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A THEORETICAL APPROACH ON DETERMINING THE GEOMETRICAL ERRORS IN CASE OF ARTICULATED ROBOT STRUCTURES

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Abstract: *The purpose of this paper is to bring in a mathematical model for determining the matrices of geometrical errors. To this effect, the matrices of input data were presented as well as a model of computing the primary errors from the driving joints, the obtained results being essential in defining the geometric errors that affect the work performances of the articulated robot structures. The assessment of robot performances can be achieved by considering the x^{th} order differential model of errors. The linear model of position and orientation errors and the cumulative errors corresponding to transformations between robot systems are also determined.*

Key words: *articulated robot, geometric errors, accuracy, geometry, differential operator.*

1. INTRODUCTION

Robot manipulators are essential components of any robotic manufacturing system, being used in most of the industrial applications. Both, design and the implementation of robots are based on the modeling, analysis as well as the accurate programming of the characteristic point in the work space. In different applications such as welding, handling or inspection of parts, it has been noticed that the accurate positioning of the robot's characteristic point was a very difficult task for the robot.

The factors that influences the positioning accuracy of the characteristic point in the work space are the variations of the geometric parameters from the robot joints, of which the dimensions of the kinetic elements and the joint orientation, are considered main sources of the position errors. The reason is that the kinematic modeling is achieved by considering the geometric parameters to determine the position of the characteristic point, as well as the generalized coordinates of each joint.

The main function of a robot is to handle different objects, materials or tools at any point in the workspace and it can perform various tasks, at a certain level of accuracy. In operating the articulated robot structures, the position and orientation of the end effector are influenced by the kinematic parameters that characterize the robot motion. Practice proved that the programmed position of the end effector does not match every time with the actual position. This is due to the fact that modeling based on kinematic parameters introduces a series of errors. These errors are a result of the poor modeling of the mechanical structure as well as of the inaccuracies caused by inappropriate choice of reference systems attached to the robot or to the workspace. Errors that emerge from a robot's mechanical modeling are a result of the differences between the designed mechanical structure and the assembled one and have a major influence on the positioning and orientation accuracy. In the ideal case, there is no difference between the designed and the assembled structure, so the robot would be able to track exactly the prescribed trajectory without any deviation from position or orientation. This is possible only in theory, because in practice, even if each component is designed and manufactured according to the specifications, during the assembly it can be subjected to deformations due to torsional moments as well as stresses in the elements of the transmission system. This paper aims to present a method of modeling the geometrical errors at the end effector, by considering the dimensional errors in robot links, which can be applied for any serial robot.

2. THE INPUT DATA

2.1 The matrices of the input data

The first step in the modeling of the kinematic accuracy of robots, according to [16] consists in computing the matrices of the input data, also known as the matrices of robot geometry.

So, considering a robot mechanical structure characterized by n degrees of freedom, for each element belonging to the nominal (errorless) and to the real (affected by errors) robot mechanical structure are established the following: the coordinates of the geometric center of each driving joint, denoted with O_i as well as the position of a certain point A_i, randomly chosen on the kinematic axis $\overline{k}_i$, both of them being referenced to the fixed system attached to the robot base $\{0\}$.

To establish the matrices of input data the following notations are considered: n which is the number of driving joints (d.o.f), m represents the number of robot distinct configurations, $q=\{n,r\}$ where n represents the nominal values and r the real values of generalized coordinates or robot links.

The matrices of the input data are established by following some steps. The first one consists in establishing the geometric model of the considered robot structure for a certain configuration. In this respect, the nominal values of the points O_i are given, with respect to $\{0\}$, according to specifications.

The collected data is entered in a $(n+1)\times 6$ matrix called the matrix of nominal geometry and denoted with M_{vn} and defined as follows:

$$M_{vn}^{(0)}=\left[\overline{p}_i^{nT} \quad \overline{k}_i^{nT}\right]^T, \quad i=1\rightarrow n+1; \quad (1)$$

where $\overline{p}_i^{nT}$ represents the nominal position of the geometric center of each driving joint and $\overline{k}_i^{nT}$ defines the orientation of each driving axis also in the nominal configuration (errorless).

The components of the real geometry matrix $M_{vr}^{(0)}$ are determined by physical measurements:

$$M_{vr}^{(0)}=\underset{[(n+1)\times 6]}{Matrix}\left\{\left[\overline{p}_i^{rT} \quad k_i^{rT}\right], \quad i=1\rightarrow n+1\right\}^T; \ (2)$$

where $\overline{p}_i^{rT}$ represents the real position of the geometric center of each driving joint and $\overline{k}_i^{rT}$ defines the orientation of each driving axis, in the real configuration (with errors). The orientation of systems attached in the geometrical center of each robot joint is established next. All the systems will have the same orientation with the fixed system $\{0\}$ attached to the robot base. For $q=n$ and $i=n+1$ the length of kinetic element (iq) is determined, according to:

$$\left\{\begin{array}{c} L_i^q=\left|\overline{p}_i^q-\overline{p}_{i-1}^q\right|= \\ =\sqrt{\left(x_i^q-x_{i-1}^q\right)^2+\left(y_i^q-y_{i-1}^q\right)^2+\left(z_i^q-z_{i-1}^q\right)^2} \end{array}\right\}; \ (3)$$

The orientation of each driving joint (iq) is established using the following expression:

$$\overline{k}_i^{qT}=\left[c\alpha_{iu}^q \quad c\beta_{iu}^q \quad c\gamma_{iu}^q\right]^T, \ u=\{x,y,z\}; \ (4)$$

Next, the $(n+2)\times 9$ matrix of nominal systems denoted with $T_n^{(0)}$ is written:

$$T_n^{(0)}=\left[\overline{p}_i^{nT} \ \overline{x}_i^{nT} \ \overline{y}_i^{nT} \ \overline{z}_i^{nT}\right]^T, \ i=1\rightarrow n+1; \ (5)$$

Based on (3) and (4) the $(n+1)\times 6$ matrix of robot nominal geometry (M_{Gn}) is established:

$$M_{Gn}=\left[x_i^n \ y_i^n \ z_i^n \ \alpha_i^n \ \beta_i^n \ \gamma_i^n\right]^T, \quad i=1\rightarrow n+1; \ (6)$$

For $q=r$ and $i=n+1$ the elements of the $M_{vr}^{(0)}$ matrix are used, previously defined with (2).

In the real configuration the $(n+2)\times 9$ matrix of real systems is defined, denoted with $T_r^{(0)}$, as:

$$T_r^{(0)}=\left[\overline{p}_i^{rT} \ \overline{x}_i^{rT} \ \overline{y}_i^{rT} \ \overline{z}_i^{rT}\right]^T, \ i=1\rightarrow n+1; \ (7)$$

The orientation of the reference systems for the real structure of the robot is established according to an algorithm developed in [16].

The matrix of robot real geometry (M_{Gr}) is determined next, whose components are defined as: $M_{Gr} = \left[x_i^r \ y_i^r \ z_i^r \ \alpha_i^r \ \beta_i^r \ \gamma_i^r \right]^T , \quad i = 1 \to n+1$; (8)

For $i = 1 \to n$, the next step consists in computing the homogenous transformations between the $\{i\}$ and $\{i-1\}$ mobile frames.

The general expression for defining the homogenous transformations is the following:

$$\left\{ \begin{array}{c} T_{ii-1}^n = \left[\begin{array}{c:c} R_{ii-1}^n & {}^{i-1}p_{ii-1}^n \\ \hdashline 0 \ 0 \ 0 & 1 \end{array} \right] = \\ = \left[\begin{array}{ccc:c} {}^{i-1}x_i^n & {}^{i-1}y_i^n & {}^{i-1}z_i^n & \left(R_{i-10}^n\right)^T \cdot \left(\bar{p}_i^{(0)n} - \bar{p}_{i-1}^{(0)n}\right) \\ \hdashline 0 & 0 & 0 & 1 \end{array} \right] \end{array} \right\}; \quad (9)$$

where T_{ii-1}^n represents the nominal value of the homogenous transformation between the $\{i\}$ and $\{i-1\}$ frames, R_{ii-1}^n is a (3×3) matrix which defines the orientation of each driving axis of the mobile frame $\{n\}$ with respect to the fixed frame $\{0\}$ attached to robot base, in the nominal configuration and ${}^{i-1}p_{ii-1}^n$ is a (3×1) column vector expressing the position of each frame $\{i\}$ with respect to the previous frame $\{i-1\}$ for the same nominal configuration.

Similarly, for $i = 1 \to n$ and $q = r$ are computed the homogenous transformations between $\{i\}$ and $\{i-1\}$ frames, where T_{ii-1}^r and its components have the same meaning as in (9) the only difference being that this time the real robot configuration is considered.

By the similarity with the properties of the time derivative of the locating matrix, according to [3] – [12], the following relation between the homogeneous transformation for robot nominal structure and the real transformation matrix T_{ii-1}^r results:

$$T_{ii-1}^r = T_{ii-1}^n + T_{ii-1}^n \cdot \delta T_{ii-1} = T_{ii-1}^n + \Delta T_{ii-1}; \quad (10)$$

$$\Delta T_{ii-1} = T_{ii-1}^n \cdot \delta T_{ii-1}; \quad (11)$$

where ΔT_{ii-1} is the first order differential error of the homogenous transformations between $\{i\}$ and $\{i-1\}$ reference frames.

In order to determine the expression for the matrix operator of errors, the following equation is used:

$$T_{ii-1}^n \cdot \delta T_{ii-1} = T_{ii-1}^r - T_{ii-1}^n; \quad (12)$$

The equation is multiplied left side by $\left(T_{ii-1}^n\right)^{-1}$. Applying the matrix properties, it results finally:

$$\left\{ \begin{array}{c} \left(T_{ii-1}^n\right)^{-1} \cdot T_{ii-1}^n \cdot \delta T_{ii-1} = \\ = \left(T_{ii-1}^n\right)^{-1} \cdot T_{ii-1}^r - \left(T_{ii-1}^n\right)^{-1} \cdot T_{ii-1}^n \end{array} \right\}; \quad (13)$$

where, $\left(T_{ii-1}^n\right)^{-1} \cdot T_{ii-1}^n = I_4$. (14)

The final expression for the matrix operator of errors is obtained in the following form:

$$\delta T_{ii-1} = \left(T_{ii-1}^n\right)^{-1} \cdot T_{ii-1}^r - I_4 . \quad (15)$$

where T_{ii-1}^n and T_{ii-1}^r were previously defined according to (9) and $\left(T_{ii-1}^n\right)^{-1}$ is the inverse matrix of the T_{ii-1}^n homogenous transformation.

It is a fact that the position and orientation errors are mainly caused by deviations of the relative positions of the driving joints, by the plays in driving joints, the deformations of the kinetic links or by the oscillation of the mechanical, driving or control system. So, for $i = 1 \to n$ the matrices of primary errors are established, which are in fact the relative errors from the robot driving joints:

$$\delta T_{ii-1} = \left[\begin{array}{c:c} \{\Delta \bar{\psi}_i \times\} & \Delta \bar{p}_{ii-1} \\ \hdashline 0 \ 0 \ 0 & 1 \end{array} \right]; \quad (16)$$

where $\{\Delta \bar{\psi}_i \times\}$ is a antisymmetric matrix that includes the relative errors for orientation $\Delta\alpha, \Delta\beta$ and $\Delta\gamma$, $\Delta \bar{p}_{ii-1}$ representing the column vector of relative errors for positioning:

$$\Delta \bar{p}_{ii-1} = \bar{p}_{ii-1}^r - \bar{p}_{ii-1}^n . \quad (17)$$

Thus, the next step in establishing the input matrices consists in eliminating the gross errors [16], in this regard using the Pearson – Hartley or Grubbs test in determining the statistics matrix $z_{n\alpha}$ (α is the level of trust and n represents in this case the number of performed measurements). The validation tests allow to determine the value of an experiment based on the probability of occurrence of a variation in the work environment. By performing these tests, it can be determined whether a certain experimental result is not an aberrant result caused by external factors.

Also it can determine if the same result can be considered within the limits of normal variability and does not negatively influence the conduct of the experiment.

Next, the δT_{ii-1} matrix operator is established by considering the following notations:

$$\bar{\varepsilon}_{yQ} = \left\{ \bar{\varepsilon}_{yS} \right\};\ y = \left\{ p_Q;\ \ e;\ \ g \right\}; \qquad (18)$$

$$p_Q = p_S = \left\{ p_{xi};\ p_{yi};\ p_{zi};\ \alpha_i;\ \beta_i;\ \gamma_i \right\}; \quad (19)$$

where y represents an index which highlights the locating errors (position and orientation errors) corresponding to: p_Q - each parameter, e - each element and g - to the whole robot mechanical structure.

The $(n+1)\times 6$ matrix of locating errors, symbolized with E_s^{pk} is established, n representing the number of elements which make up the mechanical structure of the robot.

For the nominal and real structure, the $(n\times 6)$ matrix of nominal/real configurations of the robot is determined, $M_{\theta q}\left(q = n, r\right)$ defined as following:

$$M_{\theta q} = \left[\bar{\theta}_k^{qT},\quad k = 1 \rightarrow m \right]; \qquad (20)$$

where $$\bar{\theta}_k^{qT} = \left[q_{ik}^{q},\quad i = 1 \rightarrow n \right]. \qquad (21)$$

Table 1

$M_{\theta n}$							
$M_{\theta n}^{ij}$	**Joint type**	**Nominal values**			**Mechanical restrictions**		
	R , T	q_i^n	$\dot{q}_i^n$	$\ddot{q}_i^n$	q_{in}^{max}	q_{in}^{med}	q_{in}^{min}

Table 2

$M_{\theta r}$									
$M_{\theta n}^{ij}$	**Real values**			**Errors** $\left(\Delta\bar{\theta},\ \Delta\dot{\bar{\theta}},\ \Delta\ddot{\bar{\theta}}\right)$			**Mechanical restrictions**		
	q_i^r	$\dot{q}_i^r$	$\ddot{q}_i^r$	Δq_i	$\Delta\dot{q}_i$	$\Delta\ddot{q}_i$	q_{ir}^{max}	q_{ir}^{med}	q_{ir}^{min}

In *Table 1* and *Table 2* the components of the matrix of nominal / real configurations of the robot are represented.

The time matrix is determined according to:

$$M_t = \left[t_i,\quad i = 1 \rightarrow m \right]; \qquad (22)$$

where $t_m = ...[\mathrm{s}]$ represents the time required to achieve the m configuration.

The matrix of real robot configurations is determined by performing physical measurements on the real structure.

3. MODELING OF GEOMETRICAL ERRORS

3.1 Determining of primary errors

This section aims to determine the primary errors from the driving joints, due to plays, manufacturing errors or to the generalized coordinate's errors, which are used for defining the geometric errors that affect the operation performances of the articulated robot structures.

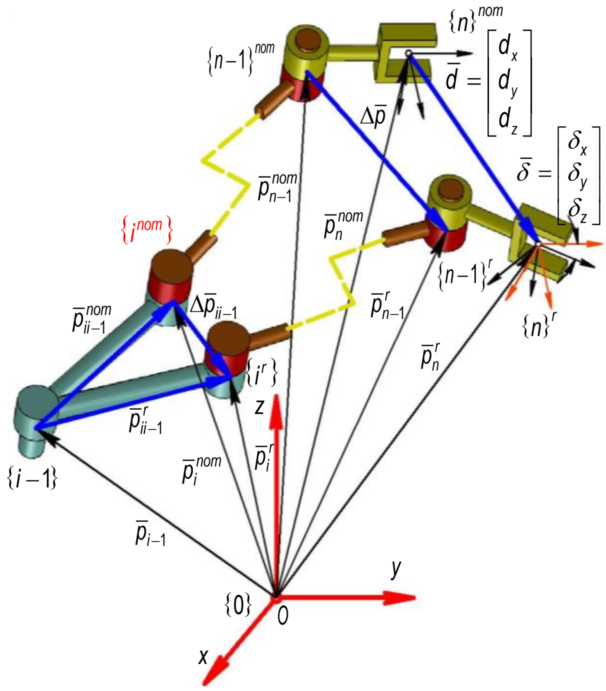

Fig. 1 The geometrical errors

The primary errors for two important sets of rotation angles – Bryant angles and Euler angles are determined further. For the set of Bryant angles $\left(\alpha_x - \beta_y - \gamma_z\right)$ the primary errors are:

$$\left.\begin{cases}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & -\Delta\alpha_x\\ 0 & \Delta\alpha_x & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 0 & \Delta\beta_y\\ 0 & 1 & 0\\ -\Delta\beta_y & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & -\Delta\gamma_z & 0\\ \Delta\gamma_z & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\\ =\begin{bmatrix}1 & -\Delta\gamma_z & \Delta\beta_y\\ \Delta\gamma_z & 1 & -\Delta\alpha_x\\ -\Delta\beta_y & \Delta\alpha_x & 1\end{bmatrix}\end{cases}\right\}. \quad (23)$$

The terms contained in (23) are identified with the terms contained in the following matrix:

$$R_{ii-1}^{nT}\cdot R_{ii-1}^{r}=\begin{bmatrix}\left({}^{i-1}\bar{x}_i^{nT}\cdot{}^{i-1}\bar{x}_i^{r}\right) & \left({}^{i-1}\bar{x}_i^{nT}\cdot{}^{i-1}\bar{y}_i^{r}\right) & \left({}^{i-1}\bar{x}_i^{nT}\cdot{}^{i-1}\bar{z}_i^{r}\right)\\ \left({}^{i-1}\bar{y}_i^{nT}\cdot{}^{i-1}\bar{x}_i^{r}\right) & \left({}^{i-1}\bar{y}_i^{nT}\cdot{}^{i-1}\bar{y}_i^{r}\right) & \left({}^{i-1}\bar{y}_i^{nT}\cdot{}^{i-1}\bar{z}_i^{r}\right)\\ \left({}^{i-1}\bar{z}_i^{nT}\cdot{}^{i-1}\bar{x}_i^{r}\right) & \left({}^{i-1}\bar{z}_i^{nT}\cdot{}^{i-1}\bar{y}_i^{r}\right) & \left({}^{i-1}\bar{z}_i^{nT}\cdot{}^{i-1}\bar{z}_i^{r}\right)\end{bmatrix}. \quad (24)$$

By identification results the orientation errors:

$$\begin{cases}\Delta\alpha_x=-\left({}^{i-1}\bar{y}_i^{nT}\cdot{}^{i-1}\bar{z}_i^{r}\right)\\ \Delta\beta_y={}^{i-1}\bar{x}_i^{nT}\cdot{}^{i-1}\bar{z}_i^{r}\\ \Delta\gamma_z=-\left({}^{i-1}\bar{x}_i^{nT}\cdot{}^{i-1}\bar{y}_i^{r}\right)\end{cases}; \quad (25)$$

For checking, the identities are written as:

$$\begin{cases}\Delta\alpha_x=-\left({}^{i-1}\bar{y}_i^{nT}\cdot{}^{i-1}\bar{z}_i^{r}\right)\\ \Delta\beta_y={}^{i-1}\bar{x}_i^{nT}\cdot{}^{i-1}\bar{z}_i^{r}\\ \Delta\gamma_z=-\left({}^{i-1}\bar{x}_i^{nT}\cdot{}^{i-1}\bar{y}_i^{r}\right)\end{cases}; \quad (26)$$

The matrix operator of the primary errors for $\left(\alpha_x-\beta_y-\gamma_z\right)$ resultant rotation is defined as:

$$\delta T_{ii-1}\left(x-y-z\right)=\left[\begin{array}{ccc|c}\multicolumn{3}{c|}{\Delta\left(\bar{\psi}\times\right)} & \Delta\,\bar{p}_{ii-1}\\ \hline 0 & 0 & 0 & 0\end{array}\right]; \quad (27)$$

$$\Delta\left(\bar{\psi}\times\right)=\begin{bmatrix}0 & -\Delta\left(\bar{\psi}_z\right) & \Delta\left(\bar{\psi}_y\right)\\ \Delta\left(\bar{\psi}_z\right) & 0 & -\Delta\left(\bar{\psi}_x\right)\\ -\Delta\left(\bar{\psi}_y\right) & \Delta\left(\bar{\psi}_x\right) & 0\end{bmatrix}; \quad (28)$$

where $\Delta\left(\bar{\psi}_x\right)=\Delta\beta_y\cdot s\gamma_z+\Delta\alpha_x\cdot c\beta_y\cdot c\gamma_z$; (29)

$$\Delta\left(\bar{\psi}_y\right)=\Delta\beta_y\cdot c\gamma_z-\Delta\alpha_x\cdot c\beta_y\cdot s\gamma_z; \quad (30)$$

$$\Delta\left(\bar{\psi}_z\right)=\Delta\gamma_z+\Delta\alpha_x\cdot s\beta_y; \quad (31)$$

$$\Delta\,\bar{p}_{ii-1}=\begin{bmatrix}\Delta p_{y_{ii-1}}\cdot(c\alpha_x\cdot s\gamma_z+c\gamma_z\cdot s\alpha_x\cdot s\beta_y)+\\ +\Delta p_{z_{ii-1}}\cdot(s\alpha_x\cdot s\gamma_z-c\alpha_x\cdot c\gamma_z\cdot s\beta_y)+\\ +\Delta p_{x_{ii-1}}\cdot c\beta_y\cdot c\gamma_z\\ \hline \Delta p_{y_{ii-1}}\cdot(c\alpha_x\cdot c\gamma_z-s\alpha_x\cdot s\beta_y\cdot s\gamma_z)+\\ +\Delta p_{z_{ii-1}}\cdot(c\gamma_z\cdot s\alpha_x+c\alpha_x\cdot s\beta_y\cdot s\gamma_z)-\\ -\Delta p_{x_{ii-1}}\cdot c\beta_y\cdot s\gamma_z\\ \hline \Delta p_{x_{ii-1}}\cdot s\beta_y+\Delta p_{z_{ii-1}}\cdot c\alpha_x\cdot c\beta_y-\\ -\Delta p_{y_{ii-1}}\cdot c\beta_y\cdot s\alpha_x\end{bmatrix}. \quad (32)$$

For the set of Euler's angles $\left(\alpha_z-\beta_x-\gamma_z\right)$ the angular errors are determined similar to the case presented in the previous example:

$$\left.\begin{cases}\begin{bmatrix}1 & \Delta\alpha_z & 0\\ -\Delta\alpha_z & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & -\Delta\beta_x\\ 0 & \Delta\beta_x & 1\end{bmatrix}\cdot\begin{bmatrix}1 & \Delta\gamma_z & 0\\ -\Delta\gamma_z & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\\ =\begin{bmatrix}1 & -\Delta\gamma_z-\Delta\alpha_z & 0\\ \Delta\alpha_z+\Delta\gamma_z & 1 & -\Delta\beta_x\\ 0 & \Delta\beta_x & 1\end{bmatrix}\end{cases}\right\}. \quad (33)$$

By identifying the terms from (33) with the corresponding terms from (24) are obtained:

$$\Delta\beta_x={}^{i-1}\bar{z}_i^{nT}\cdot{}^{i-1}\bar{y}_i^{r}; \quad (34)$$

$$\Delta\beta_x\cdot\Delta\alpha_z={}^{i-1}\bar{x}_i^{nT}\cdot{}^{i-1}\bar{z}_i^{r}. \quad (35)$$

If the condition $\Delta\beta_x\neq 0$ is satisfied, it results:

$$\Delta\alpha_z=\frac{1}{\Delta\beta_x}\cdot\left({}^{i-1}\bar{x}_i^{nT}\cdot{}^{i-1}\bar{z}_i^{r}\right)=\frac{{}^{i-1}\bar{x}_i^{nT}\cdot{}^{i-1}\bar{z}_i^{r}}{{}^{i-1}\bar{z}_i^{nT}\cdot{}^{i-1}\bar{y}_i^{r}}; \quad (36)$$

$$\Delta\beta_x\cdot\Delta\gamma_z={}^{i-1}\bar{z}_i^{nT}\cdot{}^{i-1}\bar{x}_i^{r}; \quad (37)$$

$$\Delta\gamma_z=\frac{1}{\Delta\beta_x}\cdot\left({}^{i-1}\bar{z}_i^{nT}\cdot{}^{i-1}\bar{x}_i^{r}\right)=\frac{{}^{i-1}\bar{z}_i^{nT}\cdot{}^{i-1}\bar{x}_i^{r}}{{}^{i-1}\bar{z}_i^{nT}\cdot{}^{i-1}\bar{y}_i^{r}}. \quad (38)$$

The analytical expression defining the differential operator relative to the primary errors contained in $\delta T_{ii-1}\left(z-x-z\right)$, the equivalent with the Uicker operator, is determined as follows:

$$\delta T_{ii-1}\left(z-x-z\right)=\left[\begin{array}{ccc|c}\multicolumn{3}{c|}{\Delta\left(\bar{\psi}\times\right)} & \Delta\,\bar{p}_{ii-1}\\ \hline 0 & 0 & 0 & 0\end{array}\right]. \quad (39)$$

The matrix of orientation errors is defined as:

$$\Delta(\bar{\psi}\times)=\begin{bmatrix} 0 & -\Delta(\bar{\psi}_z) & \Delta(\bar{\psi}_y) \\ \Delta(\bar{\psi}_z) & 0 & -\Delta(\bar{\psi}_x) \\ -\Delta(\bar{\psi}_y) & \Delta(\bar{\psi}_x) & 0 \end{bmatrix}; \quad (40)$$

$$\Delta(\bar{\psi}_x)=\Delta\beta_x \cdot c\gamma_z + \Delta\alpha_z \cdot s\beta_x \cdot s\gamma_z; \quad (41)$$

$$\Delta(\bar{\psi}_y)=\Delta\alpha_z \cdot s\beta_x \cdot c\gamma_z - \Delta\beta_x \cdot s\gamma_z; \quad (42)$$

$$\Delta(\bar{\psi}_z)=\Delta\gamma_z + \Delta\alpha_z \cdot c\beta_x; \quad (43)$$

$$\Delta\,\bar{p}_{ii-1}=\begin{bmatrix} \Delta p_{x_{ii-1}} \cdot (c\alpha_z \cdot c\gamma_z - c\beta_x \cdot s\alpha_z \cdot s\gamma_z) + \\ +\Delta p_{y_{ii-1}} \cdot (c\gamma_z \cdot s\alpha_z + c\alpha_z \cdot c\beta_x \cdot s\gamma_z) + \\ +\Delta p_{z_{ii-1}} \cdot s\beta_x \cdot s\gamma_z \\ \hdashline -\Delta p_{y_{ii-1}} \cdot (s\alpha_z \cdot s\gamma_z - c\alpha_z \cdot c\beta_x \cdot c\gamma_z) - \\ -\Delta p_{x_{ii-1}} \cdot (c\alpha_z \cdot s\gamma_z + c\beta_x \cdot c\gamma_z \cdot s\alpha_z) + \\ +\Delta p_{z_{ii-1}} \cdot c\gamma_z \cdot s\beta_x \\ \hdashline \Delta p_{z_{ii-1}} \cdot c\beta_x - \Delta p_{y_{ii-1}} \cdot c\alpha_z \cdot s\beta_x + \\ +\Delta p_{x_{ii-1}} \cdot s\alpha_z \cdot s\beta_x \end{bmatrix}. \quad (44)$$

The expressions (32) and (44) represent the position errors from every driving joint which is a function of the orientation angles.

3.2 The matrices of geometrical errors

In this section are presented the matrices of geometrical errors according to [16], [3] - [12]. The first order differential error corresponding to homogenous transformation between $\{i\}$ and $\{i-1\}$ reference systems is determined using the following expression:

$$\Delta T_{ii-1}^{(1)} = T_{ii-1}^{n} \cdot \delta T_{ii-1}; \quad (45)$$

where T_{ii-1}^{n} and δT_{ii-1} are defined with (9) and (16).

For $i=1 \rightarrow n+1$ and $j=1 \rightarrow i$ the following first-order error differentiation matrix operators are determined: $\delta^{(j-1)}T_{ii-1}$, $\delta T_{ij-1}^{(1)}$, δT_{ij-1}^{1}, ΔT_{ij-1}^{1}.

$$\delta^{(j-1)}T_{ii-1} = T_{ij-1}^{-1} \cdot \delta T_{ii-1} \cdot T_{ij-1}; \quad (46)$$

where $\delta^{(j-1)}T_{ii-1}$ represents the error differentiation matrix operator with respect to the errors from the driving joint (i), which include the local error differentiation operator. The T_{ij-1} along with its inverse is obtained as:

$$T_{ij-1} = \prod_{k=j}^{i}(T_{kk-1}); \qquad T_{ij-1}^{-1} = (T_{ij-1})^{T}. \quad (47)$$

The assessment of the robot performances is achieved by considering the x^th^ order differential model of errors, where $x=\{1,2,3\}$.

The differential model of errors is first applied on the homogenous transformation between the systems $\{0\}$ and $\{n\}$. The first order differential matrix operator $\delta T_{ij-1}^{(1)}$ is determined. It includes the cumulative error operator, as it results from:

$$\delta T_{ij-1}^{(1)} = \sum_{k=j}^{i} \delta^{(j-1)}T_{kk-1}. \quad (48)$$

Thus, according to [5], [16], the matrix of geometrical errors which characterizes the linear model is defined by the expressions:

$$\delta^{0}T_{n}^{(1)} = \sum_{i=1}^{n}\left({}_{i}^{0}[T] \cdot \delta^{i-1}T_{ii-1} \cdot {}_{i}^{0}[T]^{-1}\right); \quad (49)$$

$$\delta^{0}T_{n}^{1} = \sum_{i=1}^{n}\delta^{0}T_{ii-1} = \sum_{i=1}^{n}\left({}_{i-1}^{0}[T] \cdot \delta^{i-1}T_{ii-1} \cdot {}_{i-1}^{0}[T]^{-1}\right); \quad (50)$$

where $\delta^{0}T_{n}^{1}$ is the linear model of position and orientation errors, being defined as follows:

$$\left\{\begin{aligned} \delta^{0}T_{n}^{1} &= \left[\begin{array}{ccc:c} \multicolumn{3}{c:}{\bar{\delta}\times} & \bar{d} \\ \hdashline 0 & 0 & 0 & 0 \end{array}\right] = \\ &= \left[\begin{array}{ccc:c} 0 & -\delta z & \delta y & d_x \\ \delta z & 0 & -\delta x & d_y \\ -\delta y & \delta x & 0 & d_z \\ \hdashline 0 & 0 & 0 & 0 \end{array}\right] \end{aligned}\right\}; \quad (51)$$

where $\bar{d}=\begin{bmatrix} d_x & d_y & d_z \end{bmatrix}^{T}$ is the linear position error and $\bar{\delta}=\begin{bmatrix} \delta_x & \delta_y & \delta_z \end{bmatrix}^{T}$ is the orientation error.

Finally, the first order cumulative errors corresponding to homogenous transformation between $\{i\}$ and $\{j-1\}$ systems, are defined by means of the expression presented below:

$$\Delta T_{ij-1}^{1} = T_{ij-1} \cdot \delta T_{ij-1}^{1}. \tag{52}$$

In order to increase the positioning and orientation accuracy of a serial structure robot, it is recommended to compute the higher order differential errors, which are going to be presented in a future paper.

The equation (51) defines the forward model and can be applied in case that the primary geometric errors of the parameters which characterizes each element are known.

4. CONCLUSION

The main purpose of the industrial robots is to achieve the positions and orientations of the manipulated parts, which were previously prescribed, its accuracy being influenced by the values of the positional deviations, to which are added the deviations in displacements, speed, accelerations and forces respectively. In other words, the accuracy of any industrial robot can be evaluated by means of errors. The errors are defined as differential quantities which are represented by the difference between a nominal value that have to be achieved and that have been previously programmed and the reached value (actual or real value). The accuracy of industrial robots can be evaluated by means of two types of errors: systematic and random errors. The position and orientation errors are random errors while geometric (structural) errors falls into the category of systematic errors. The position error is a vector whose origin is represented by the coordinates of the point specified in the robot program and as application point the coordinates of the point that is reached in reality by the robot (the difference between the programed position and the achieved position). The orientation error is represented by means of an angle which has as its sides the programmed position and the achieved position of the characteristic straight line. The present paper aims to show a mathematical model for determining the matrices of geometrical errors. To this effect, first, the matrices of input data were presented. It was also presented a model of computing the primary errors from the driving joints, the results being essential in defining the geometric errors that affect the working performances of the articulated robot structures. The assessment of robot performances can be achieved by considering the x^{th} order differential model of errors, where $x = \{1, 2, 3\}$. In this paper the linear model of position and orientation was determined, as well as the cumulative errors corresponding to the transformations between the systems. In order to increase the positioning and orientation accuracy of a serial robot structure, it is recommended to compute the higher order differential errors, which are going to be presented in a future paper.

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O abordare teoretică privind determinarea erorilor geometrice in cazul structurilor articulate de roboţi

Scopul acestei lucrări este de a propune un model matematic pentru determinarea matricelor erorilor geometrice. În acest sens, au fost prezentate matricele datelor de intrare, precum şi un model de calcul al erorilor primare care apar în cuplele motoare, rezultatele obţinute fiind esenţiale în definirea erorilor geometrice care afectează performanţele de lucru ale structurilor articulate de roboţi. Evaluarea performanţelor unui robot se realizează luând în considerare modelul diferenţial al erorilor având ordinul x. Se determină, de asemenea, modelul liniar al erorilor de poziție și orientare și erorile cumulate de ordinul întâi corespunzătoare transformărilor omogene care au loc între sistemele atașate robotului.

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