right] \\ for\ \left\{q_k(t)\cdot{}^{k}\bar{k}_k;\ q_j(t)\cdot{}^{j}\bar{k}_j\right\}\end{matrix}\right\}\end{matrix}\right\}$$

In case $j=i$, $i=n$ and $k=1\rightarrow i$, the expressions for the differential matrices of second order that are applied in these cases are:

$$\left\{\begin{array}{c} A_{nij}\left[\bar{\theta}(t)\right]=A_{nij}\left[\bar{\theta}(t)\right]= \\ \left\{\begin{array}{c} {}^{0}_{j-1}[T]\left[\bar{\theta}_{j-1}(t)\right]\cdot U_j\cdot {}^{j-1}_{i-1}[T]\left[\bar{\theta}_{i-1j-1}(t)\right]\cdot U_i\cdot {}^{i-1}_{n}[T]\left[\bar{\theta}_{ni-1}(t)\right] \\ for\ \left\{ q_j(t)\cdot {}^{j-1}\bar{k}_j\ ;q_i(t)\cdot {}^{i-1}\bar{k}_i \right\} \end{array}\right\} \\ \left\{\begin{array}{c} {}^{0}_{j}[T]\left[\bar{\theta}_{j}(t)\right]\cdot U_j\cdot {}^{j}_{i}[T]\left[\bar{\theta}_{ij}(t)\right]\cdot U_i\cdot {}^{i}_{n}[T]\left[\bar{\theta}_{ni}(t)\right] \\ for\ \left\{ q_j(t)\cdot {}^{j}\bar{k}_j\ ;q_i(t)\cdot {}^{i}\bar{k}_i \right\} \end{array}\right\} \end{array}\right\}$$

The differential matrices presented in this section can be applied in the study of operational velocities and accelerations as well as for the computing of the Jacobian matrix also known as the velocities transfer matrix.

4. THE JACOBIAN MATRIX BASED ON DIFFERENTIAL MATRICES

4.1. The Jacobian matrix with respect to the fixed reference system $\{0\}$

Each column (6×1) included in the Jacobian matrix as well as its first order time derivative, with respect to fixed and mobile systems, according to [6] – [10] can be defined as follows:

$$ {}^{0}J_i=\left[\begin{array}{c} A_{ni}\left(\bar{p}\right) \\ \hline {}^{0}_{i}[R]\cdot {}^{i}\bar{k}_i\cdot\Delta_i \end{array}\right]; \quad (19)$$

$$ {}^{\dot{0}}J_i=\left[\begin{array}{c} \sum_{j=1}^{n} A_{nij}(\bar{p})\cdot q_j \\ \left\{\sum_{j=1}^{i} A_{ij}(R)\cdot q_j\right\}\cdot\ {}^{\bar{\imath}}k_i\cdot\Delta_i \end{array}\right]; \quad (20)$$

The components of the Jacobian matrix projected on the $\{n\}$ mobile system are defined:

$$ {}^{n}J_i={}^{n}R\cdot {}^{0}J_i=\left[\begin{array}{cc} {}^{0}_{n}[R]^T & [0] \\ [0] & {}^{0}_{n}[R]^T \end{array}\right]\cdot {}^{0}J_i; \quad (21)$$

$$ {}^{n}\dot{J}_i={}^{n}R\cdot {}^{0}\dot{J}_i=\left[\begin{array}{cc} {}^{0}_{n}[R]^T & [0] \\ [0] & {}^{0}_{n}[R]^T \end{array}\right]\cdot {}^{0}\dot{J}_i; \quad (22)$$

where ${}^{n}R$ represents the differential matrix operator which ensures the transfer between the fixed $\{0\}$ and mobile system $\{n\}$.

The Jacobian matrix with respect to the fixed reference system is defined according to [ ], as:

$$ {}^{0}J\left[\bar{\theta}(t)\right]=\left[{}^{0}J_i\left[\bar{\theta}_{i0}(t)\right];\ i=1\rightarrow n\right]; \quad (23)$$

where, $${}^{0}J_i\left[\bar{\theta}_{i0}(t)\right]=\left[\begin{array}{c} {}^{0}\bar{d}_i\left[\bar{\theta}_{i0}(t)\right] \\ \hline {}^{0}\bar{\delta}_i\left[\bar{\theta}_{i0}(t)\right] \end{array}\right]; \quad (24)$$

The components of the column (i) from the componence of the velocities transfer matrix are:

$$\left\{\begin{array}{c} {}^{0}\bar{d}_i=\bar{V}_i=\dfrac{\partial\bar{p}_n}{\partial q_i}=A_{ni}\left(\bar{p}\right)= \\ ={}^{0}\bar{k}_i\times\left(\bar{p}_n-\bar{p}_i\right)\cdot\Delta_i+{}^{0}\bar{k}_i\cdot\left(1-\Delta_i\right) \end{array}\right\}; \quad (25)$$

$$\left\{\begin{array}{c} {}^{0}\bar{\delta}_i=\bar{\Omega}_i={}^{0}\bar{k}_i\cdot\Delta_i= \\ ={}^{0}_{i}[R]\cdot {}^{i}\bar{k}_i\cdot\Delta_i \end{array}\right\}; \quad (26)$$

In the expressions (25) and (26), ${}^{0}\bar{d}_i$ and ${}^{0}\bar{\delta}_i$ represent the components of the differential vector describing the motion in the Cartesian space.

4.2. The Jacobian matrix with respect to the mobile reference system $\{n\}$

When the Jacobian matrix and its time derivative are known, this matrices can be transferred to the mobile system $\{n\}$. The mathematical model based on differential matrices allows the direct formulation with respect to the mobile system for the Jacobian matrix. The same Jacobian matrix with projections on the mobile reference system is:

$$ {}^{n}J\left[\bar{\theta}(t)\right]=\left[{}^{n}J_i=\left(\begin{array}{c} {}^{n}\bar{d}_i \\ \hline {}^{n}\bar{\delta}_i \end{array}\right);\ i=1\rightarrow n\right]; \quad (27)$$

where the components ${}^{n}\bar{d}_i$ and ${}^{n}\delta_i$ are defined:

$$\left\{\begin{array}{c} {}^{n}\bar{d}_i={}^{n}\bar{V}_i= \\ ={}^{i}_{n}[R]^{-1}\left\{{}^{i}\bar{k}_i\times {}^{i}\bar{p}_{ni}\cdot\Delta_i+{}^{i}\bar{k}_i\cdot\left(1-\Delta_i\right)\right\} \end{array}\right\}; \quad (28)$$

$$\left\{{}^{n}\bar{\delta}_i\equiv {}^{n}\bar{\Omega}_i={}^{i}_{n}[R]^{-1}\cdot {}^{i}\bar{k}_i\cdot\Delta_i\right\}; \quad (29)$$

The expressions (24) and (27) define the Jacobian matrix based on differential matrices.

If the Jacobian matrix with projections on the mobile system is known, it can be obtained its absolute derivative with respect time:

$$\left\{ \begin{array}{c} {}^{\dot{n}}J(\bar{\theta}) = {}^{n}R \cdot {}^{\dot{0}}J(\bar{\theta}) = \\ = \begin{bmatrix} \{ {}^{\bar{n}}\omega_n^{**} \times \} & [0] \\ [0] & \{ {}^{\bar{n}}\omega_n \times \} \end{bmatrix} \cdot {}^{n}J(\bar{\theta}) + \frac{\partial}{\partial t}\{ {}^{n}J(\bar{\theta}) \} \end{array} \right\}; \quad (30)$$

The mathematical model presented in this paper aims to the computing of the Jacobian matrix also known as the matrix of partial derivatives of the locating equations or the velocity transfer matrix. The Jacobian matrix establishes the connection between the generalized velocities and operational velocities, both defining the forward kinematics equations. According to the model presented in this paper it is observed that the Jacobian matrix can be determined by using locating equations, linear and angular transfer matrices or by applying the differential matrices of the homogenous transformations.

5. CONCLUSION

The present paper aims to determine a mathematical model which can be applied to compute the most important differential matrix from robot kinematics known as Jacobian matrix or the velocities transfer matrix. The velocity of a robot link with respect to the previous link usually depends on the type of joint that connects them. The velocity of the end effector is a result of the contribution made by the local velocities from each joint of the robot. that the Jacobian matrix is a function of joint variables and it comprises the kinematical parameters. The expressions for the Jacobian matrix were developed using the differential matrices of first and second order. The differential matrices are obtained by differentiating the expressions for the locating matrices that define the transformations between the systems. Based on these, were determined the expressions for the Jacobian matrix with projections on the fixed and mobile system.

The role of Jacobian matrix is to establish the connection between the generalized velocities and operational velocities, both defining the forward kinematics equations.

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Determinarea matricei Jacobiene pe baza matricelor diferențiale

Lucrarea are drept scop prezentarea unui model matematic de determinare a matricei Jacobiene bazat pe matricele diferențiale ale transformărilor omogene. Expresiile matricei Jacobiene au fost dezvoltate utilizând matricele diferențiale de ordinul întâi și de ordinul al doilea, obținute prin derivarea matricelor de situare ce definesc transformările ce au loc între sistemele de referință atașate fiecărei cuple cinematice a robotului. Aceste matrice diferențiale stau la baza determinării expresiilor pentru matricea Jacobiană și a derivatei ei atât în raport cu sistemul de referință fix cât și în raport cu sistemul de referință mobil. Modelul matematic propus în cadrul acestei lucrări evidențiază rolul matricei Jacobiene de a realiza legătura între vitezele generalizate și cele operaționale, ce definesc ecuațiile cinematicii directe pentru orice structură de robot.

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