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FORMULATIONS ON ACCURACY IN ADVANCED ROBOT MECHANICS

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Abstract: *The assessment of the robot accuracy is achieved by means of the geometrical, kinematic and dynamic errors. The main source of appearance of these errors consists in the dimensional deviations and irregularities from every driving joint of the robot. The main author has published a series of mathematical models devoted to accuracy. They were based on classical transformations from robot kinematics and dynamics. The main author has included them in the first variant of the* ***SimMEcRob Simulator****. Within of this paper about the robot kinematic and dynamic accuracy of the robot structure will be applied formulations, having at basis the matrix exponentials of direct kinematics.*

Key words: *Robotics; Mechanics; Kinematics; Dynamics; Accuracy.*

1. INTRODUCTION

The essential parameters on assessment of the kinematic accuracy are geometrical and kinematic errors. In the size of geometrical errors enter linear and angular deviations that define the relative positions among the links of the mechanical robot structure *(MRS)*. With these, the differences between nominal and actual fact values of the generalized variables, which express the relative motions from every driving joint of the robot, are also added. The kinematic accuracy is estimated by means of the position and orienting errors, likewise called *locating errors* of the end-effector. Beside these, in the assessment to kinematic accuracy, essential parts have the errors of the velocities and accelerations. They correspond to relative motions in every driving joint and absolute motion of the end-effector. For define the equations answerable to direct geometry and kinematics *(DKM)* of the robots, the matrix exponentials *(ME),* with important advantages are applied. In this paper, at beginning the error matrices with classical transformations, and in the following the matrix exponentials are applied in the kinematic accuracy of the robot.

1.1 The Matrices of the Input Data

Every kinetic link $(i = 1 \rightarrow n)$ is physically connected through driving joints $i = \{R;T\}$, where (R) is *rotation* and (T) *prismatic* joint. In keeping with Fig.1, the position of the geometry center

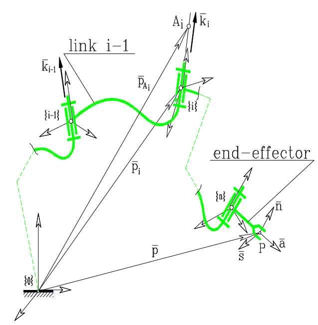

Fig. 1 Geometrical Parameters of *MRS*

O_i from every driving joint, as well as the position of a point A_i arbitrarily chosen on the kinematic axis $\bar{k}_i^{(0)}$, is defined with respect to fixed frame $\{0\}$. According to [4] and [5] they are defined for the nominal (errorless) as well as actual mechanical robot structure *(MRS)*. The matrix of the (nominal and actual) geometry $M_{IDq}^{(0)} = \left\{ M_{IDr}^{(0)}; M_{IDn}^{(0)} \right\}$ is symbolically written as:

$$\underset{[(n+1)\times 6]}{M_{IDq}^{(0)}} = \left\{ \begin{matrix} \left[\left[\bar{p}_{iq}^{(0)T} \quad \bar{p}_{A_i q}^{(0)T} \right] \quad i = 1 \rightarrow n+1 \right]^T \\ \left[\left[\bar{p}_{iq}^{(0)T} \quad \bar{k}_{iq}^{(0)T} \right] \quad i = 1 \rightarrow n+1 \right]^T \end{matrix} \right\}; \quad (1)$$

$$\bar{k}_{iq}^{(0)} = \left(\bar{p}_{A_i q}^{(0)} - \bar{p}_{iq}^{(0)} \right) \Big/ \left| \bar{p}_{A_i q}^{(0)} - \bar{p}_{iq}^{(0)} \right|. \quad (2)$$

The notation $q = \{n\,;r\}$ refers to nominal values, and actual values respectively. In the same stage, the matrix of the (nominal and actual) configurations $M_{\theta q}$ is also accepted under form:

$$\underset{(m\times n)}{M_{\theta q}} = \begin{Bmatrix} \{M_{\theta n}\,;M_{\theta r}\} = \left[\bar{\theta}_{qk}^{T}\; k = 1 \to m\right] \\ \bar{\theta}_{qk}^{T} = \left[q_{iqk}\;\; \dot{q}_{iqk}\;\; \ddot{q}_{iqk}\; i = 1 \to n\right]^{T} \end{Bmatrix}. \quad (3)$$

In the above matrix, q_{iqk} is called the generalized variable in the every driving axis, while $(k = 1 \to m)$ is the number of robot configurations taken into study. Considering the above matrices, the primary errors are included in the parameters:

$$\bar{p}_{ir}^{(0)} = \bar{p}_{in}^{(0)} \pm \Delta\bar{p}_{i}^{(0)};\; \bar{p}_{A_i r}^{(0)} = \bar{p}_{A_i n}^{(0)} \pm \Delta\,\bar{p}_{A_i}^{(0)}; \quad (4)$$

$$\bar{k}_{ir}^{(0)} = \bar{k}_{in}^{(0)} \pm \Delta\bar{k}_{i}^{(0)};\;\; q_{irk} = q_{ink} \pm \Delta q_{ik}\,. \quad (5)$$

The input data matrices are established in the design or simulating stage of the mechanical structure. The primary errors are the result of the dimensional deviations and link elasticity, as well as clearances, wears and frictions respectively. They are given in every driving joint and link.

1.2 The Matrices of the Geometrical Errors

This section is devoted to establishment of geometrical errors between the adjoining kinetic links of *MRS* with $n\,d.o.f.$, due to the parameters from the input data matrices. It is known that the relative position and (locating) orientation, between two neighboring links $(i-1) \to (i)$ can be expressed by means of *the DH-type* (Denavit-Hartenberg) or *GP-type* (generalized) operators. In keeping with [4] and [5], their actual values are characterized by geometrical *DH-type* and *GP-type* errors respectively, all included in the *SimMEcROb Simulator.* In keeping with this, a few matrix expressions devoted to geometrical errors of the kinetic links are presented.

$$\bar{\varepsilon}_{yQk}^{T} = \left\{\left\{\bar{\varepsilon}_{yDk}^{T};\bar{\varepsilon}_{yGk}^{T}\right\}\; y = \{p_{Q}\,;\, e\,;\, g\}\;\; k = 1 \to m\right\}; \quad (6)$$

$$\bar{\varepsilon}_{yQk}^{T} = \begin{Bmatrix} \underset{[1\times 5(n+1)]}{\bar{\varepsilon}_{yDk}^{T}} = \left[\Delta\bar{\beta}_{k}^{T}\; \Delta\bar{\alpha}_{k}^{T}\; \Delta\bar{a}_{k}^{T}\; \Delta\bar{d}_{k}^{T}\; \Delta\bar{\theta}_{k}^{T}\right] \\ \underset{[1\times 6(n+1)]}{\bar{\varepsilon}_{yGk}^{T}} = \left[\Delta\bar{a}_{k}^{T}\; \Delta\bar{b}_{k}^{T}\; \Delta\bar{c}_{k}^{T}\; \Delta\bar{\alpha}_{k}^{T}\; \Delta\bar{\beta}_{k}^{T}\; \Delta\bar{\gamma}_{k}^{T}\right] \end{Bmatrix}. \quad (7)$$

In this paper the classical transformations based on *DH- or GP-type operators* are substituted by means of the matrix exponentials, with a few significant advantages, in keeping with [6], [7] and [8]. With the view of this, at beginning the *MRS* (mechanical robot structure) is taken in the initial configuration: $\bar{\theta}^{(0)} = [q_i = 0; i = 1 \to n]^{T}$.

The Fig. 2 shows that the screw parameters to every driving axis *(i)*, are defined with respect to frame $\{0\}$ as: $\left\{\bar{k}_{i}^{(0)}\,;\bar{v}_{i}^{(0)}\right\}$. The first, $\bar{k}_{i}^{(0)}$ is the unit vector of the driving axis, while the second:

$$\bar{v}_{i}^{(0)} = \left\{\bar{p}_{i}^{(0)}\times\right\}\bar{k}_{i}^{(0)}\cdot\sigma_i + (1-\sigma_i)\cdot\bar{k}_{i}^{(0)}; \quad (8)$$

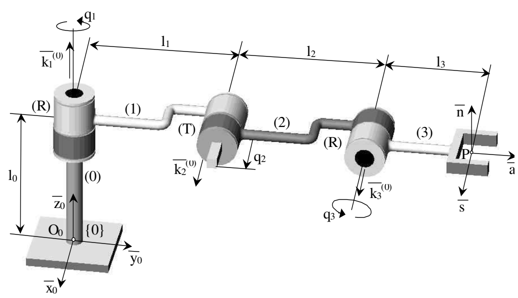

Fig. 2 The screw Parameters for *MRS*

Above, $\sigma_i = \{\{1\;\;, i = R\};\{0\;\;, i = T\}\}$ represents an operator, which defines the driving joint type.

In the equation (8), $\left\{\bar{p}_{i}^{(0)}\times\right\}$ expresses the skew-symmetric matrix associated to position vector.

Considering (4) and (5), the new geometrical errors of the kinetic links are defined with:

$$\Delta\,\bar{v}_{i}^{(0)} = \Delta\,\left\{\left\{\bar{p}_{i}^{(0)}\times\right\}\bar{k}_{i}^{(0)}\cdot\sigma_i\right\} + (1-\sigma_i)\cdot\Delta\,\bar{k}_{i}^{(0)}; \quad (9)$$

$$\bar{\varepsilon}_{yMEk} = \left\{\begin{bmatrix} \Delta\bar{p}_{i}^{(0)} = [dx_i\;\; dy_i\;\; dz_i\,]^{T} \\ \Delta\bar{k}_{i}^{(0)} = [\delta\,x_i\; \delta\,y_i\;\; \delta\,z_i\,]^{T} \end{bmatrix}\;\; i = 1 \to n\right\}; \quad (10)$$

The generalized matrices of kinematical errors (6) and (7) are completed with exponentials, thus:

$$\bar{\varepsilon}_{yQEk}^{T} = \begin{Bmatrix} \left\{\bar{\varepsilon}_{yDk}^{T}\,;\bar{\varepsilon}_{yGk}^{T}\,:\,\bar{\varepsilon}_{yMEk}^{T}\right\} \\ y = \{p_{QE}\,;\; e\,;\; g\}\; k = 1 \to m \end{Bmatrix}; \quad (11)$$

$$\bar{\varepsilon}_{\theta k}^{T} = \left[\,\left[\Delta\,\dot{q}_{ik}\,;\; i = 1 \to n\right]\left[\Delta\,\ddot{q}_{ik}\,;\; i = 1 \to n\right]\right]; \quad (12)$$

$$\bar{\varepsilon}_{\theta yQEk} = \left\{\left[\bar{\varepsilon}_{yQEk}^{T}\;\;\; \bar{\varepsilon}_{\theta k}^{T}\right]^{T}\;\; \bar{\varepsilon}_{\theta k}^{T} = \left[\Delta\,\dot{\bar{\theta}}_{k}^{T}\;\;\; \Delta\,\ddot{\bar{\theta}}_{k}^{T}\right]\right\}. \quad (13)$$

Remarks. In the above matrices, the symbols: $\bar{\varepsilon}_{yQEk}$ and $\bar{\varepsilon}_{\theta yQEk}$ are the column vectors of the geometrical and generalized variable errors. They are answerable of DH-type, GP-type operators, and matrix exponentials. Unlike the classical transformations, the primary errors are direct implemented in the expressions based on the matrix exponentials, this representing an important model advantage.

THE KINEMATIC ERROR MATRICES

This section is devoted to apply the matrix exponentials for define the direct kinematics transformations and matrices of the errors. They lie at the basis to determination of the kinematic accuracy for any robot type taken into study.

2.1 Matrix Exponentials in Direct Kinematics

On the basis of the *ME Algorithm* from [6], [7] and [8], in this section a few essential expressions of the matrix exponentials in the direct geometry and kinematics *(DKM Algorithm)* are shortly presented.

The orienting and locating matrices, between the two adjoining frames $\{i-1\} \to \{i\}$ are defined by means of the matrix exponentials as follows:

$$\left\{\begin{array}{l} exp\left\{\left\{\bar{k}_i^{(0)}\times\right\}q_i\cdot\sigma_i\right\}=\left\{R_{ii-1}=R\left(\bar{k}_i^{(0)};\,q_i\cdot\sigma_i\right)\right\} \\ =I_3\cdot cq_i+\left\{\bar{k}_i^{(0)}\times\right\}sq_i+\bar{k}_i^{(0)}\cdot\bar{k}_i^{(0)T}\cdot(1-cq_i) \end{array}\right\}; \quad (14)$$

$$T_{ii-1}=exp\left\{T_{ii-1}^{(0)}\cdot(U_i\cdot q_i)\cdot\left\{T_{ii-1}^{(0)}\right\}^{-1}\right\}\cdot T_{ii-1}^{(0)}. \quad (15)$$

The matrix-deriving operator is substituted as:

$$U_i=\tau\cdot\begin{bmatrix}\left\{{}^i\bar{k}_i\cdot\sigma_i\times\right\} & {}^i\bar{k}_i\cdot(1-\sigma_i)\\ 0\ 0\ 0 & 0\end{bmatrix};\quad \tau=sgn(q_i). \quad (16)$$

On the basis of the Rodrigues' formula, devoted to position and orientation, the (3×1) column vector is established by means of the expression:

$$\bar{b}_i=\left\{\begin{array}{l} I_3\cdot q_i+\left\{\bar{k}_i^{(0)}\times\right\}\left[1-c(q_i\cdot\sigma_i)\right]+ \\ +\bar{k}_i^{(0)}\cdot\bar{k}_i^{(0)T}\cdot\left[q_i-s(q_i\cdot\sigma_i)\right] \end{array}\right\}\cdot\bar{v}_i^{(0)}; \quad (17)$$

Using the above expressions and the screw parameters, another matrix exponential shows as:

$$e^{A_i\cdot q_i}=exp\left[A_i\cdot q_i\right]=\begin{bmatrix} e^{\left\{\bar{k}_i^{(0)}\times\right\}q_i\cdot\sigma_i} & \bar{b}_i\\ 0\quad 0\quad 0 & 1\end{bmatrix};$$

$$A_i=\begin{bmatrix}\left\{{}^i\bar{k}_i\cdot\sigma_i\times\right\} & \bar{v}_i^{(0)}\\ 0\ 0\ 0 & 0\end{bmatrix} \quad (18)$$

Considering (14), (15) and (18), the exponentials for the locating matrices of the end effector are:

$$\left\{T_{x0}=\begin{bmatrix}R_{x0} & \bar{p}\\ 0\ 0\ 0 & 1\end{bmatrix}\right\}=\prod_{i=1}^{n}exp\left[A_i\cdot q_i\right]\cdot T_{x0}^{(0)}. \quad (19)$$

This expression contains the orienting matrix and position vector. The first is expressed as:

$$R_{x0}=\prod_{i=1}^{n}exp\left\{\left\{\bar{k}_i^{(0)}\times\right\}\cdot q_i\cdot\sigma_i\right\}\cdot R_{x0}^{(0)};\ x=\{n;\,n+1\}; \quad (20)$$

The exponential of the position vector is defined:

$$\bar{p}=\left\{\begin{array}{l} \sum_{i=1}^{n}\left\{\prod_{j=0}^{i-1}exp\left\{\left\{\bar{k}_j^{(0)}\times\right\}\cdot q_j\cdot\sigma_j\right\}\right\}\cdot\bar{b}_i+ \\ +\prod_{i=1}^{n}exp\left\{\left\{\bar{k}_j^{(0)}\times\right\}\cdot q_i\cdot\sigma_i\right\}\cdot\bar{p}^{(0)} \end{array}\right\} \quad (21)$$

In the following, the algorithm contains the main expressions of the *Jacobian matrix exponentials*, belonging to direct kinematics, in keeping with [6], [7] and [8]. At beginning, the three new kinematics matrices, based on exponentials, are determined as below:

$$\underset{(3x3)}{ME}(V_{i1})=\prod_{j=0}^{i-1}exp\left\{\left\{\bar{k}_j^{(0)}\times\right\}\cdot q_j\cdot\sigma_j\right\}; \quad (22)$$

$$\underset{(3x6)}{ME}(V_{i2})=\left\{I_3\quad \sigma_i\cdot\left\{\bar{k}_i^{(0)}\times\right\}\right\};$$

$$\underset{\{6x[9+3\cdot(n-i)]\}}{ME(V_{i3})}=\begin{bmatrix}I_3 & [0] & [0]\\ [0] & ME1(V_{i3}) & ME2(V_{i3})\end{bmatrix}; \quad (23)$$

The matrices from the last expression show as:

$$ME1(V_{i3})=\left\{\begin{array}{l}\prod_{m=i-1}^{k-1}exp\left\{\left\{\bar{k}_m^{(0)}\times\right\}\cdot q_m\cdot\delta_m\cdot\sigma_m\right\}\\ k=i\to n;\ \delta_m=\left\{\begin{array}{l}\{0;\,m=i-1\}\\ \{1;\,m\ge i\}\end{array}\right\}\end{array}\right\};$$

$$ME2(V_{i3})=\prod_{k=i}^{n}exp\left\{\left\{\bar{k}_k^{(0)}\times\right\}\cdot q_k\cdot\sigma_k\right\}. \quad (24)$$

On the basis of the matrices (22) and (23) other new kinematics matrices are also implemented:

$$\underset{(6\times 6)}{ME}\{J_{i1}\}=\begin{bmatrix}ME\{V_{i1}\} & [0]\\ [0] & ME\{V_{i1}\}\end{bmatrix}; \quad (25)$$

$$\underset{(6\times 9)}{ME}\{J_{i2}\}=\begin{bmatrix}ME\{V_{i2}\} & [0]\\ [0] & I_3\end{bmatrix};\qquad \underset{\{9\times[12+3\cdot(n-i)]\}}{ME}\{J_{i3}\}=\begin{bmatrix}ME\{V_{i3}\} & [0]\\ [0] & I_3\end{bmatrix};$$

$$\underset{\{[12+3\cdot(n-i)]\times 1\}}{ME_{iv\omega}}=\begin{bmatrix}ME_{iv}\\ ME_{i\omega}\end{bmatrix}=\begin{bmatrix}\begin{bmatrix}\bar{v}_i^T & \left[\bar{b}_k\ k=i\to n\right]^T & \bar{p}_n^{(0)T}\end{bmatrix}^T\\ \sigma_i\bar{k}_i^{(0)}\end{bmatrix}.$$

The Jacobian matrix with its sub-matrices is determined by means of the below expressions:

$${}^0J(\bar{\theta})=\begin{bmatrix}\underset{(6\times 1)}{{}^0J_i}=\begin{bmatrix}{}^0J_{iv}\\ {}^0J_{i\omega}\end{bmatrix} & i=1\to n\end{bmatrix}; \quad (26)$$

$$ {}^{0}J_i = Matrix\{ME\{J_{i1}\} \cdot ME\{J_{i2}\} \cdot ME\{J_{i3}\} \cdot ME_{iv\omega}\}. \quad (27)$$

The next kinematics matrices lie at the basis to calculus of the Jacobian matrix time-derivative:

$$\underset{(6\times 6)}{ME}\{\dot{J}_{i1}\} = \begin{bmatrix} ME\{\dot{V}_{i1}\} & [0] \\ [0] & ME\{\dot{V}_{i1}\} \end{bmatrix}; ME\{\dot{J}_{i2}\} = \begin{bmatrix} ME\{\dot{V}_{i2}\} & [0] \\ [0] & I_2 \end{bmatrix};$$

$$\left\{ \begin{array}{c} \dfrac{d}{dt}\{ME[V_{i1}]\} = \\ = \sum_{j=1}^{i-1} \left\{ \prod_{k=0}^{j-1} exp\left\{ \left\{ \bar{k}_k^{(0)} \times \right\} q_k \cdot \delta_{jk} \cdot \sigma_k \right\} \right\} \cdot ME1 \\ where \ \ \delta_{jk} = \{\{0; j > i-1\}; \{1; j \le i-1\}\} \end{array} \right\} \quad (28)$$

$$ME1 = \left\{ \bar{k}_j^{(n)} \times \right\} \dot{q}_j \cdot \sigma_j \cdot \prod_{m=j}^{i-1} exp \left\{ \left\{ \bar{k}_m^{(n)} \times \right\} q_m \cdot \delta_m \cdot \sigma_m \right\}$$

;

$$\left\{ \begin{array}{c} \dfrac{d}{dt}\{ME1[V_{i3}]\} = \\ = \sum_{l=i-1}^{k-1} \left\{ \prod_{p=i-1}^{l-1} exp\left\{ \left\{ \bar{k}_p^{(0)} \times \right\} q_p \cdot \delta_{lp} \cdot \sigma_p \right\} \right\} \cdot MEV1 \\ \left\{ \begin{array}{c} l = 0 \to i \\ p = i-1 \to n \end{array} \right\}; \ \ \delta_{lp} = \left\{ \begin{array}{c} \{0; l > k-1\} \\ \{1; l \le k-1\} \end{array} \right\} \end{array} \right\}; \quad (29)$$

$$MEV1 = \left\{ \bar{k}_l^{(0)} \times \right\} \dot{q}_l \cdot \sigma_l \cdot \prod_{r=l}^{k-1} exp \left\{ \left\{ \bar{k}_r^{(0)} \times \right\} q_r \cdot \delta_r \cdot \sigma_r \right\};$$

$$ME2\{\dot{V}_{i3}\} = \sum_{k=1}^{n} \left\{ \prod_{l=1}^{k-1} exp \left\{ \left\{ \bar{k}_l^{(0)} \times \right\} q_l \cdot \delta_l \cdot \sigma_l \right\} \right\} \cdot MEV2 \quad (30)$$

$$MEV2 = \left\{ \bar{k}_k^{(0)} \times \right\} \dot{q}_k \cdot \sigma_k \cdot \prod_{m=k}^{n} exp \left\{ \left\{ \bar{k}_m^{(0)} \times \right\} q_m \cdot \sigma_m \right\}.$$

Through the application of the time-derivative the column matrix $ME_{iv\omega}$ is changed as follows:

$$ME_i = [ME_{iv\omega} \quad ME_{iv\omega}^{*} \quad \dot{M}E_{iv\omega}]; \quad (31)$$

$$ME_{iv\omega}^{*} = \left[\bar{v}_i^{(0)T} \quad \left[\bar{b}_k ; k = i \to n \right]^T \quad \bar{p}^{(0)T} \quad \bar{0}^T \right]^T;$$

$$\dot{M}E_{iv\omega} = \left[\bar{0}^T \quad \left[\dot{\bar{b}}_k ; k = i \to n \right]^T \quad \bar{0}^T \quad \bar{0}^T \right]^T;$$

and $\quad \dot{\bar{b}}_k^{*} = \left\{ exp \left\{ \left\{ \bar{k}_k^{(0)} \times \right\} q_k \cdot \sigma_k \right\} \right\} \cdot \bar{v}_k^{(0)} \cdot \dot{q}_k.$

As a result, every column from Jacobian matrix time-derivative is determined with exponentials:

$$\underset{(i=1\to n)}{{}^{0}\dot{J}_i} = Tr \left\{ \begin{bmatrix} ME\{\dot{J}_{i1}\} & ME\{J_{i2}\} & ME\{J_{i3}\} \\ ME\{J_{i1}\} \cdot & ME\{\dot{J}_{i2}\} \cdot & ME\{J_{i3}\} \\ ME\{J_{i1}\} \cdot & ME\{J_{i2}\} \cdot & ME\{\dot{J}_{i3}\} \end{bmatrix} \cdot ME_i \right\}. \quad (32)$$

On the basis of the same papers [6], [7] and [8] the differential matrices of first and second order, applied about the locating transformations, are determined by means of the exponentials expressions as below:

$$A_{ki} = \left\{ \prod_{j=0}^{i-1} exp\left[A_j \cdot q_j\right] \right\} \cdot A_i \cdot \left\{ \prod_{l=i}^{k} exp\left[A_l \cdot q_l\right] \right\} \cdot T_{k0}^{(0)} \quad (33)$$

$$\left\{ \begin{array}{c} A_{kjm} = \left\{ \prod_{l=0}^{m-1} exp\left[A_l \cdot q_l\right] \right\} \cdot A_m \cdot B_{kjm} \cdot T_{k0}^{(0)} \\ B_{kjm} = \left\{ \left\{ \prod_{i=m}^{j-1} exp\left[A_i \cdot q_i\right] \right\} \cdot A_i \cdot \left\{ \prod_{p=i}^{k} exp\left[A_p \cdot q_p\right] \right\} \right\} \end{array} \right\}.$$

Using the results from (19), (20) and (21), the (locating) direct geometry equations are defined:

$${}^{0}\bar{X}(t) = \begin{bmatrix} \bar{p}(t) \\ .... \\ \bar{\psi}(t) \end{bmatrix} = \left\{ \begin{array}{l} \left[f_j\left(q_i(t) \cdot \delta_i ; i = 1 \to n\right); j = 1 \to 6 \right]^T \\ \delta_i = \{\{1; j = 1 \to 3\}; \{\sigma_i ; j = 4 \to 6\}\} \end{array} \right\};$$

$$\underset{(m\times 6)}{M_{XQE}} = Matrix \begin{bmatrix} p_{xk} \ \ p_{yk} \ \ p_{zk} \ \ \alpha_{Ak} \ \ \beta_{Bk} \ \ \gamma_{Ck} \\ k = 1 \to m \end{bmatrix}. \quad (34)$$

Using the Jacobian matrix with its time derivative, above written by means of the exponentials, the DKM equations with respect to {0} and {n}, are defined with the next matrix:

$$\begin{bmatrix} {}^{0}\dot{\bar{X}} \\ ... \\ {}^{0}\ddot{\bar{X}} \end{bmatrix} = \begin{bmatrix} \left[{}^{0}\bar{v}_n^T \ \ {}^{0}\bar{\omega}_n^T \right]^T \\ \cdots\cdots\cdots\cdots \\ \left[{}^{0}\dot{\bar{v}}_n^T \ \ {}^{0}\dot{\bar{\omega}}_n^T \right]^T \end{bmatrix} = \begin{bmatrix} [0] & {}^{0}J(\bar{\theta}) \\ \cdots\cdots\cdots\cdots \\ {}^{0}J(\bar{\theta}) & {}^{0}\dot{J}(\bar{\theta}) \end{bmatrix} \cdot \begin{bmatrix} \dot{\bar{\theta}} \\ .... \\ \ddot{\bar{\theta}} \end{bmatrix};$$

$$\underset{(m\times 12)}{M_{XVQE}} = Matrix \begin{bmatrix} {}^{(n)0}\bar{X}_{VQEk}^T = \left[{}^{(n)0}\dot{\bar{X}}_{QEK}^T \quad {}^{(n)0}\ddot{\bar{X}}_{QEK}^T \right]^T \\ where \quad k = 1 \to m \end{bmatrix}. \quad (35)$$

Above, M_{XQE} is the generalized matrix of the locating equations, while M_{XVQE} represents the generalized matrix of the operational velocities and accelerations. They describe the absolute motion of the end-effector. On the basis of the polynomial interpolating functions, as well as the above matrices, the kinematic control functions (generalized variables) are symbolically defined in the configuration space, with the next matrix:

$$\underset{(m\times n)}{M_{\theta}^{CF}} = Matrix \begin{bmatrix} q_{jk}(\tau) \quad \dot{q}_{jk}(\tau) \quad \ddot{q}_{jk}(\tau) \\ k = 1 \to m \quad j = 1 \to n \end{bmatrix}. \quad (36)$$

Remarks. It comes out that the matrix exponentials enjoy important advantages specifically its compact form, and especially they avoid the frames answerable to every kinetic link. It is obvious that all the above matrices are expressing the absolute motion of the mechanical robot structure, defined in the nominal state, without to take the primary errors in this calculus. At the same time, all locating and differential matrix functions, symbolically described within of this section, lie at the basis to determination of the robot kinematics accuracy.

2.2 The Differential Matrices of Errors

A new model for the kinematic error matrices is established. The geometrical errors: (11) and (13) can, essentially, influence the kinematic accuracy. This is dignified by means of the differentials applied about the matrix exponentials, defined in the previous section. In keeping with [5], applying a few differential transformations the error matrices are, symbolically, established:

$$[\Delta T]_{QE} = \left\{ \begin{matrix} {}^{i-1}_{\ i}\left[\delta\, {}^{j-1}T\right]_{yQEk} \ ; \ {}^{j-1}_{\ i}\left[\delta T^{(x)}\right]_{yQEk} \\ {}^{j-1}_{\ i}\left[\delta T^{x}\right]_{yQEk} \ ; \ {}^{j-1}_{\ i}\left[\Delta T^{x}\right]_{yQEk} \\ where\ \ x=\{1;2;3\}\ and\ \ k=1\rightarrow m \\ i=1\rightarrow n \quad j=1\rightarrow i \end{matrix} \right\}; \quad (37)$$

$$\Delta J_{QE} = \left\{ \begin{matrix} \Delta^{0}J(\bar{\theta})^{xy}_{QEk}\ ;\ \ \Delta^{0}\dot{J}(\bar{\theta})^{xy}_{QEk};\ \left\{\Delta^{0}J(\bar{\theta})^{xy}_{QEk}\right\}^{-1} \\ x=\{1;2;3\}\ \ k=1\rightarrow m \end{matrix} \right\}; \quad (38)$$

$$\Delta A_{QE} = \left\{ \begin{matrix} \Delta A^{xy}_{ijQEk} = \Delta A^{xy}_{jiQEk};\ \Delta A^{xy}_{ijlQEk} = \Delta A^{xy}_{iljQEk} \\ k=1\rightarrow m\ ;\ i=1\rightarrow n;\ j=1\rightarrow i;\ l=1\rightarrow j \end{matrix} \right\}. \quad (39)$$

The above expressions (37), (38) and (39) characterize the kinematic error matrices answerable to locating, Jacobian matrices and differentials of the locating transformations. For increase the kinematic accuracy of the robot, the differentiating order is dignified by means of the index: $x=\{1;2;3\}$. Using [9], the error matrices from (37) are developed below:

$${}^{i-1}_{\ i}\left[\delta\ {}^{j-1}T\right]_{yQE} = \left\{ \begin{matrix} {}^{j-1}_{i-1}[T]_{QE}\cdot {}^{i-1}_{\ i}[\delta\, T]_{yQE}\cdot {}^{j-1}_{i-1}[T]^{-1}_{QE} \\ where\ i=1\rightarrow n+1;\ j=1\rightarrow i \end{matrix} \right\}; \ (40)$$

$${}^{i-1}_{\ i}[\ \delta\, T]_{yQE} = \left\{ \begin{matrix} \dfrac{\partial}{\partial p_{QE}}\left\{ {}^{i-1}_{\ i}[T]_{QE} \right\}\cdot {}^{i-1}_{\ i}[T]^{-1}_{QE} = \\ = \Delta T_{i\,i-1QE}\cdot T^{-1}_{ii-1QE} \end{matrix} \right\}; \ (41)$$

$${}^{j-1}_{\ i}\left[\delta T^{(x)}\right]_{yQE} = \sum_{k=j}^{i-(x-1)} {}^{k-1}_{\ k}\left[\delta\ {}^{j-1}T\right]_{yQE}\cdot {}^{k}_{i}\left[\delta T^{(x-1)}\right]_{yQE}; \ (42)$$

$${}^{j-1}_{\ i}\left[\Delta\, T^{x}\right]_{yQE} = \left\{ \sum_{k=1}^{x} {}^{j-1}_{\ i}\left[\delta T^{(k)}\right]_{yQE} \right\}\cdot {}^{j-1}_{\ i}[T]_{QE}\ ; \ (43)$$

where $p_{QE} = \left\{\Delta\bar{p}^{(0)T}_{i};\Delta\bar{k}^{(0)T}_{i};\Delta q_{ik}\right\} \subset \bar{\varepsilon}_{\theta yQEk}$; (44)

and $\Delta T_{i\,i-1E} = \left\{ \begin{matrix} \Delta T^{(0)}_{i\,i-1E}\cdot exp\{U_i\cdot q_i\} + \\ + T^{(0)}_{i\,i-1}\cdot \{exp\{U_i\cdot q_i\}\}\cdot U_i\cdot \Delta q_i \end{matrix} \right\}$. (45)

As a result, substituting, directly, the primary errors from (44) and error matrices expressed with exponentials (45), in the (40)-(43), the new matrix expressions of the locating errors are obtained. It is obvious that the expressions: (42) and (43) show the differential model of x order. In the following, using the same model from [9], the error matrices, from (38) and (39), show in the symbolically form as:

$$\Delta\left\{exp[A_i\cdot q_i]\right\} = \begin{bmatrix} \Delta\left\{exp\left\{\left\{\bar{k}^{(0)}_{i}\times\right\}q_i\cdot\sigma_i\right\}\right\} & \Delta\bar{b}_i \\ 0\quad 0\quad 0 & 0 \end{bmatrix}; \ (46)$$

$$\Delta\bar{b}_i = \Delta\left\{ \left\{ \begin{matrix} I_3\cdot q_i + \left\{\bar{k}^{(0)}_{i}\times\right\}[1-c(q_i\sigma_i)] + \\ + \bar{k}^{(0)}_{i}\cdot\bar{k}^{(0)T}_{i}\cdot[q_i - s(q_i\sigma_i)] \end{matrix} \right\}\cdot\bar{v}^{(0)}_{i} \right\}; \ (47)$$

$$\Delta A_i = \begin{bmatrix} \Delta\left\{{}^{i}\bar{k}_i\cdot\sigma_i\times\right\} & \Delta\bar{v}^{(0)}_{i} \\ 0\ 0\ 0 & 0 \end{bmatrix}. \quad (48)$$

$$\Delta A^{xy}_{ijEk} = \Delta\left\{ \left\{ \prod_{l=0}^{j-1} exp[A_l\cdot q_l] \right\}\cdot A_j\cdot \left\{ \prod_{p=j}^{i} exp[A_p\cdot q_p] \right\} T^{(0)}_{i0} \right\};$$

$$\left\{ \begin{matrix} \Delta A^{xy}_{ijlEk} = \left\{ \prod_{m=0}^{l-1} exp[A_m\cdot q_m] \right\}\cdot A_l\cdot B_{ijl}\cdot T^{(0)}_{i0} \\ B_{ijl} = \left\{ \prod_{p=l}^{j-1} exp[A_p\cdot q_p]\cdot A_p\cdot \prod_{r=p}^{i} exp[A_r\cdot q_r] \right\} \end{matrix} \right\}; \ (49)$$

$$\Delta\ {}^{0}J(\bar{\theta})^{xy}_{QE} = \underset{(6\times n)}{Matrix}\left[\Delta\ {}^{0}J^{xy}_{jiQE}\ \ j=1\rightarrow 6\ \ i=1\rightarrow n\right]; \ (50)$$

$$\underset{(j=1\rightarrow 6)}{\Delta\ {}^{0}J^{xy}_{jiQ}} = \left\{ \begin{matrix} \Delta A^{xy}_{niQj4} & j=1\rightarrow 3 \\ \left\{\sigma_i\cdot {}^{0}_{i}\left[\Delta R^{x}\right]_{yQ}\cdot {}^{i}\bar{k}_i\right\} & j=4\rightarrow 6 \end{matrix} \right\}; \quad (51)$$

$$\Delta\ {}^{0}\dot{J}(\bar{\theta})^{xy}_{QE} = \underset{(6\times n)}{Matrix}\left[\Delta\ {}^{0}\dot{J}^{xy}_{jiQE}\ \ j=1\rightarrow 6\ \ i=1\rightarrow n\right]; (52)$$

$$\underset{(j=1\rightarrow 6)}{\Delta\ {}^{0}\dot{J}^{xy}_{jiQ}} = \left\{ \begin{matrix} \sum_{k=1}^{n}\Delta A^{xy}_{nikQj4}\cdot\dot{q}_k & j=1\rightarrow 3 \\ \left\{\sigma_i\cdot\left\{\sum_{k=1}^{i}\Delta A^{xy}_{ikQj3}\cdot\dot{q}_k\right\}\cdot {}^{i}\bar{k}_i\right\} & \end{matrix} \right\}; \quad (53)$$

Therefore, substituting the above locating errors under exponential form in the (49)-(53) the new expressions for: ΔA^{xy}_{ijek}, ΔA^{xy}_{ijlEk}, $\Delta\ {}^{0}J(\bar{\theta})^{xy}_{QE}$ are determined as error matrices. But the error matrix functions for the Jacobian matrix and its time-derivative are directly obtained, as follows:

$$\Delta^0 J_{iE}^{xy} = \Delta\left\{\underset{(n\times 1)}{Matrix}\left\{ME\{J_{i1}\}\cdot ME\{J_{i2}\}\cdot ME\{J_{i3}\}\cdot ME_{iv\omega}\right\}\right\};$$

$$\Delta ME\left\{J_{iE}^{xy}\right\} = \left\{\begin{array}{c} \left\{\Delta ME\left\{V_{ijE}^{xy}\right\};\ \ j=1,2,3\right\} \\ \left\{\Delta ME\left\{J_{ijE}^{xy}\right\};\ \ j=1,2,3\right\} \\ \Delta ME_{ivE}^{xy};\ \Delta ME_{i\omega E}^{xy};\ \Delta ME_{iv\omega E}^{xy} \end{array}\right\}; \quad (54)$$

$$\Delta^0 \dot{J}_{iE}^{xy} = \Delta\left\{Tr\left\{\begin{bmatrix} ME\{\dot{J}_{i1}\}\cdot ME\{J_{i2}\}\cdot ME\{J_{i3}\} \\ ME\{J_{i1}\}\cdot ME\{J_{i2}\}\cdot ME\{\dot{J}_{i3}\} \\ ME\{J_{i1}\}\cdot ME\{J_{i2}\}\cdot ME\{J_{i3}\} \end{bmatrix}\cdot M_i\right\}\right\};$$

$$\Delta ME\{\dot{J}_{iE}^{xy}\} = \left\{\begin{array}{c} \Delta ME\{\dot{V}_{i1E}^{xy}\};\Delta ME\{\dot{V}_{i2E}^{xy}\};\Delta ME\{\dot{V}_{i3E}^{xy}\} \\ \Delta ME\{\dot{J}_{i1E}^{xy}\};\Delta ME\{\dot{J}_{i2E}^{xy}\};\Delta ME\{\dot{J}_{i3E}^{xy}\} \\ \Delta ME_{iv\omega E}^{xy};\ \ \Delta ME_{iv\omega E}^{*xy};\ \ \Delta \dot{M}E_{iv\omega E}^{xy} \end{array}\right\} \quad (55)$$

Above, the error matrices are defined by means of (22)-(25) and (28)-(31), through applying the differentials in function of errors: (10) and (12), as well as the differentiating order $x=\{1;2;3\}$.

Remarks. Unlike the classical transformations, the calculus of the geometrical parameters, based on the primary errors, between the two adjoining links is eliminated. They ensure the determination of whole error and differential matrices asked in the kinematic accuracy.

4. THE KINEMATIC ROBOT ACCURACY

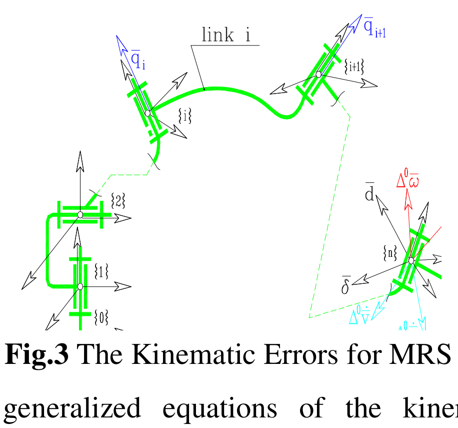

Fig.3 The Kinematic Errors for MRS

The generalized equations of the kinematic accuracy are, symbolically, described below by means of a few expressions, in keeping with the *SimMEcROb Simulator,* fully described in the [4] and [5].The main symbols of the kinematics errors, devoted to assessment of the accuracy, are shown in Fig.3.

The direct modeling of the locating errors, as well as velocity and acceleration errors of the end-effector in the Cartesian space is shown as:

$$\underset{(k=1\to m)}{\Delta^0 \bar{X}_{QEk}^{xy}} = \left[\left\{\bar{d}_{QEk}^{xy}\right\}^T \quad \left\{\bar{\delta}_{QEk}^{xy}\right\}^T\right]^T = E_{d\delta QEk}^{xy}\cdot\bar{\varepsilon}_{yQEk}; \ (56)$$

$$\begin{bmatrix} \bar{d}_{QEk}^{xy} \\ \bar{\delta}_{QEk}^{xy} \end{bmatrix} = \begin{bmatrix} \left[d^{xy}x_{QEk} \quad d^{xy}y_{QEk} \quad d^{xy}z_{QEk}\right]^T \\ \left[\delta^{xy}x_{QEk} \quad \delta^{xy}y_{QEk} \quad \delta^{xy}z_{QEk}\right]^T \end{bmatrix};$$

$$\underset{(k=1\to m)}{\Delta^0 \bar{X}_{vQEk}^{xy}} = \left[\left\{\Delta^0 \dot{\bar{X}}_{QEk}^{xy}\right\}^T \left\{\Delta^0 \ddot{\bar{X}}_{QEk}^{xy}\right\}^T\right]^T = E_{vaQEk}^{xy}\cdot\bar{\varepsilon}_{\theta yQEk};$$

$$\begin{bmatrix} \Delta^0\dot{\bar{X}}_{QEk}^{xy} \\ \Delta^0\ddot{\bar{X}}_{QEk}^{xy} \end{bmatrix} = \begin{bmatrix} \left[\left\{\Delta^0\bar{v}_{QEk}^{xy}\right\}^T \quad \left\{\Delta^0\bar{\omega}_{QEk}^{xy}\right\}^T\right]^T \\ \left[\left\{\Delta^0\dot{\bar{v}}_{QEk}^{xy}\right\}^T \quad \left\{\Delta^0\dot{\bar{\omega}}_{QEk}^{xy}\right\}^T\right]^T \end{bmatrix}. \ (57)$$

Above, the $\bar{\varepsilon}_{yQEk}$ column vector of the errors is determined with new expressions based on the matrix exponentials, according to (10) and (13). The above expressions also contain $E_{d\delta Qk}^{xy}$ and E_{vaQk}^{xy}. The first is called the transfer matrix of the geometrical errors, whose size is defined as: $\{\{6\times 5\cdot n;Q=D\};\{6\times 6\cdot n;Q=G\};\{6\times 6\cdot n;Q=ME\}\}$. The second is the transfer matrix of the kinematics errors with $(12\times N_Q\cdot n)$ size. They are developed on the basis of the above matrix exponentials and likewise error matrix functions.

Considering the optimizing model analyzed in the papers [4], [5] and [9], the matrix of the turning values for kinematics errors is symbolically presented below:

$$E_{YQE} = Matrix\begin{bmatrix} Values\{\{max;min\};\{x\,;y\,;k\}\} \\ Y_{QE} = \left\{\Delta^0\bar{X}_{QE}\,;\Delta^0\bar{X}_{vQE}\right\} \end{bmatrix}. \quad (58)$$

The inverse modeling of the generalized errors of (position-orientation) locating, as well as velocity acceleration from the end-effector is defined as:

$$\bar{\varepsilon}_{yQek}^{x} = \left\{E_{d\delta QEk}^{xy}\right\}^{-1}\cdot\left\{\Delta^0\bar{X}_{QEk}^{xy} = \left[\left\{\bar{d}_{QEk}^{xy}\right\}^T \ \left\{\bar{\delta}_{QEk}^{xy}\right\}^T\right]^T\right\};$$

$$\bar{\varepsilon}_{\theta yQek}^{x} = \left\{E_{vaQEk}^{xy}\right\}^{-1}\cdot\left\{\Delta^0\bar{X}_{vQEk}^{xy} = \left[\left\{\Delta^0\dot{\bar{X}}_{QEk}^{xy}\right\}^T\left\{\Delta^0\ddot{\bar{X}}_{QEk}^{xy}\right\}^T\right]^T\right\};$$

$$E_{YQE}^{*} = Matrix\left[Values\ \{x\,;y\,;k\}\ \ \bar{\varepsilon}_{yQEk}^{x}\ \ \bar{\varepsilon}_{\theta yQEk}^{x}\right] \ (59)$$

where $\left\{E^{xy}_{d\delta QEk}\right\}^{-1}$ and $\left\{E^{xy}_{vaQEk}\right\}^{-1}$ represent the inverses of the transfer matrices of the geometrical and kinematic errors respectively, and E^{*}_{YQE} is the generalized matrix of the errors. These models are included in the generalized algorithm devoted to assessment and simulation of the kinematic and dynamic robot accuracy.

5. DYNAMIC ACCURACY OF ROBOTS

The generalized equations of the dynamic accuracy are, symbolically, described below by means of a few expressions, in keeping with the *SimMEcROb Simulator*, fully described in [4] and [5]. The main symbols of the kinematics and dynamics errors are shown in the Fig.2.

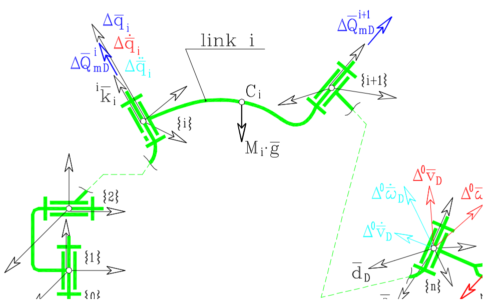

Fig.4 The Dynamic Errors for MRS

At beginning, the generalized matrices of the kinematics errors are considered (11) – (13).
The direct modeling of the locating errors, as well as velocity and acceleration errors in the Cartesian space (56) and (57) are considered.

On the basis of the mathematical model, in keeping with [1] - [30], the generalized matrices of dynamics have been established. In this paper, the symbolical expressions for dynamics matrices of errors will be defined. First of all, generalized matrix of the MD-type errors is:

$$E_{MDQ} = \begin{Bmatrix} E_{MDQk} \\ (nx10) \\ \bar{\varepsilon}_{MDQk} \end{Bmatrix} = \left[\begin{bmatrix} \Delta M_{ik} & \left(\Delta^{i}\bar{r}_{CiQk}\right)^{T} & \Delta^{i} I_{\delta Qk} \\ k=1\to m; & & i=1\to n \end{bmatrix}\right]^{T} \quad (28)$$

$$\left\{\begin{array}{l} \Delta^{i} I_{\delta Qk} \subset \Delta^{i} I_{iQk} = \underset{(3\times3)}{Matrix}\begin{pmatrix} \Delta I_{jlk} = \Delta I_{ljk} \\ j=1\to3 \quad l=1\to j \end{pmatrix} \\ E_{MDQk} \subset \Delta^{i} I_{psiQ\,k} = \underset{(4\times4)}{Matrix}\begin{pmatrix} \Delta I_{jl\,k} = \Delta I_{l\,jk} \\ j=1\to4 \quad l=1\to j \end{pmatrix} \end{array}\right\};$$

where $\Delta^{i} I_{iQk}$ and $\Delta^{i} I_{psiQ\,k}$ represent the error matrices of the inertial and pseudo-inertial tensor respectively, while $\bar{\varepsilon}_{MDQ}$ is the column vector of the *MD-type errors (mass distribution errors).*
The generalized matrix of dynamic errors is determined, and its symbolical expression is:

$$\Delta M^{xy}_{\mathbf{0}} = \left\{\begin{array}{l} \Delta M^{xy}_{\mathbf{0}Q} = \left\{\Delta M^{xy}_{\mathbf{0}L}; \Delta M^{xy}_{\mathbf{0}D}; \Delta M^{xy}_{\mathbf{0}G}; \Delta M^{xy}_{\mathbf{0}ME}\right\} \\ \left\{\Delta M^{xy}_{\mathbf{0}Qyk}; z = \{\Delta_m ; \Delta_\theta\} ; k=1\to m\right\} \end{array}\right\};$$

$$\Delta M^{xy}_{\mathbf{0}Qzk} = \left\{\begin{array}{c} \Delta M(\bar{\theta})^{xy}; \Delta\bar{V}\left(\bar{\theta};\dot{\bar{\theta}}\right)^{xy}; \Delta B(\bar{\theta})^{xy} \\ \Delta C(\bar{\theta})^{xy}; \Delta Q_g(\bar{\theta})^{xy}; \Delta\bar{Q}^{xy}_{SU}; \Delta\bar{Q}^{xy}_{fd} \end{array}\right\}_{Qzk} \quad (29)$$

where $\Delta M^{xy}_{\mathbf{0}Qzk}$ represent the differentials of the dynamic matrices with respect to $\bar{\varepsilon}_{yQk}$ and $\bar{\varepsilon}_{MDQ}$.

$$\Delta M_{ij} = \Delta\left\{\sum_{k=i}^{n} Tr\left\{\left[\exp\left\{\sum_{j=0}^{i-1} A_j\cdot q_j\right\}\right]\cdot A_i\cdot A_{lkj}\right\}\right\}; \quad (30)$$

$$where \quad A_{lkj} = \left\{\exp\left\{\sum_{l=i}^{k} A_l\cdot q_l\right\}\right\}\cdot T^{(0)}_{k0}\cdot {}^{k}I_{psk}\cdot A^{T}_{ijk}$$

$$A^{T}_{ijk} = \left\{\left[\exp\left\{\sum_{i=0}^{j-1} A_i\cdot q_i\right\}\right]\cdot A_j\cdot\left\{\exp\left\{\sum_{l=j}^{k} A_l\cdot q_l\right\}\right\}\cdot T^{(0)}_{k0}\right\}^{T}$$

$$\Delta V_{ijm} = \Delta\left\{\sum_{k=\max(i;j)}^{n} Tr\left\{\exp\left\{\sum_{j=0}^{i-1} A_j\cdot q_j\right\}\right\}\cdot A_i\cdot A_{lkjm}\right\} \quad (31)$$

$$\left\{\begin{array}{l} where\ A_{lkjm} = \left\{\exp\left\{\sum_{l=i}^{k} A_l\cdot q_l\right\}\right\}\cdot T^{(0)}_{k0}\cdot {}^{k}I_{psk}\cdot A^{T}_{kjm}; \\ A_{kjm} = \left\{\exp\left\{\sum_{l=0}^{m-1} A_l\cdot q_l\right\}\right\}\cdot A_m\cdot A_{kj}\cdot T^{(0)}_{k0}; \\ and\ A_{kj} = \left\{\exp\left\{\sum_{i=m}^{j-1} A_i\cdot q_i\right\}\cdot A_i\cdot\exp\left\{\sum_{p=i}^{k} A_p\cdot q_p\right\}\right\} \end{array}\right\}.$$

The dynamic errors (30) and (31) are included in the dynamics matrices, [1] and [2], as below:

$$\Delta M(\bar{\theta}) = \begin{bmatrix} \Delta M_{ij} = \Delta\left\{\sum_{k=max(i;j)}^{n} Tr\left[A_{ki}\cdot {}^{k}I_{psk}\cdot A^{T}_{kj}\right]\right\} \\ where\ i=1\to n\ and\ j=1\to n \end{bmatrix} \quad (32)$$

$$\left\{\begin{array}{l} \Delta V\left(\bar{\theta};\dot{\bar{\theta}}\right) = \Delta\left\{\dot{\bar{\theta}}^{T}\cdot\left\{\left[V_{ijm}; \quad i=1\to n\right]\cdot\dot{\bar{\theta}}\right\}^{T}\right\} = \\ \Delta\left\{\left[\dot{\bar{\theta}}^{T}\cdot\begin{bmatrix} =\sum_{k=max(i;j)}^{n} Tr\left[A_{ki}\cdot {}^{k}I_{psk}\cdot A^{T}_{kjm}\right] \\ j=1\to n and m=1\to n \end{bmatrix}\cdot\dot{\bar{\theta}}\right]^{T}\right\} \end{array}\right\} \quad (33)$$

$$\Delta B(\bar{\theta}) = \begin{bmatrix} \Delta\left\{\sum_{k=max(i;j;m)}^{n} Tr\left[A_{ki}\cdot {}^{k}I_{psk}\cdot A^{T}_{kjm}\right]\right\} \\ i=1\to n,\ j=1\to n-1,\ m=j+1\to n \end{bmatrix} \quad (34)$$

$$\Delta C(\bar{\theta}) = \begin{bmatrix} \Delta \left\{ \sum_{k=max(i;j)}^{n} Tr\left[A_{ki} \cdot {}^{k}I_{psk} \cdot A_{kjj}^{T} \right] \right\} \\ i = 1 \to n,\ j = 1 \to n \end{bmatrix} \quad (35)$$

The above dynamic errors are answerable to inertia matrix (32), column matrix (33) which is containing the matrix of the Coriolis terms (34) and centrifugal terms (35). Analyzing (30) and (31), it comes out that their components are expressed by means of the matrix exponentials. Therefore, the dynamic error of the generalized driving forces is symbolically defined with:

$$\Delta Q_m(\bar{\theta}) = \Delta \begin{Bmatrix} \sigma_m^2 \cdot \sigma_\theta \cdot \left\{ M(\bar{\theta}) \cdot \ddot{\bar{\theta}} + V\left(\bar{\theta}; \dot{\bar{\theta}}\right) \right\} + \\ +\sigma_m^2 \cdot Q_g(\bar{\theta}) + \\ +(-1)^{\sigma_m} \frac{1-\sigma_m}{1+2\cdot\sigma_m} \cdot Q_{SU}(\bar{\theta}) \end{Bmatrix}. \quad (37)$$

According to algorithm of [4] and [5], and considering the above symbolically expressions, the matrix equation for the inverse model of the dynamics errors is written as symbolical form:

$$\begin{Bmatrix} \bar{Y}_{dQk}^{xy} = \mathrm{E}_{YdQk}^{xy} \cdot \bar{\varepsilon}_{ydQK} \\ where\ \bar{Y}_{dQk}^{xy} = \left\{ \Delta M_{DEQzk}^{xy}(\tau);\ \Delta M_{XDFQzk}^{xy}(\tau) \right\} \end{Bmatrix}; \quad (38)$$

$$\bar{Y}_{dQ} = \begin{Bmatrix} \Delta \bar{Q}_{mQ};\ \Delta \bar{X}_{dQ} = \left[\Delta \dot{\bar{v}}_{dQ}\ \Delta \dot{\bar{\omega}}_{dQ} \right]^T \\ \bar{\varepsilon}_{ydQk} = \left[\bar{\varepsilon}_{yQk}^T \quad \bar{\varepsilon}_{MDQk}^T \right]^T \end{Bmatrix}. \quad (39)$$

In the above expressions, the symbol $\left\{ E_{YdQk}^{xy} \right\}$ is devoted to transfer matrix of the dynamics errors, while $\bar{Y}_{dQ}$ is answerable to $\Delta \bar{Q}_{mQ}$ generalized driving forces and $\Delta \bar{X}_{dQ}$ operational variables. On the basis of the above dynamics error functions, the generalized expressions symbolically written:

$$\Delta M_{DE}^{xy} = \begin{Bmatrix} \Delta M_{DEQ}^{xy} = \left\{ \Delta M_{DED(G)}^{xy}\ ; \Delta M_{DEME}^{xy} \right\} \\ \begin{Bmatrix} \Delta M_{DEQzk}^{xy};\ z = \{\Delta_m\ ;\ \Delta_\theta\} \\ k = 1 \to m \end{Bmatrix} \end{Bmatrix};$$

$$\begin{Bmatrix} \Delta M_{XDF}^{xy}(\tau) = \\ = \Delta M_{XDFQ}^{xy}(\tau) = \left\{ \Delta M_{XDFD(G)}^{xy}(\tau); \Delta M_{XDFME}^{xy}(\tau) \right\} \end{Bmatrix};$$

$$\begin{Bmatrix} \Delta M_{XDF}^{xy}(\tau) = \\ \begin{Bmatrix} \Delta M_{XDFQzk}^{xy}(\tau);\ z = \{\Delta_m\ ; \Delta_\theta\} \\ k = 1 \to m \end{Bmatrix} \end{Bmatrix}$$

$$\Delta M_{XDFQzk}^{xy}(\tau) = \left\{ \Delta^0 \bar{X}_{Qzk}^{xy}(\tau); \Delta^0 \dot{\bar{X}}_{Qzk}^{xy}(\tau); \Delta^0 \ddot{\bar{X}}_{Qzk}^{xy}(\tau) \right\};$$

$$\bar{Y}_{dQk}^{xy} = \left\{ \Delta M_{DEQzk}^{xy}(\tau);\ \Delta M_{XDFQzk}^{xy}(\tau) \right\} \quad (40)$$

where $\Delta M_{DE}^{xy}(\tau)$ and $\Delta M_{XDF}^{xy}(\tau)$ are the generalized matrices of the errors corresponding to generalized dynamic forces and dynamic functions of operational variables respectively.

The optimization model of the dynamic accuracy consists in the determination of the global maximum and minimum of the dynamic errors corresponding to the generalized driving forces and operational variables respectively. The results are included in the generalized matrices:

$$E_{YdQ} = \begin{bmatrix} Values\ \ \{\{max; min\}; \{x\ ;\ y\ ;\ i\ ;\ k\}\} \\ \Delta Q_{mQ} \quad \Delta X_{dQ} \end{bmatrix}; \quad (41)$$

$$\Delta M_{dQ} = \begin{Bmatrix} \Delta M_{dQk} \\ k = 1 \to m \end{Bmatrix}; \begin{Bmatrix} \Delta M_{dYQ} \\ E_{YdQ} \end{Bmatrix} = \begin{Bmatrix} E_{YdQk}^{xy};\ \bar{Y}_{dQk}^{xy} \\ k = 1 \to m \end{Bmatrix}.$$

Remark. In the generalized matrices are included all dynamic errors. They have been determined through the application of the matrix exponentials. Considering the other papers of the author, it remarks that they assure the directly assessment of the whole differential matrices of kinematic and dynamic errors, while calculus of locating errors between links is avoided.

5. CONCLUSIONS

In the assessment of kinematic accuracy, an essential role is played by the locating, velocity and acceleration errors. To define the equations answerable to direct geometry and kinematics of robots, the matrix exponentials, have been applied. This paper is devoted to new formulations based on matrix exponentials in the kinematic accuracy. The matrix exponentials enjoy important advantages given by its compact form, and especially by the fact that by their use in mathematical modeling the frames answerable to every kinetic link are avoided. Unlike the classical transformations they ensure the direct computation of all error matrices, and therefore the calculus of the geometrical errors between the kinetic links is avoided.

In the assessment of the dynamic accuracy in Robotics, the dynamic errors have an essential role. They are referring, on the one hand, to the locating, velocities and accelerations. On the other hand the dynamic errors have been extended about the generalized dynamics forces. For defining the generalized equations answerable to kinematics and dynamics of robots, the matrix exponentials have been applied. This paper aims to the developing of a new mathematical model based on matrix exponentials in the dynamics of robot accuracy. It comes out that the matrix exponentials enjoy important advantages given by its compact form, and especially by the fact that they avoid the frames answerable to every kinetic link. Unlike the classical modeling, they ensure the direct assessment of all differential matrices of errors, while the calculus of the locating errors between the links is avoided. That is why, from viewpoint of the calculus and implementing of the algorithm in the robot dynamic control, the study of the dynamics accuracy is dominated by a few advantages. As a result, the generalized equations symbolically presented in this paper will complete the *SimMEcROb Simulator*. This is devoted to the assessment of geometry, kinematics, dynamics and accuracy for any mechanical robot structure regardless of its complexity and configuration.

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Formulări asupra preciziei în mecanica avansată a roboților

Rezumat: Evaluarea preciziei roboților se realizează prin intermediul erorilor geometrice, cinematice și dinamice. Principala sursă de apariție a acestor erori o reprezintă abaterile dimensionale și neregularitățile care apar la un moment dat în cuplele motoare ale robotului. Autorul principal al acestei lucrări a publicat un număr mare de articole dedicate modelării matematice a preciziei roboților industriali. Aceste modele se bazează pe transformări clasice din cinematica și dinamica roboților ele fiind incluse în prima variantă a simulatorului SimMEcRob. În cadrul acestei lucrări vor fi aplicate o serie de formulări asupra preciziei cinematice și dinamice a structurilor de roboți care au la bază având la bază modelele matematice cu exponențiale de matrice din cinematică directă.

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