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MATRIX EXPONENTIALS IN ROBOT ELASTOKINEMATICS

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Abstract: *The main objective of this paper consists in the establishment of the generalized elastokinematics equations for robot structures with flexible links. For kinematics and differential matrices in the case of the robot structures with rigid and elastic links will be applied the matrix exponentials, in accordance with the algorithm developed by the main author. Consequently, the matrix exponentials will stay at the basis of establishment the linear and angular transfer matrices. By means of the same matrix exponentials will be also determined all kinematic parameters. In the second part of this paper elastokinematics structure of the serial robot will be analyzed. Applying the properties of the matrix exponentials, the locating matrices and their time derivatives corresponding to small deformations will be also established. On the basis of these differential transformations, in the final part of the paper, will be determined the linear and angular velocities and accelerations, as well as Jacobian matrix.*
Key words: *elastokinematics, elastodynamics, advanced mechanics, robotics.*

1. INTRODUCTION

According to [2] – [7], transfer equations of any kinematic chain, with *(R)-rotation* or *(T)-prismatic* joints, typical of *MRS* can be expressed WITH *the locating transformations*.
The locating term substitutes the position and orientation between two kinetic links, shown in the Fig.1 and Fig.2. For analyze the transfer equations the mechanical robot structure *(MRS)* is represented in the initial configuration: $\bar{\theta}^{(0)} = [q_i = 0; i = 1 \rightarrow n]^T$. In the robot kinematics the linear and angular transfer matrices will be defined, in the following, by means of matrix exponentials. As a result *the DKM Equations* answerable to linear and angular velocities and accelerations will be presented by their defining expressions. Considering these transfer matrices, in the following, the study with matrix exponentials will be extended about robotic structures with flexible links. The kinematic transformations for structures with rigid and elastic links will be compulsively applied in the second part of this paper devoted to the establishment of the elastokinematics equations. In the view of this, new formulations about matrix exponentials and differential transformations will be applied.

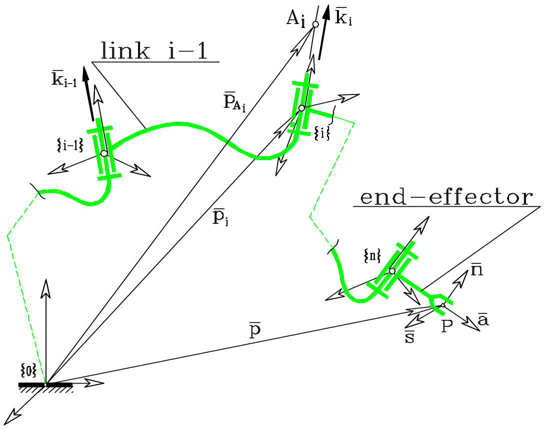

Fig. 1 Geometrical Parameters of *MRS*

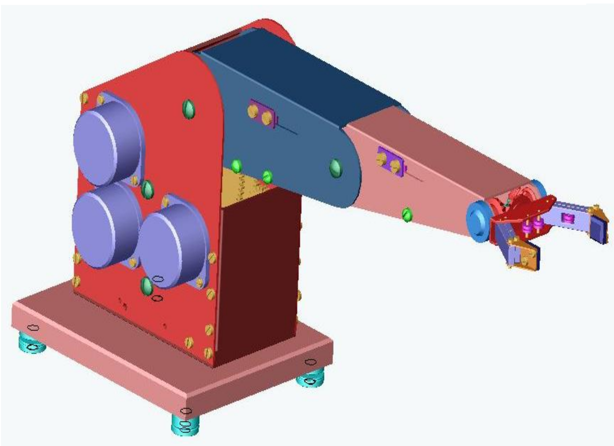

Fig.2 Mechanical Robot Structure

2. LINEAR TRANSFER MATRICES

In this section, according to [8]-[19] and [21], the expressions from *the MEK Algorithm (MEK—Matrix Exponentials in Kinematics)* will be presented. At beginning, the partial derivative, with respect to (q_i) of the locating matrix between $\{0\} \rightarrow \{n\}$ is established as:

$$\left\{ \begin{array}{l} \left\{ {}^{0}_{n}[T] = T_{n0} = \prod_{i=1}^{n} T_{ii-1}(q_i) \right\} = \\ = \left(\prod_{j=1}^{i-1} \delta_{jj-1} \right)^{-1} \cdot \left(\prod_{j=1}^{i} T^{(0)}_{jj-1} \right) \cdot e^{U_i \cdot q_i} \cdot E_{jj-1} \\ where\ E_{jj-1} = \left(\prod_{j=1}^{i} T^{(0)}_{jj-1} \right)^{-1} \cdot \left(\prod_{j=1}^{i} T^{(0)}_{jj-1} \right) \end{array} \right\}; \qquad (1)$$

$$where \qquad \delta_{jj-1} = \left\{ \left\{ T^{(0)}_{jj-1};\ i \geq 2 \right\}; \left\{ I_4;\ i = 1 \right\} \right\}$$

$$\left\{ \begin{array}{l} \left\{ T_{x0} = \prod_{i=1}^{x} T_{ii-1} \right\} = \begin{bmatrix} R_{x0} & \bar{p} \\ 0\ 0\ 0 & 1 \end{bmatrix} = \\ = \left\{ \begin{array}{l} \left\{ \prod_{i=1}^{n} exp\left[A_i \cdot q_i \right] \right\} \cdot T^{(0)}_{x0} \\ where \qquad x = \{ n ; n+1 \} \end{array} \right\} \end{array} \right\}; \qquad (2)$$

$$R_{x0} = \left\{ \prod_{i=1}^{n} exp\left\{ \bar{k}_i^{(0)} \times \right\} q_i \cdot \Delta_i \right\} \cdot R^{(0)}_{x0}$$

$$\bar{p} = \left\{ \begin{array}{l} \sum_{i=1}^{n} \left\{ \prod_{j=0}^{i-1} exp\left\{ \bar{k}_j^{(0)} \times \right\} q_j \cdot \Delta_j \right\} \cdot \bar{b}_i + \\ + \left\{ \prod_{i=1}^{n} exp\left\{ \bar{k}_i^{(0)} \times \right\} q_i \cdot \Delta_i \right\} \cdot \bar{p}^{(0)} \cdot \delta_x \end{array} \right\}$$

$$where\ \delta_x = \left\{ \{ 0;\ x = n \}; \{ 1;\ x = n+1 \} \right\}$$

$$\frac{\partial \{ T_{n0} \}}{\partial q_i} = \begin{bmatrix} A_{ni}(R) & A_{ni}(\bar{p}) \\ 0\ 0\ 0 & 0 \end{bmatrix} = \qquad (3)$$

$$= \left\{ \left\{ \prod_{j=0}^{i-1} exp\left[A_j \cdot q_j \right] \right\} \cdot A_i \cdot \left\{ \prod_{k=i}^{n} exp\left[-A_k \cdot q_k \right] \right\} \cdot T_n^{(0)} \right\}$$

The last column from (3) is taken into account for establish the exponential of the linear transfer matrix. According to [3], first and second matrix exponential from (3) shows as:

$$\prod_{j=0}^{i-1} exp\left[A_j \cdot q_j \right] = \begin{bmatrix} & exp[R] & & exp[\bar{p}] \\ 0 & 0 & 0 & 1 \end{bmatrix}; \qquad (4)$$

$$\left\{ \begin{array}{l} where\ exp[R] = \prod_{j=0}^{i-1} exp\left\{ \left\{ \bar{k}_j^{(0)} \times \right\} q_j \cdot \Delta_j \right\},\ and \\ exp[\bar{p}] = \sum_{j=0}^{i-1} \left\{ \prod_{k=0}^{i-1} exp\left\{ \left\{ \bar{k}_k^{(0)} \times \right\} q_k \cdot \Delta_k \right\} \right\} \cdot \bar{b}_{j+1} \end{array} \right\}$$

$$exp\left\{ \sum_{k=i}^{n} A_k \cdot q_k \right\} = \begin{bmatrix} & exp[R_k] & & exp[\bar{p}_k] \\ 0 & 0 & 0 & 1 \end{bmatrix}; \qquad (5)$$

$$\left\{ \begin{array}{l} where\ exp[R_k] = \prod_{k=i}^{n} exp\left\{ \left\{ \bar{k}_k^{(0)} \times \right\} q_k \cdot \Delta_k \right\},\ and \\ exp[\bar{p}_k] = \sum_{k=i}^{n} \left\{ \prod_{m=i-1}^{k-1} exp\left\{ \left\{ \bar{k}_m^{(0)} \times \right\} q_m \cdot \delta_m \right\} \right\} \cdot \bar{b}_k \end{array} \right\};$$

$$\delta_m = \left\{ \{ 0;\ m = i-1 \}; \{ 1;\ m \geq i \} \right\}.$$

In keeping with the transfer matrix algorithm [5], it is known that the *ith* column from the linear transfer sub-matrix will represent the last column from (3) expression:

$$\left\{ \begin{array}{l} \left\{ \bar{V}_i = {}^{0}J_{iv} \right\} \equiv \left\{ \frac{\partial}{\partial q_i}(\bar{p}_n) = A_{ni}(\bar{p}) \right\} = \\ = \left\{ \prod_{j=0}^{i-1} exp\left\{ \left\{ \bar{k}_j^{(0)} \times \right\} q_j \cdot \Delta_j \right\} \right\} \cdot \bar{v}_i^{(0)} + \\ + \left\{ \prod_{k=i}^{n} exp\left\{ \left\{ \bar{k}_k^{(0)} \times \right\} q_k \cdot \Delta_k \right\} \right\} \cdot \bar{p}_n^{(0)} + \\ + \Delta_i \cdot \left\{ \prod_{j=0}^{i-1} exp\left\{ \left\{ \bar{k}_j^{(0)} \times \right\} q_j \cdot \Delta_j \right\} \right\} \cdot \left\{ \bar{k}_i^{(0)} \times \right\} \cdot A_{mk}, \\ and\ A_{mk} = \sum_{k=i}^{n} \left\{ \prod_{m=i-1}^{k-1} exp\left\{ \left\{ \bar{k}_m^{(0)} \times \right\} q_m \cdot \delta_m \right\} \cdot \bar{b}_k \right\} \end{array} \right\}. \qquad (6)$$

Considering [4] and [6], the linear component ${}^{0}J_{iv}$ can be written under another matrix form:

$$\underset{(3x3)}{ME}(V_{i1}) = \prod_{j=0}^{i-1} exp\left\{ \left\{ \bar{k}_j^{(0)} \times \right\} q_j \cdot \Delta_j \right\}; \qquad (7)$$

$$\underset{(3\times 6)}{ME}(V_{i2}) = I_3 \quad \Delta_i \cdot \left\{ \bar{k}_i^{(0)} \times \right\}; \qquad (8)$$

$$\underset{[9+3\cdot(n-i)]\times 1}{M_{iv}} = \left[\bar{v}_i^{(0)T} \quad \left[\bar{b}_k;\ k = i \rightarrow 3 \right]^T \quad \bar{p}_n^{(0)T} \right]^T; \qquad (9)$$

$$\underset{\{6x[9+3\cdot(n-i)]\}}{ME(V_{i3})} = \begin{bmatrix} I_3 & [0] & [0] \\ [0] & ME(V_{i322}) & ME(V_{i323}) \end{bmatrix}; \qquad (10)$$

$$ME(V_{i322}) = \left\{ \begin{array}{l} \begin{bmatrix} \prod_{m=i-1}^{k-1} exp\left\{ \left\{ \bar{k}_m^{(0)} \times \right\} q_m \cdot \delta_m \cdot \Delta_m \right\} \\ where \quad k = i \rightarrow n \end{bmatrix} \\ \delta_m = \left\{ \{ 0;\ m = i-1 \}; \{ 1;\ m \geq i \} \right\} \end{array} \right\},$$

$$ME\left(V_{i323}\right)=\prod_{k=i}^{n} exp\left\{\left\{\bar{k}_k^{(0)}\times\right\}\ q_k\cdot\Delta_k\right\}.$$

The above expressions are swinging for: $i=1\rightarrow n$. So, the exponential of the linear matrix ${}^{0}J_V\left(\bar{\theta}\right)=V\left(\bar{\theta}\right)$, from Jacobian matrix ${}^{0}J\left(\bar{\theta}\right)$, will be characterized by the expression:

$$\left\{\begin{array}{c} V\left(\bar{\theta}\right)={}^{0}J_V\left(\bar{\theta}\right)= \\ =\left[\begin{array}{c} {}^{0}J_{iv} \quad where \quad i=1\rightarrow n \\ (3x3) \end{array}\right]= \\ =ME\left(V_{i1}\right)\cdot ME\left(V_{i2}\right)\cdot ME\left(V_{i3}\right)\cdot M_{iv} \end{array}\right\}. \qquad (11)$$

Remark: *The second expression from (11) is matrix one. It can be easily applied in the DKM generalized algorithm. As a result, this will dignify a few advantages of the matrix calculus.*

3. ANGULAR TRANSFER MATRICES

In keeping with *MEK Algorithm* from [8]-[19] and [21], at beginning, the first time derivative for the exponential (2) of the locating matrix between $\{0\}\rightarrow\{n\}$ is defined, that is:

$$\left\{\begin{array}{l} {}^{0}_{n}\left[\dot{T}\right]=\dot{T}_{n0}=\sum_{i=1}^{n}\left\{\prod_{j=0}^{i-1} exp\left\{A_j\cdot q_j\right\}\right\}\cdot A^{*} \\ where\ \ A^{*}=\left(A_i\cdot\dot{q}_i\right)\cdot\left\{\prod_{k=i}^{n} exp\left\{A_k\cdot q_k\right\}\right\}\cdot T_{n0}^{(0)} \end{array}\right\};(12)$$

$$and\ \ {}^{0}_{n}\left[T\right]^{-1}=T_{n0}^{-1}=\left\{T_{n0}^{(0)}\right\}^{-1}\cdot\prod_{i=n}^{1}\left\{-exp\left[A_i\cdot q_i\right]\right\}.$$

Performing the matrix product between the two matrices from (10), the expression is obtained:

$$\left\{\begin{array}{l} {}^{0}_{n}\left[\dot{T}\right]\cdot{}^{0}_{n}\left[T\right]^{-1}=\dot{T}_{n0}\cdot T_{n0}^{-1}= \\ =\sum_{i=1}^{n}\left\{\prod_{j=0}^{i-1} exp\left[A_j\cdot q_j\right]\right\}\cdot A_i\cdot A_j^{*} \\ where\ A_j^{*}=\left\{\prod_{j=i-1}^{0}\left\{exp\left[-A_j\cdot q_j\right]\right\}\right\}\cdot\dot{q}_i \end{array}\right\}; \qquad (13)$$

$$\left\{\begin{array}{l} \frac{\partial}{\partial q_i}\left\{{}^{0}_{n}\left[T\right]\right\}{}^{0}_{n}\left[T\right]^{-1}=\frac{\partial}{\partial q_i}\left\{T_{n0}\right\}\cdot T_{n0}^{-1}= \\ =\left\{\prod_{j=0}^{i-1} exp\left[A_j\cdot q_j\right]\right\}\cdot A_i\cdot\left\{\prod_{j=i-1}^{0}\left\{exp\left[-A_j\cdot q_j\right]\right\}\right\} \end{array}\right\}$$

The inverses of the exponentials (4) and (5) are characterized by the following expressions:

$$\left\{\begin{array}{l} \left\{\prod_{i=1}^{n} exp\left[A_i\cdot q_i\right]\right\}^{-1}=\prod_{i=n}^{1}\left\{exp\left[-A_i\cdot q_i\right]\right\}= \\ =\left[\begin{array}{cccc} & exp\left[R_i\right] & & exp\left[\bar{p}_i\right] \\ 0 & 0 & 0 & 1 \end{array}\right] \end{array}\right\}. \quad (14)$$

$$\left\{\begin{array}{l} where\ exp\left[R_i\right]=\prod_{i=n}^{1} exp\left\{-\left\{\bar{k}_i^{(0)}\times\right\}\ q_i\cdot\Delta_i\right\},\ and \\ exp\left[\bar{p}_i\right]=-\sum_{i=n}^{i}\left\{\prod_{j=n}^{i} exp\left\{-\left\{\bar{k}_j^{(0)}\times\right\}q_j\cdot\Delta_j\right\}\right\}\cdot\bar{b}_i \end{array}\right\}.$$

In keeping with *MEK Algorithm*, to above expressions another is compulsory added:

$$\prod_{j=0}^{i-1} exp\left[A_j\cdot q_j\right]=\left[\begin{array}{cccc} & exp\left[R_j\right] & & exp\left[\bar{p}_k\right] \\ 0 & 0 & 0 & 1 \end{array}\right]; \qquad (14)$$

$$\left\{\begin{array}{l} where\ exp\left[R_j\right]=\prod_{j=0}^{i-1} exp\left\{\left\{\bar{k}_j^{(0)}\times\right\}\ q_j\cdot\Delta_j\right\},\ and \\ exp\left[\bar{p}_k\right]=\sum_{j=0}^{i-1}\left\{\prod_{k=0}^{i-1} exp\left\{\left\{\bar{k}_k^{(0)}\times\right\}q_k\cdot\Delta_k\right\}\right\}\cdot\bar{b}_{j+1} \end{array}\right\};$$

$$\prod_{j=i-1}^{0} exp\left[-A_j\cdot q_j\right]=\left[\begin{array}{cccc} & exp\left[-R_k\right] & & exp\left[-\bar{p}_k\right] \\ 0 & 0 & 0 & 1 \end{array}\right].(15)$$

$$\left\{\begin{array}{l} where\ exp\left[-R_k\right]=\prod_{j=i-1}^{0} exp\left\{-\left\{\bar{k}_j^{(0)}\times\right\}q_j\Delta_j\right\},\ and \\ exp\left[-\bar{p}_k\right]=-\sum_{j=i-1}^{0}\left\{\prod_{k=i-1}^{j} exp\left\{-\left\{\bar{k}_k^{(0)}\times\right\}\ q_k\Delta_k\right\}\right\}\bar{b}_j \end{array}\right\}.$$

The skew-symmetric matrix associated to column vector $\bar{\Omega}_i$ of the angular component from the velocity transfer matrix is the result of the following partial derivative:

$$\left\{\begin{array}{l} \left\{\bar{\Omega}_i\times\right\}=\left\{\prod_{j=0}^{i-1} exp\left\{\left\{\bar{k}_j^{(0)}\times\right\}q_j\cdot\Delta_j\right\}\right\}\cdot\Omega_j^{*}, \\ \Omega_j^{*}=\left\{\bar{k}_i^{(0)}\times\right\}\cdot\Delta_i\cdot\prod_{j=i-1}^{0} exp\left\{-\left\{\bar{k}_j^{(0)}\times\right\}\ q_j\cdot\Delta_j\right\} \end{array}\right\}. \ (16)$$

Performing the product in (16), the (3×1) column vector $\bar{\Omega}_i$ is a matrix exponential:

$$\underset{(3\times 1)}{\bar{\Omega}_i}=\left\{\prod_{j=0}^{i-1} exp\left\{\left\{\bar{k}_j^{(0)}\times\right\}\ q_j\cdot\Delta_j\right\}\right\}\cdot\bar{k}_i^{(0)}. \qquad (17)$$

The angular matrix $\Omega\left(\bar{\theta}\right)={}^{0}J_{\Omega}\left(\bar{\theta}\right)$, from the Jacobian matrix ${}^{0}J\left(\bar{\theta}\right)$, shows as:

$$\left\{\begin{array}{c} \Omega\left(\bar{\theta}\right)={}^{0}J_{\Omega}\left(\bar{\theta}\right)= \\ \left[\bar{\Omega}_i=\left\{\prod_{j=0}^{i-1} exp\left\{\left\{\bar{k}_j^{(0)}\times\right\}\ q_j\cdot\Delta_j\right\}\right\}\cdot\bar{k}_i^{(0)}\ \ i=1\rightarrow n\right] \end{array}\right\} \ (18)$$

***Remark**: When the driving joint (j) is prismatic one ($\Delta_j = 0$), then it obtains:* $exp\{0\} = I_3$.

Taking into consideration the same *MEK Algorithm* from [8] - [20], *the Jacobian matrix*, also named *the velocity transfer matrix*, can be determined by means of the matrices (6) - (11) and (18). In the view of this, the other new matrices are implemented as follows:

$$\underset{\{6\times[12+3\cdot(n-i)]\}}{ME\left\{{}^{0}J_i\right\}} = ME\{J_{i1}\}\cdot ME\{J_{i2}\}\cdot ME\{J_{i3}\}; \qquad (19)$$

$$\left\{\begin{array}{l} \text{where } \underset{(6\times6)}{ME}\{J_{i1}\} = \begin{bmatrix} ME\{V_{i1}\} & [0] \\ [0] & ME\{V_{i1}\} \end{bmatrix}, \\ \underset{(6\times9)}{ME}\{J_{i2}\} = \begin{bmatrix} ME\{V_{i2}\} & [0] \\ [0] & I_3 \end{bmatrix}, \\ \text{and } \underset{\{9\times[12+3\cdot(n-i)]\}}{ME\{J_{i3}\}} = \begin{bmatrix} ME\{V_{i3}\} & [0] \\ [0] & I_3 \end{bmatrix} \end{array}\right\};$$

$$\underset{\{[12+3\cdot(n-i)]\times1\}}{M_{iv\omega}} = \begin{bmatrix} \bar{v}_i^{(0)T} & \left[\bar{b}_k\,;\, k=i\to n\right]^T & \bar{p}_n^{(0)T} & \Delta_i\cdot\bar{k}_i^{(0)T} \end{bmatrix}^T.$$

Considering the above notations, the new expression of the Jacobian matrix is:

$${}^{0}J\left(\bar{\theta}\right) = \left\{\begin{pmatrix} {}^{0}J_{iV} \\ {}^{0}J_{i\Omega} \end{pmatrix} \quad i=1\to n\right\} = ME\left\{{}^{0}J_i\right\}\cdot M_{iv\omega}. \qquad (20)$$

and $\quad {}^{0}\dot{J}_i = \dfrac{d}{dt}\left\{ME\left\{{}^{0}J_i\right\}\cdot M_{iv\omega}\right\},\ i=1\to n; \qquad (21)$

where (21) is the first time derivative for every column from Jacobian matrix as exponentials.

4. KINEMATICS EQUATIONS

Considering the same *MEK Algorithm* from [8] and [20], *the DKM Equations* can be likewise defined by means of the matrix exponentials. So, for every $i=1\to n$ the next expressions are:

$${}^{0}\bar{\omega}_i = \Sigma_{j=1}^{i}\left\{\Pi_{k=1}^{j-1} exp\ \left\{\{\bar{k}_k^{(0)}\times\}q_k\cdot\Delta_k\right\}\right\}\cdot\bar{k}_j^{(0)}\cdot\dot{q}_j\cdot\Delta_j \qquad (22)$$

$${}^{i}\bar{\omega}_i = \left\{\left\{R_{i0}^{(0)}\right\}^{-1}\cdot\prod_{j=i}^{1} exp\left\{-\left\{\bar{k}_j^{(0)}\times\right\}q_j\cdot\Delta_j\right\}\right\}\cdot{}^{0}\bar{\omega}_i;$$

$$\left\{\begin{array}{l} {}^{0}\dot{\bar{\omega}}_i = \displaystyle\sum_{j=1}^{i}\left\{\prod_{k=1}^{j-1} exp\ \left\{\{\bar{k}_k^{(0)}\times\}q_k\cdot\Delta_k\right\}\right\}\cdot\bar{k}_j^{(0)}\cdot\ddot{q}_j\cdot\Delta_j + \\ +\displaystyle\sum_{j=1}^{i}\sum_{k=0}^{j-1}\left\{\prod_{m=0}^{k} exp\ \left\{\{\bar{k}_m^{(0)}\times\}q_m\cdot\Delta_m\right\}\cdot\bar{k}_k^{(0)}\right\}^T\cdot\Omega_{jk}^{*}, \text{and} \end{array}\right.$$

$$\Omega_{jk}^{*} = \left\{\prod_{k=1}^{j-1} exp\ \left\{\left\{\bar{k}_k^{(0)}\times\right\}q_k\cdot\Delta_k\right\}\right\}\cdot\Delta_j\cdot\Delta_k\cdot\bar{k}_j^{(0)}\cdot\dot{q}_j\cdot\dot{q}_k$$

$$\left\{\begin{array}{l} {}^{0}\bar{v}_i = \Sigma_{j=1}^{i}\left\{\Pi_{k=0}^{j-1} exp\ \left\{\left\{\bar{k}_k^{(0)}\times\right\}q_k\cdot\Delta_k\right\}\cdot\bar{v}_j^{(0)} + \right. \\ \quad +\Pi_{l=j}^{i} exp\ \left\{\left\{\bar{k}_l^{(0)}\times\right\}q_l\cdot\Delta_l\right\}\cdot\bar{p}_i^{(0)} + \\ \quad \left. +\Delta_j\cdot\Pi_{k=0}^{j-1} exp\ \left\{\left\{\bar{k}_k^{(0)}\times\right\}q_k\cdot\Delta_k\right\}\cdot C_j\right\}\cdot\dot{q}_j \end{array}\right\}; \qquad (23)$$

$$C_j = \left\{\bar{k}_j^{(0)}\times\right\}\cdot\left\{\sum_{k=j}^{i}\left\{\prod_{m=j-1}^{k-1} exp\left\{\left\{\bar{k}_m^{(0)}\times\right\}q_m\cdot\delta_m\right\}\cdot\bar{b}_k\right\}\right\};$$

$$\left\{\begin{array}{l} {}^{0}\dot{\bar{v}}_i = \Sigma_{j=1}^{i}\left\{ME(V_{j1})\cdot ME(V_{j2})\cdot ME(V_{j3})\cdot M_{jv}\right\}\cdot\ddot{q}_j + \\ +\Sigma_{j=1}^{i}\left\{\frac{d}{dt}\left\{ME(V_{j1})\cdot ME(V_{j2})\cdot ME(V_{j3})\cdot M_{jv}\right\}\right\}\cdot\dot{q}_j \end{array}\right\} \qquad (24)$$

On the basis of the same papers [3], [4] and [6], the differential matrices of first and second order, are determined with the exponentials:

$$A_{ki} = \left\{\prod_{j=0}^{i-1} exp\left[A_j\cdot q_j\right]\right\}\cdot A_i\cdot\left\{\prod_{l=i}^{k} exp\left[A_l\cdot q_l\right]\right\}\cdot T_{k0}^{(0)};$$

$$\left\{\begin{array}{l} A_{kjm} = \left\{\displaystyle\prod_{l=0}^{m-1} exp\left[A_l\cdot q_l\right]\right\}\cdot A_m\cdot B_{kjm}\cdot T_{k0}^{(0)} \\ B_{kjm} = \left\{\displaystyle\prod_{i=m}^{j-1} exp\left[A_i\cdot q_i\right]\right\}\cdot A_i\cdot\left\{\displaystyle\prod_{p=i}^{k} exp\left[A_p\cdot q_p\right]\right\} \end{array}\right\} \qquad (25)$$

***Remarks**: The matrix exponentials (ME) enjoy important advantages due to their compact form, easy geometric visualization and especially they avoid the frames typical to every kinetic link. As a result the matrix exponentials will stay at the basis of defining the dynamic control functions for whatever mechanical robot structure, regardless of its building complexity.*

5. EXPONENTIALS AT FLEXIBLE ROBOT

This section is devoted to define the generalized elastodynamics equations, when the robot links are dominated of flexibility properties. At first, a few kinematic transformations are described. In the aria of the small deflections, and considering the aspects from Fig.3, the time functions for the angular and linear deformations of the link (i) are written, according to [30] and [22] – [29] as follows:

$${}^{i}\bar{\delta}_i = \begin{pmatrix} \delta_{xi} \\ \delta_{yi} \\ \delta_{zi} \end{pmatrix} = \left\{\sum_{j=1}^{m_i} q_{ij}(t)\cdot{}^{i}\bar{\delta}_{ij}\right\} = \sum_{j=1}^{m_i} q_{ij}(t)\cdot\begin{pmatrix} \delta_{xij} \\ \delta_{yij} \\ \delta_{zij} \end{pmatrix} \qquad (26)$$

$${}^{i}\bar{d}_i = \begin{pmatrix} u_{xi} \\ v_{yi} \\ w_{zi} \end{pmatrix} = \left\{\sum_{j=1}^{m_i} q_{ij}(t)\cdot{}^{i}\bar{d}_{ij}\right\} = \sum_{j=1}^{m_i} q_{ij}(t)\cdot\begin{pmatrix} u_{ij} \\ v_{ij} \\ w_{ij} \end{pmatrix} \qquad (27)$$

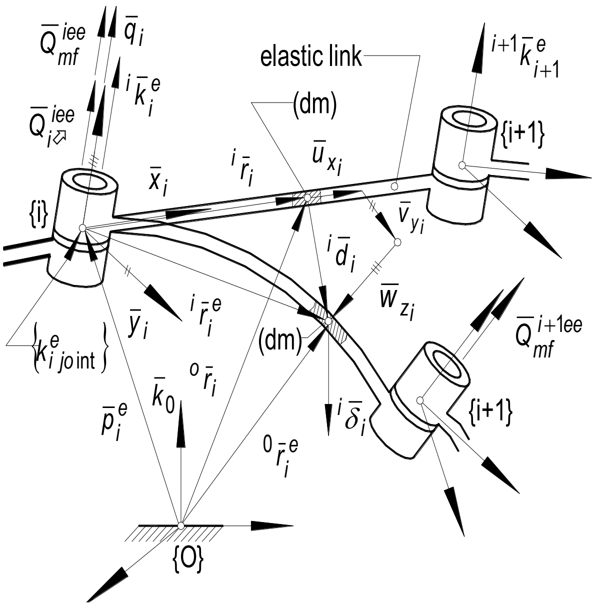

Fig. 3 Elastic Link from MRS

The functions: $q_{ij}(t)$ are time amplitude of the proper modes $j=1 \rightarrow m_i$, and they are completing the generalized variables $q_i(t)$. The position vector for an elementary mass $d\,m$ is:

$$ {}^i\bar{r}_i^e = {}^i\bar{r}_i + {}^i\bar{d}_i\,;\; {}^0\bar{r}_i^e = \bar{p}_i^e + R_{i0}^e \cdot \left({}^i\bar{r}_i + {}^i\bar{d}_i \right). \qquad (28) $$

The symbol (e) highlights the elasticity of the kinetic link. After a few kinematic transformations, the new locating matrix, between adjoining elastic links, shows as:

$$ T_{i\,i-1}^e = T_{ii-1} \cdot \left\{ \begin{bmatrix} I_3 & {}^i\bar{r}_i \\ 0\;0\;0 & 1 \end{bmatrix} + \begin{bmatrix} \left\{ {}^i\bar{\delta}_i \times \right\} & {}^i\bar{d}_i \\ 0\;0\;0 & 0 \end{bmatrix} \right\}; \qquad (29) $$

$$ \Delta T_{ij}^e = \left\{ \begin{array}{c} \begin{bmatrix} I_3 & {}^i\bar{r}_i \\ 0\;0\;0 & 1 \end{bmatrix} + \begin{bmatrix} \left\{ {}^i\bar{\delta}_i \times \right\} & {}^i\bar{d}_i \\ 0\;0\;0 & 0 \end{bmatrix} = \\ \begin{bmatrix} I_3 & {}^i\bar{r}_i \\ 0\;0\;0 & 1 \end{bmatrix} + \sum_{j=1}^{m_i} q_{ij} \begin{bmatrix} \left\{ {}^i\bar{\delta}_{ij} \times \right\} & {}^i\bar{d}_{ij} \\ 0\;0\;0 & 0 \end{bmatrix} \end{array} \right\}; \qquad (30) $$

$$ \left\{ \begin{array}{c} T_{i\,i-1}^e = T_{ii-1} \cdot \Delta T_{ij}^e = \\ = T_{i\,i-1}^{ee} + T_{i\,i-1} \cdot \left\{ \sum_{j=1}^{m_i} q_{ij} \cdot \begin{bmatrix} \left\{ \bar{\delta}_{ij} \times \right\} & \bar{d}_{ij} \\ 0\;0\;0 & 0 \end{bmatrix} \right\} = \\ = T_{i\,i-1}^{ee} + T_{i\,i-1} \cdot \left\{ \sum_{j=1}^{m_i} q_{ij} \cdot \Delta T_{ij} \right\} \end{array} \right\}; \qquad (31) $$

$$ T_{i0}^e = \prod_{j=1}^{i-1} T_{j\,j-1}^e \cdot T_{i\,i-1}\,; $$

$$ \left\{ \begin{array}{c} T_{i0}^e = \prod_{j=1}^{i-1} T_{j\,j-1}^e \cdot T_{i\,i-1} = \\ \prod_{j=1}^{i-1} \left\{ T_{j\,j-1}^{ee} + T_{j\,j-1} \cdot \left\{ \sum_{k=1}^{m_j} q_{jk} \cdot \begin{bmatrix} \left\{ \bar{\delta}_{jk} \times \right\} & \bar{d}_{jk} \\ 0\;0\;0 & 0 \end{bmatrix} \right\} \right\} \cdot T_{i\,i-1} \end{array} \right\} \qquad (32) $$

The locating matrix $T_{i\,i-1}$ is answerable to rigid link, while ΔT_{ij}^e to small deformations of link.

The above *kinematic transformations* can be also obtained by means of *the matrix exponentials*. In keeping with [8], [12] and [30], the exponentials are applied for elastic links:

$$ \left\{ \begin{array}{l} T_{i\,i-1}^e = T_{i\,i-1}^{ee(0)} \cdot e^{U_i \cdot q_i} + T_{i\,i-1}^{(0)} \cdot exp\left\{ \sum_{j=1}^{m_i} \Delta T_{ij} \cdot q_{ij} \right\} = \\ = T_{i\,i-1}^{ee(0)} \cdot exp\left\{ U_i \cdot q_i \right\} + T_{i\,i-1}^{(0)} \cdot \prod_{j=1}^{m_i} exp\left\{ \Delta T_{ij} \cdot q_{ij} \right\} \end{array} \right\}; \qquad (33) $$

$$ \left\{ \begin{array}{c} exp\left\{ \Delta T_{ij} \cdot q_{ij} \right\} = exp\left\{ \begin{bmatrix} \left\{ \bar{\delta}_{ij} \times \right\} & \bar{d}_{ij} \\ 0\;0\;0 & 0 \end{bmatrix} \cdot q_{ij} \right\} \\ = exp\left\{ \begin{bmatrix} exp\left\{ \bar{\delta}_{ij} \times \right\} q_{ij} & \bar{b}_{ij} \\ 0\;0\;0 & 1 \end{bmatrix} \right\} \end{array} \right\}; \qquad (34) $$

$$ exp\left\{ \begin{bmatrix} 0 & -\delta_{zij} & \delta_{yij} \\ \delta_{zij} & 0 & -\delta_{xij} \\ -\delta_{yij} & \delta_{xij} & 0 \end{bmatrix} \cdot q_{ij} \right\} = exp\left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \cdot \delta_{xij} \cdot q_{ij} + \right. $$

$$ \left. + \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \cdot \delta_{yij} \cdot q_{ij} + \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \delta_{zij} \cdot q_{ij} \right\}; \qquad (35) $$

$$ exp\left\{ \bar{\delta}_{ij} \times \right\} q_{ij} = exp\left\{ \begin{bmatrix} 0 & -\delta_{zij} & \delta_{yij} \\ \delta_{zij} & 0 & -\delta_{xij} \\ -\delta_{yij} & \delta_{xij} & 0 \end{bmatrix} \cdot q_{ij} \right\} = ; \qquad (36) $$

$$ = exp\left\{ U_{ix} \cdot \delta_{xij} \cdot q_{ij} \right\} \cdot exp\left\{ U_{iy} \cdot \delta_{yij} \cdot q_{ij} \right\} \cdot exp\left\{ U_{iz} \cdot \delta_{zij} \cdot q_{ij} \right\}; $$

$$ \left\{ \begin{array}{l} exp\left\{ U_{iu} \cdot \delta_{uij} \cdot q_{ij} \right\} \\ where\ u = \left\{ x; y; z \right\} \end{array} \right\} = \bar{u}_i^{(0)} \cdot \bar{u}_i^{(0)T} \cdot \left[1 - c\left(\delta_{xij} \cdot q_{ij} \right) \right] + $$

$$ + I_3 \cdot c\left(\delta_{xij} \cdot q_{ij} \right) + \left\{ \bar{u}_i^{(0)} \times \right\} s\left(\delta_{uij} \cdot q_{ij} \right); \qquad (37) $$

$$ \left\{ \begin{array}{l} T_{i0}^e = \prod_{j=1}^{i-1} \left\{ T_{j\,j-1}^{ee(0)} \cdot exp\left\{ U_j \cdot q_j \right\} \right\} \cdot T_{i\,i-1}^{(0)} \cdot exp\left\{ U_i \cdot q_i \right\} \\ + \prod_{j=1}^{i-1} \left\{ T_{j\,j-1}^{(0)} \cdot \prod_{k=1}^{m_j} \left\{ q_{jk} \cdot \Delta T_{jk} \right\} \right\} \cdot T_{i\,i-1}^{(0)} \cdot exp\left\{ U_i \cdot q_i \right\} \end{array} \right\} \qquad (38) $$

On the matrix (38) which it expresses the locating of the frame $\{i\}$ with respect to fixed basis, are applied the time derivatives of first and second order as follows:

$$\dot{T}_{i0}^{e} = \begin{bmatrix} \dot{R}_{i0}^{e} & \dot{\bar{p}}_{i}^{e} \\ 000 & 0 \end{bmatrix};$$

$$\dot{T}_{i0}^{e} = \sum_{k=1}^{i} T_{k0}^{e} \cdot U_k \cdot T_{ik}^{e} \cdot \dot{q}_k + \sum_{k=1}^{i-1} \sum_{l=1}^{m_k} T_{kk-1}\, \Delta T_{kl}^{e} \cdot T_{il}^{e} \cdot \dot{q}_{kl} \quad (39)$$

$$\ddot{T}_{i0}^{e} = \begin{bmatrix} \ddot{R}_{i0}^{e} & \ddot{\bar{p}}_{i}^{e} \\ 000 & 0 \end{bmatrix}; \quad \ddot{T}_{i0}^{e} = \ddot{T}_{i0A}^{e} + \ddot{T}_{i0B}^{e} + \ddot{T}_{i0C}^{e};$$

$$\ddot{T}_{i0A}^{e} = \sum_{k=1}^{i} T_{k0}^{e} \cdot U_k \cdot T_{ik}^{e} \cdot \ddot{q}_k + \sum_{k=1}^{i-1} \sum_{l=1}^{m_i} T_{kk-1} \cdot \Delta T_{kl}^{e} \cdot T_{il}^{e} \cdot \ddot{q}_{kl} \quad (40)$$

$$\ddot{T}_{i0B}^{e} = \left\{ \begin{aligned} &\sum_{k=1}^{i} \sum_{m=1}^{k} T_{m0}^{e} \cdot U_m \cdot T_{km}^{e} \cdot U_k \cdot T_{ik}^{e} \cdot \dot{q}_m \cdot \dot{q}_k + \\ &+ \sum_{k=1}^{i} \sum_{l=k}^{i} T_{k0}^{e} \cdot U_k \cdot T_{lk}^{e} \cdot U_l \cdot T_{il}^{e} \cdot \dot{q}_l \cdot \dot{q}_k + \\ &+ \sum_{k=1}^{i-1} \sum_{l=1}^{m_i} \dot{T}_{kk-1} \cdot \Delta T_{kl}^{e} \cdot T_{il}^{e} \cdot \dot{q}_k \cdot \dot{q}_{kl} \end{aligned} \right\}; \quad (41)$$

$$\ddot{T}_{i0C}^{e} = \sum_{k=1}^{i-1} \sum_{l=1}^{m_i} \sum_{m=l}^{i} T_{kk-1} \cdot \Delta T_{kl}^{e} \cdot T_{mk}^{e} \cdot U_m \cdot T_{il}^{e} \cdot \dot{q}_{kl} \cdot \dot{q}_m \quad (42)$$

In these expressions are substituted the matrices defined by means of the exponentials above shown. Considering (41) and (42), the angular rotation velocity and acceleration are defined as:

$$\left\{ {}^{0}\bar{\omega}_{i}^{e} \times \right\} = \dot{R}_{i0}^{e} \cdot \{R_{i0}^{e}\}^{T};$$

$$vect\left\{ {}^{0}\bar{\omega}_{i}^{e} \times \right\} = \left[{}^{0}\omega_{ix}^{e} \quad {}^{0}\omega_{iy}^{e} \quad {}^{0}\omega_{iz}^{e} \right]^{T};$$

$${}^{0}\bar{\omega}_{i}^{e} = vect\{\dot{R}_{i0}^{e} \cdot \{R_{i0}^{e}\}^{T}\};$$

$${}^{0}\dot{\bar{\omega}}_{i}^{e} = vect\left\{ \ddot{R}_{i0}^{e} \cdot \{R_{i0}^{e}\}^{T} + \dot{R}_{i0}^{e} \cdot \{\dot{R}_{i0}^{e}\}^{T} \right\}. \quad (43)$$

The linear velocity and acceleration of the elementary mass dm are defined by means of the time derivative applied on the position vector (3). As a result, the next expressions are:

$${}^{0}\bar{r}_{i}^{e} = \bar{p}_{i}^{e} + R_{i0}^{e} \cdot \left({}^{i}\bar{r}_{i} + {}^{i}\bar{d}_{i} \right) + R_{i0}^{e} \cdot {}^{i}\bar{d}_{i};$$

$$\left\{ \begin{aligned} &{}^{0}\dot{\bar{r}}_{i}^{e} = \dot{\bar{p}}_{i}^{e} + \dot{R}_{i0}^{e} \cdot \left({}^{i}\bar{r}_{i} + {}^{i}\bar{d}_{i} \right) + R_{i0}^{e} \cdot {}^{i}\dot{\bar{d}}_{i} = \\ &= \dot{\bar{p}}_{i}^{e} + {}^{0}\bar{\omega}_{i}^{e} \times R_{i0}^{e} \cdot \left({}^{i}\bar{r}_{i} + {}^{i}\bar{d}_{i} \right) + R_{i0}^{e} \cdot {}^{i}\dot{\bar{d}}_{i} \end{aligned} \right\}; \quad (44)$$

$$\left\{ \begin{aligned} &{}^{0}\ddot{\bar{r}}_{i}^{e} = \ddot{\bar{p}}_{i}^{e} + 2 \cdot {}^{0}\bar{\omega}_{i}^{e} \times R_{i0}^{e} \cdot {}^{i}\dot{\bar{d}}_{i} + R_{i0}^{e} \cdot {}^{i}\ddot{\bar{d}}_{i} + \\ &{}^{0}\dot{\bar{\omega}}_{i}^{e} \times R_{i0}^{e} \left({}^{i}\bar{r}_{i} + {}^{i}\bar{d}_{i} \right) + {}^{0}\bar{\omega}_{i}^{e} \times {}^{0}\bar{\omega}_{i}^{e} \times R_{i0}^{e} \cdot \left({}^{i}\bar{r}_{i} + {}^{i}\bar{d}_{i} \right) \end{aligned} \right\} \quad (45)$$

Unlike *MRS* dominated of stiffness hypothesis, the column vector of the generalized variables, in the case of the structures with flexible links, is completed with (26) and (27) as below:

$$\bar{\theta}^{e}(t) = \left[\left[\theta_{ij}^{eT}(t) \quad j = 0 \rightarrow m_i \right] \quad i = 1 \rightarrow n \right]^{T}; \quad (46)$$

$$\theta_{ij}^{eT}(t) = \left\{ \{ q_i(t) \text{ if } j = 0 \}; \{ q_{ij}(t) \text{ if } j \geq 1 \} \right\};$$

$$\dot{\bar{\theta}}^{e}(t) = \left[\dot{\bar{\theta}}_{ij}^{eT}(t) \left[\dot{\theta}_{ij}^{eT}(t) \quad j = 0 \rightarrow m_i \right] \quad i = 1 \rightarrow n \right]^{T};$$

$$\dot{\theta}_{ij}^{eT}(t) = \left\{ \{ \dot{q}_i(t) if j = 0 \}; \{ \dot{q}_{ij}(t) if j \geq 1 \} \right\}; \quad (47)$$

$$\ddot{\bar{\theta}}^{e}(t) = \left[\ddot{\bar{\theta}}_{ij}^{eT}(t) = \left[\ddot{\theta}_{ij}^{eT}(t) \quad j = 0 \rightarrow m_i \right] \quad i = 1 \rightarrow n \right]^{T};$$

$$\ddot{\theta}_{ij}^{eT}(t) = \left\{ \{ \ddot{q}_i(t) if j = 0 \}; \{ \ddot{q}_{ij}(t) if j \geq 1 \} \right\}; \quad (48)$$

$$\left\{ {}^{0}J_{i}^{e} \begin{pmatrix} q_j(t) & q_{jk}(t) \\ j = 1 \rightarrow i & k = 1 \rightarrow m_j \end{pmatrix} = \begin{bmatrix} {}^{0}J_{iv}^{e} \\ {}^{0}J_{i\omega}^{e} \end{bmatrix} \right\} \in {}^{0}J(\bar{\theta})^{e}. \quad (49)$$

The above expression shows that every column of Jacobian matrix is function of generalized variables. Considering [8] and [30], its expression is defined by means of the classical transformations or matrix exponentials.

6. CONCLUSIONS

Within of this paper, the generalized elastokinematics equations have been analyzed for robot structures with flexible links. For define the kinematics and differential matrices functions in the case of the robot structures with rigid and elastic links have been applied the matrix exponentials, in accordance with the *ME Algorithm*. They are characterized through important advantages with respect to classical transformations. So, the matrix exponentials (ME) enjoy important advantages due to their compact form, easy geometric visualization and especially they avoid the frames typical to every kinetic link. As a result the matrix exponentials will stay at the basis of defining the linear and angular transfer matrices. By means of the matrix exponentials have been also determined all kinematic parameters. They characterize the equations of direct and control kinematics for any mechanical robot structure, regardless of its constructive complexity.

In the second part of this paper an elastic structure of the serial robot was analyzed from view point of elastokinematics behavior. As a result, using the properties of the matrix exponentials, the locating matrices and their time derivatives corresponding to small deformations been established. On the basis of these differential transformations, in the final part of the paper, have been determined the linear and angular velocities and accelerations, as well as Jacobian matrix as function of the column vector of the generalized variables.

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Exponențiale de matrice în elastocinematica roboților

Rezumat: Obiectivul principal al lucrării constă în stabilirea ecuațiilor generalizate ale elastocinematicii structurilor de roboți cu elemente flexibile. Pentru cinematica și matricele diferențiale ale structurilor de robot cu elemente rigide și elastice se vor aplica exponențiale de matrice, în conformitate cu algoritmul dezvoltat de autorul principal. Ca urmare, exponențialele matrice vor sta la baza stabilirii matricelor de transfer liniare și unghiulare, determinându-se toți parametrii cinematici. În partea a doua a lucrării se va analiza structura elastocinematică a unui robot serial. Utilizând aceleași proprietăți ale exponențialelor de matrice, se vor stabili matricele de situare și derivatele în raport cu timpul corespunzătoare deformațiilor mici. Pe baza transformărilor diferențiale în partea finală a lucrării se vor determina vitezele și accelerațiile lineare și unghiulare, precum și matricea Jacobiană.

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