TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering
Vol. 62, Issue I, March, 2019

GEOMETRIC MODEL OF FANUC LR Mate 100iB CONSIDERING THE DIMENSIONAL AND GENERALIZED COORDINATES ERRORS

Ovidiu-Aurelian DETEȘAN, Adina Veronica CRIȘAN

Abstract: *The methods of geometric modeling of industrial robots, regularly used in the literature, are strictly related to robot's nominal geometry, the robot being considered as a system of rigid bodies, dimensionally and positionally perfect. This paper outlines the influence of dimensional and generalized coordinates errors on the position and orientation of the end-effecter of FANUC LR Mate 100iB industrial robot. The dimensional-constructive errors and the errors of generalized coordinates are included in the equations of the direct geometric model, determined by the method of situation matrices. By substituting numerical values in these equations, the influence of these errors on the operational position and orientation parameters is put in evidence.*

Key words: *articulated robot, geometric errors, symbolic computation, FANUC.*

1. INTRODUCTION

When modeling a robot, usually the methods applied do not consider the geometric, kinematic and dynamic errors [1]. Some simulators, as SimMEcROb [2] are complex applications that analyze various parameters of serial industrial robots, including precision, but they are difficult to be used because of the software requirements such as: operating system and other related applications that are required to be installed. A major drawback of pure numerical applications is that the results are numeric only and the algebraic form of the geometric, kinematic or dynamic model of the robot put into analysis cannot be determined.

The goal of this paper is to establish the equations of the geometric model of an articulated industrial robot, by symbolic computations, considering the dimensional errors and the errors of the generalized coordinates.

2. FANUC LR Mate 100iB

The robot which is the object of this study is FANUC LR Mate 100iB [3] and it is used with didactic and research purpose at the Laboratory of Applied Mechanics in Robotics, Faculty of Machine Building, Technical University of Cluj-Napoca, Romania. It is an articulated robot of a small to medium size, having five degrees of freedom (fig. 1). The kinematic scheme is presented in fig. 2 and it will be the source of input data in the algorithm of defining the mechanical structure of the robot.

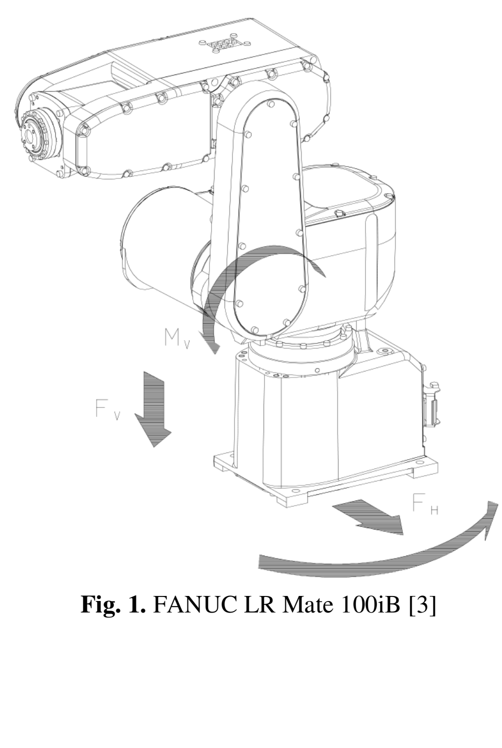

Fig. 1. FANUC LR Mate 100iB [3]

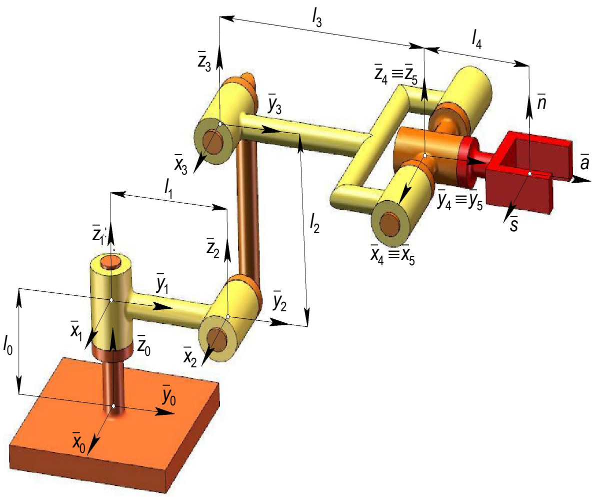

Fig. 2. FANUC LR Mate 100iB. Kinematic scheme [2]

3. THE METHOD OF SITUATION MATRICES

Achieving the definition of the robot's mechanical structure [4] is made possible by describing the position and orientation of the frames $\{i\}$ attached to each of the robot's joints (i), as well as to the end-effecter (n+1). These geometric amounts are expressed by the matrices of position-orientation, also called matrices of situation, ${}^{i-1}_{i}[T], i = 1 \div n+1$:

$$ {}^{i-1}_{i}[T] = \left[\begin{array}{ccc|c} \multicolumn{3}{c|}{{}^{i-1}_{i}[R]} & {}^{i-1}\bar{p}_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right]. \qquad (1) $$

These matrices contain the rotation submatrices, ${}^{i-1}_{i}[R]$, expressing the orientation of the frame axes $\{i\}$ with respect to the frame $\{i\text{-}1\}$ and the position vectors ${}^{i-1}\bar{p}_i$, expressing the position of the origins (O_i) of frames $\{i\}$ with respect to the origins (O_{i-1}) of the frames $\{i\text{-}1\}$ [5]. Using iterative matrix multiplications as:

$$ {}^{0}_{i}[T] = {}^{0}_{i-1}[T] \cdot {}^{i-1}_{i}[T], \qquad (2) $$

the position-orientation matrices of frames $\{i\}$, with respect to the fixed frame $\{0\}$ are determined. Among them, the most important is the matrix ${}^{0}_{n+1}[T]$. It expresses the position of the gripper's characteristic point, by the position vector $\bar{p}_{n+1}$ and the orientation of the frame $\{n+1\}$ axes, with respect to the fixed frame $\{0\}$.

4. THE FUNCTIONS rotx(), roty(), rotz()

To define the mechanical structure of the robot, based on its kinematic scheme (fig. 2), three functions are written in MATLAB. They calculate the expressions of the rotation matrices with respect to the axes *x, y* and *z*. The functions are:

rotx(q) – determines the rotation matrix with respect to *x*-axis with the angle *q*, expressed in radians.

Function file rotx.m

```
function Rx = rotx( q )
Rx = [0 pi/2 pi/2
    pi/2 q pi/2+q
    pi/2 pi/2-q q];
Rx = cos(Rx);
end
```

roty(q) – computes the rotation matrix with respect to *y*-axis with the angle *q*, expressed in radians.

Function file roty.m

```
function Ry = roty( q )
Ry = [q pi/2 pi/2-q
    pi/2 0 pi/2
    pi/2+q pi/2 q];
Ry = cos(Ry);
end
```

rotz(q) – computes the rotation matrix with respect to *z*-axis with the angle *q*, expressed in radians.

Function file rotz.m

```
function Rz = rotz( q )
Rz = [q pi/2+q pi/2
    pi/2-q q pi/2
    pi/2 pi/2 0];
Rz = cos(Rz);
end
```

5. DEFINING THE ROBOT'S MECHANICAL STRUCTURE

The script file *FANUC_geo_err.m* was created with the purpose of determining the equations of the direct geometric model of FANUC robot, considering the geometric and generalized coordinates errors. The first section of the script handles the working environment preparation. With this regard, all the variables existing in the current MATLAB workspace are cleared and the file *FANUC_geo_err.txt* is created and open to be written, recording the content of the *Command Window* associated to the current MATLAB session. The declaration of the symbolic variables *q1…q5* follows, representing the generalized coordinates, as well as the variables *l0…l4*, which are the geometric-constructive parameters.

Along them, the symbolic variables *dq1…dq5* are defined, representing the errors of the generalized parameters, also *dl0…dl4*, the dimensional parameters errors. The variables *q1r…q5r* are then initialized, being the real generalized coordinates (considering the errors) and *l0r…l4r* also, which are the real geometric-constructive parameters.

```
clear variables
diary FANUC_geo_err.txt
syms q1 q2 q3 q4 q5 real
syms l0 l1 l2 l3 l4 real
syms dq1 dq2 dq3 dq4 dq5 real
syms dl0 dl1 dl2 dl3 dl4 real
%% r for real, d for delta
q1r = q1 + dq1;
q2r = q2 + dq2;
q3r = q3 + dq3;
q4r = q4 + dq4;
q5r = q5 + dq5;
l0r = l0 + dl0;
l1r = l1 + dl1;
l2r = l2 + dl2;
l3r = l3 + dl3;
l4r = l4 + dl4;
```

Using the functions *rotx(), roty()* and *rotz()*, the situation matrices of the frames $\{i\}$ with respect to the frames $\{i\text{-}1\}$ are expressed, using the relations (1):

```
T10 = [simplify(rotz(q1r)) [0;0;l0r]
    0 0 0 1];
display(T10)
T21 = [simplify(rotx(q2r)) [0;l1r;0]
    0 0 0 1];
display(T21)
T32 = [simplify(rotx(q3r)) [0;0;l2r]
    0 0 0 1];
display(T32)
T43 = [simplify(rotx(q4r)) [0;l3r;0]
    0 0 0 1];
display(T43)
T54 = [simplify(roty(q5r)) [0;0;0]
    0 0 0 1];
display(T54)
T65 = [eye(3) [0;l4r;0]
    0 0 0 1];
display(T65)
```

The effect of these code lines is to generate the matrices ${}^{i-1}[T]_i, i = 1 \dots 6$.

```
T10 =
[c(dq1 + q1),-s(dq1 + q1),0,         0]
[s(dq1 + q1), c(dq1 + q1),0,         0]
[          0,            0,1, dl0 + l0]
[          0,            0,0,         1]
T21 =
[1,           0,            0,         0]
[0,c(dq2 + q2),-s(dq2 + q2), dl1 + l1]
[0,s(dq2 + q2), c(dq2 + q2),         0]
[0,           0,            0,         1]
T32 =
[ 1,          0,            0,         0]
[0,c(dq3 + q3),-s(dq3 + q3),         0]
[0,s(dq3 + q3), c(dq3 + q3), dl2 + l2]
[0,           0,            0,         1]
T43 =
[1,           0,            0,         0]
[0,c(dq4 + q4),-s(dq4 + q4), dl3 + l3]
[0,s(dq4 + q4), c(dq4 + q4),         0]
[0,           0,            0,         1]
T54 =
[ c(dq5 + q5), 0, s(dq5 + q5), 0]
[           0, 1,          0 , 0]
[-s(dq5 + q5), 0, c(dq5 + q5), 0]
[           0, 0,           0, 1]
T65 =
[ 1, 0, 0,        0]
[ 0, 1, 0, dl4 + l4]
[ 0, 0, 1,        0]
[ 0, 0, 0,        1]
```

6. THE SITUATION MATRICES WITH RESPECT TO THE FIXED FRAME {0}

By applying the iterative relation (2), the script determines the situation matrices of the frames attached to the robot's joints and to the end-effecter, ${}^{0}[T]_i, i = 2 \dots 6$, with respect to the fixed frame $\{0\}$.

```
T20 = simplify(T10*T21);
display(T20)
T30 = simplify(T20*T32);
display(T30)
T40 = simplify(T30*T43);
display(T40)
T50 = simplify(T40*T54);
display(T50)
T60 = simplify(T50*T65);
display(T60)
```

The *simplify()* function, used for simplifying the expressions of the situation matrices, is very useful for obtaining more compact symbolic expressions. The obtained matrices are:

```
T20 =
[c(dq1 + q1),-c(dq2 + q2)*s(dq1 + q1), s(dq1 + q1)*s(dq2 + q2),-s(dq1 + q1)*(dl1 + l1)]
[s(dq1 + q1), c(dq1 + q1)*c(dq2 + q2),-c(dq1 + q1)*s(dq2 + q2), c(dq1 + q1)*(dl1 + l1)]
[          0,             s(dq2 + q2),             c(dq2 + q2),               dl0 + l0]
[          0,                       0,                       0,                      1]

R30 =
[c(dq1 + q1),-c(dq2 + dq3 + q2 + q3)*s(dq1 + q1), s(dq2 + dq3 + q2 + q3)*s(dq1 + q1)]
[s(dq1 + q1), c(dq2 + dq3 + q2 + q3)*c(dq1 + q1),-s(dq2 + dq3 + q2 + q3)*c(dq1 + q1)]
[          0,             s(dq2 + dq3 + q2 + q3),             c(dq2 + dq3 + q2 + q3)]

p3 =
 s(dq1 + q1)*s(dq2 + q2)*(dl2 + l2) - s(dq1 + q1)*(dl1 + l1)
 c(dq1 + q1)*(dl1 + l1) - c(dq1 + q1)*s(dq2 + q2)*(dl2 + l2)
                           dl0 + l0 + c(dq2 + q2)*(dl2 + l2)
R40 =
[c(dq1+q1),-c(dq2+dq3+dq4+q2+q3+q4)*s(dq1+q1), s(dq2+dq3+dq4+q2+q3+q4)*s(dq1+q1)]
[s(dq1+q1), c(dq2+dq3+dq4+q2+q3+q4)*c(dq1+q1),-s(dq2+dq3+dq4+q2+q3+q4)*c(dq1+q1)]
[        0,           s(dq2+dq3+dq4+q2+q3+q4),           c(dq2+dq3+dq4+q2+q3+q4)]

p4 =
s(dq1+q1)*s(dq2+q2)*(dl2+l2)-c(dq2+dq3+q2+q3)*s(dq1+q1)*(dl3+l3)-s(dq1+q1)*(dl1+l1)
c(dq1+q1)*(dl1+l1)+c(dq2+dq3+q2+q3)*c(dq1+q1)*(dl3+l3)-c(dq1+q1)*s(dq2+q2)*(dl2+l2)
                                dl0+l0+s(dq2+dq3+q2+q3)*(dl3+l3)+c(dq2+q2)*(dl2+l2)
R50 =
[c(dq1+q1)*c(dq5+q5)-s(dq2+dq3+dq4+q2+q3+q4)*s(dq1+q1)*s(dq5+q5),-c(dq2+dq3+dq4+q2+q3+
          +q4)*s(dq1+q1),c(dq1+q1)*s(dq5+q5)+s(dq2+dq3+dq4+q2+q3+q4)*c(dq5+q5)*s(dq1+q1)]
[------------------------------------------------------------------------------------]
[c(dq5+q5)*s(dq1+q1)+s(dq2+dq3+dq4+q2+q3+q4)*c(dq1+q1)*s(dq5+q5),c(dq2+dq3+dq4+q2+q3+q4)*
            *c(dq1+q1),s(dq1+q1)*s(dq5+q5)-sin(dq2+dq3+dq4+q2+q3+q4)*c(dq1+q1)*c(dq5+q5)]
[------------------------------------------------------------------------------------]
[-c(dq2+dq3+dq4+q2+q3+q4)*s(dq5+q5), s(dq2+dq3+dq4+q2+q3+q4), c(dq2+dq3+dq4+q2+q3+q4)*
                                                                          *c(dq5+q5)]
p5 =
s(dq1+q1)*s(dq2+q2)*(dl2+l2)-c(dq2+dq3+q2+q3)*s(dq1+q1)*(dl3+l3)-s(dq1+q1)*(dl1+l1)
c(dq1+q1)*(dl1+l1)+c(dq2+dq3+q2+q3)*c(dq1+q1)*(dl3+l3)-c(dq1+q1)*s(dq2+q2)*(dl2+l2)
                                dl0+l0+s(dq2+dq3+q2+q3)*(dl3+l3)+c(dq2+q2)*(dl2+l2)
R60 =
[c(dq1+q1)*c(dq5+q5)-s(dq2+dq3+dq4+q2+q3+q4)*s(dq1+q1)*s(dq5+q5),-c(dq2+dq3+dq4+q2+q3+
          +q4)*s(dq1+q1),c(dq1+q1)*s(dq5+q5)+s(dq2+dq3+dq4+q2+q3+q4)*c(dq5+q5)*s(dq1+q1)]
[------------------------------------------------------------------------------------]
[c(dq5+q5)*s(dq1+q1)+s(dq2+dq3+dq4+q2+q3+q4)*c(dq1+q1)*s(dq5+q5),c(dq2+dq3+dq4+q2+q3+q4)*
            *c(dq1+q1),s(dq1+q1)*s(dq5+q5)-sin(dq2+dq3+dq4+q2+q3+q4)*c(dq1+q1)*c(dq5+q5)]
[------------------------------------------------------------------------------------]
[-c(dq2+dq3+dq4+q2+q3+q4)*s(dq5+q5), s(dq2+dq3+dq4+q2+q3+q4), c(dq2+dq3+dq4+q2+q3+q4)*
                                                                          *c(dq5+q5)]
p6 =
s(dq1+q1)*s(dq2+q2)*(dl2+l2)-c(dq2+dq3+dq4+q2+q3+q4)*s(dq1+q1)*(dl4+l4)-c(dq2+dq3+q2+q3)*
*s(dq1+q1)*(dl3+l3)-s(dq1+q1)*(dl1+l1)
--------------------------------------------------------------------------------------
c(dq1+q1)*(dl1+l1)+c(dq2+dq3+dq4+q2+q3+q4)*c(dq1+q1)*(dl4+l4)+c(dq2+dq3+q2+q3)*c(dq1+
+q1)*(dl3+l3)-c(dq1+q1)*s(dq2+q2)*(dl2+l2)
--------------------------------------------------------------------------------------
dl0+l0+s(dq2+dq3+q2+q3)*(dl3+l3)+c(dq2+q2)*(dl2+l2)+s(dq2+dq3+dq4+q2+q3+q4)*(dl4+l4)
```

Because of the complexity of the obtained equations, the situation matrices *T30…T60* are split into the rotation matrices *R30… R60* and the position vectors *p3…p6*.

Among the above matrices, the most important are *T50* and *T60*. They are the equations of the direct geometric model of FANUC LR Mate 100iB, considering the geometric errors and they express the position and orientation of the frame {5} attached to the 5th joint, as well as the frame {6}, attached to the end-effecter, with respect to the fixed frame{0}.

7. NUMERIC DATA

In order to make numeric substitutions into the equations of the DGM, one defines the parameters *l0n…l4n* and they are initialized with the nominal values of the constructive dimensions of FANUC LR Mate 100iB, taken from [2], expressed in millimeters.

```
l0n = 350;
l1n = 150;
l2n = 250;
l3n = 220;
l4n = 225;
```

Some values of generalized coordinate from a robot program [6] are then stored into the variables *q1n…q5n*, expressed in degrees and converted to radians, using the function *deg2rad()*:

```
q1n = deg2rad(45);
q2n = deg2rad(12);
q3n = deg2rad(22.5);
q4n = deg2rad(-15);
q5n = deg2rad(90);
```

The following step is to define by numerical values the errors of constructive dimensions *dl0n…dl4n*, as well as the errors of the generalized coordinates, *dq1n…dq5n*:

```
dl0n = .235;
dl1n = .12;
dl2n = -.07;
dl3n = .115;
dl4n = -.089;
dq1n = deg2rad(.35);
dq2n = deg2rad(.151);
dq3n = deg2rad(.255);
dq4n = deg2rad(.009);
dq5n = deg2rad(.425);
```

All numeric data will be substituted in the basic form of *T60*, yielding two forms of the transformation matrix: one for the nominal mechanical structure (*T60nn*) and one for the real mechanical structure, including the errors (*T60nr*).

```
T60n = subs(T60, [l0,l1,l2,l3,l4],...
 [l0n, l1n, l2n, l3n, l4n]);
T60n = subs(T60n, [q1,q2,q3,q4,q5],...
 [q1n, q2n, q3n, q4n, q5n]);
T60nn = subs(T60n, [dl0,dl1,dl2,...
 dl3, dl4], zeros(1,5));
T60nn = eval(subs(T60nn, [dq1,...
 dq2, dq3, dq4, dq5], zeros(1,5)));
T60nr = subs(T60n, [dl0,dl1,dl2,...
 dl3,dl4],[dl0n,dl1n,dl2n,dl3n,dl4n]);
T60nr=eval(subs(T60nr,[dq1,dq2,dq3,...
dq4,dq5],[dq1n,dq2n,dq3n,dq4n,dq5n]));
disp('Nom. numeric situation matrix:')
display(T60nn)
disp('Real numeric situation matrix:')
display(T60nr)
dT60n = T60nn - T60nr;
disp('Delta numeric sit. matrix:')
display(dT60n)
save FANUC_geo_err
diary off
```

The difference of the two matrices will mark the operational coordinates errors (the Cartesian components of the position vector, the first three elements from the 4th column, as well as the values of the errors of the rotation matrix):

```
T60nn =
-0.2360   -0.6665   0.7071  -347.4894
 0.2360    0.6665   0.7071   347.4894
-0.9426    0.3338        0   794.2528
      0         0        0     1.0000

T60nr =
 -0.2475   -0.6689   0.7010  -348.2283
  0.2341    0.6607   0.7132   343.9996
 -0.9402    0.3406  -0.0070   797.1329
       0         0        0     1.0000

dT60n =
 0.0115    0.0023   0.0061    0.7389
 0.0019    0.0058  -0.0061    3.4897
-0.0025   -0.0068   0.0070   -2.8801
      0         0        0         0
```

The last two lines of script save the workspace into the data file *FANUC_geo_err.mat* and close the diary file *FANUC_geo_err.txt*.

8. CONCLUSION

The open chain architecture of serial robots leads to geometric error propagation from link to link and from joint to joint. To keep the robot in its prescribed operating parameters, some important error compensating measures have to be taken. As a further development of this work, the analysis of kinematic and dynamic [7, 8] errors must be performed, with the goal of detecting and minimizing the source of operational errors [9, 10]. Problems like path planning [6] and vibration analysis [11, 12] can also be considered.

9. REFERENCES

[1] Deteşan, O.A., *Cercetări privind modelarea, simularea şi construcţia miniroboţilor,* Ph.D. Thesis, Technical University of Cluj-Napoca, 2007.

[2] Negrean, I., Duca, A., Negrean, C., Kacso, K., *Mecanică avansată în robotică*, U.T. Press, ISBN 978-973-662-420-9, Cluj-Napoca, 2008.

[3] FANUC Robot Series, LR Mate 100iB, Maintenance Manual, B-81595EN/01, 2001.

[4] Deteșan, O.A., *The Definition of the Robot Mechanical Structure Using the Symbolic Computation in MATLAB*, Acta Technica Napocensis, Series: Appplied Mathematics and Mechanics, Vol 51, No 4, ISSN 1221-5872, Cluj-Napoca, 2008.

[5] Deteșan, O.A., Bugnar, F., *Using the Symbolic Computation in MATLAB for Determining the Geometric Model of Serial Robots*, Acta Technica Napocensis, Series: Appplied Mathematics, Mechanics and Engineering, Vol 55, No 3, ISSN 1221-5872, Cluj-Napoca, 2012.

[6] Deteșan, O.A., *The Path Planning of Industrial Robots Using Polynomial Interpolation, with Applications to Fanuc LR-Mate 100iB*, Acta Technica Napocensis, Series: Appplied Mathematics, Mechanics and Engineering, Vol 56, No 4, ISSN 1221-5872, Cluj-Napoca, 2013.

[7] Negrean, I., *New Approaches on Notions from Advanced Mechanics*, Acta Technica Napocensis, Series: Appplied Mathematics, Mechanics and Engineering, Vol 61, No 2, ISSN 1221-5872, Cluj-Napoca, 2018.

[8] Negrean, I., *Advanced Notions and Differential Principles of Motion in Analytical Dynamics*, Journal of Engineering Sciences and Innovation, Technical Sciences Academy of Romania, Vol 1, No 1, 49-72, ISSN 2537-320X, Bucharest, 2016.

[9] Antal, T.A., *Considerations on the Serial PC - Arduino Uno R3 Interaction, in Java, Using Jdeveloper, for a 3R Serial Robot, Based on the Ardulink Library*, Acta Technica Napocensis, Series: Appplied Mathematics, Mechanics and Engineering, Vol 61, No 1, pp. 7-10, ISSN 1221-5872, Cluj-Napoca, 2018.

[10] Antal, T.A., *3R Serial Robot Control Based on Arduino/Genuino Uno, in Java, Using Jdeveloper and Ardulink*, Acta Technica Napocensis, Series: Appplied Mathematics, Mechanics and Engineering, Vol 61, No 1, pp. 11-16, ISSN 1221-5872, Cluj-Napoca, 2018.

[11] Cristea, A.F., *Mechanical stress and strain properties, regarding the elbow joint*, Acta Technica Napocensis, series Applied Mathematics, Mechanics and Engineering, Vol. 57, No 2, June 2014, ISSN 1221-5872.

[12] Cristea A.F., Bălcău, M., *Linear hand-arm system stability,* Acta Technica Napocensis Cluj-Napoca, series: Applied Mathematics, Mechanics and Engineering, Vol. 55, No 4, 2012, pp. 745-748, ISSN 1221-5872.

Modelul geometric al robotului FANUC LR Mate 100iB, ținând seama de erorile dimensionale și ale coordonatelor generalizate

Rezumat: Metodele de modelare geometrică a roboților industriali, utilizate de regulă în literatura de specialitate, se referă strict la geometria nominală a acestora, robotul fiind considerat un sistem de solide rigide, perfect din punct de vedere dimensional sau pozițional. Lucrarea de față surprinde influența erorilor dimensionale și de coordonate generalizate asupra poziției și orientării efectorului final al robotului FANUC LR Mate 100iB. În ecuațiile modelului geometric direct, determinate prin metoda matricelor de situare, sunt introduse erorile dimensional-constructive ale robotului, respectiv erorile coordonatelor generalizate. Particularizând aceste ecuații prin valori numerice, este pusă în evidență influența acestor erori asupra parametrilor operaționali de poziție și orientare.

Ovidiu-Aurelian DETEȘAN, Ph.D., Assoc. Prof., Eng., Technical University of Cluj-Napoca, Faculty of Machine Building, Dept. of Mechanical System Eng, Ovidiu.Detesan@mep.utcluj.ro, phone: +40-64-401667, B-dul Muncii no.103-105, Cluj-Napoca, Romania.

Adina Veronica CRIȘAN, Ph.D., Senior Lecturer, Eng., Technical University of Cluj-Napoca, Faculty of Machine Building, Dept. of Mechanical System Engineering, aduca@mail.utcluj.ro, phone: +40-64-401751, B-dul Muncii no.103-105, Cluj-Napoca, Romania.