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**CONTRIBUTION TO THE DYNAMIC STUDY
OF THE MILL WITH HORIZONTAL AXIS.
PART II: DIFFERENTIAL EQUATIONS SYSTEM**

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***Abstract:** The work is part of a group of three works, in which the dynamics of a fodder grain mill, equipped with a horizontal shaft, are studied. The mill shaft is welded directly onto the drive shaft. The present work represents the second part of this paper, whereby the mechanical system corresponding to the mill is established, the mechanical characteristics, with which the System of differential equations, which governs the dynamics of the mill.*

***Key words:** mill with horizontal axis, mechanical characteristics, differential equations system*

1. GENERAL CONSIDERATIONS

The mill for the shredding of grain in a feed mixture, for feeding the animals from the experimental group of the USAMV, is presented in dimensional form to the dynamic study, in the first part of the works [1].

This work is structured on three sides, which was imagined as follows:

- I. The first part of the work is done a preliminary study, establishing the:
 - a. component elements of the mill,
 - b. correlation between them,
 - c. how the mill is trained,
 - d. grated mixture granulation;
 - e. and escape mode [1].
- II. In the second part shall be established:
 1. the reference systems to which the mill is reported [9], [10];
 2. reporting of constructive dimensions to reference systems;
 3. mechanical characteristics of the rotating mill with horizontal shaft;
 4. differential equation system.
- III. In the third part is drawn up:
 - Matlab program that achieves the integration of the system of dynamic equations of the mill;

- the solution related to the law of movement of the mill and the dynamic reactions in the system;
- Interpretation of the results.

2. MECHANICAL MODEL

The mechanical model of the horizontal shaft mill for the milling of the fodder mixture is considered a system, which admits a symmetry axis, placed along the rolling bearings of the drive motor.

The rotor of the mill is welded to the flange [1] which makes the joint body with the rotor ring of the mill, on the free end of the electric motor drive shaft.

It shows the tabulated dimensions [23] of the engine symbolized on the catalogue with the designation HJN 132 SX-2, to correlate with the size of the mill, for the preparation of the mechanical scheme.

Table 1.
Electric motor HJN 132 SX-2 of the mill [23]

CRT No.	Name	U.M.	Value
1.	Distance between supports	mm	140
2.	Distance from camp to exit	mm	89
3.	Engine shaft output	mm	80
4.	Shaft diameter	m	38
5.	Electric motor mass	kg	45
6.	Electric rotor mass	kg	18

7.	Shaft mass	kg	2,75
8.	Engine power	kw	7,5
9.	Engine speed	rpm	2900
10.	Torque engine	Nm	24,7

The electric motor valves are considered: spherical articulation – the one on the free end and a cylindrical joint, which is found to the mill rotor [2], [3], [4], [5].

2.1. Walking Mill

The mechanical scheme of the mill on idle is given in Figure 1. The adopted reference system has its origin in the spherical articulation, and the axis O_1y_1 is the axis of symmetry for the mill.

The dimensions adapted for the mechanical diagram correspond to the mill and the electric drive motor.

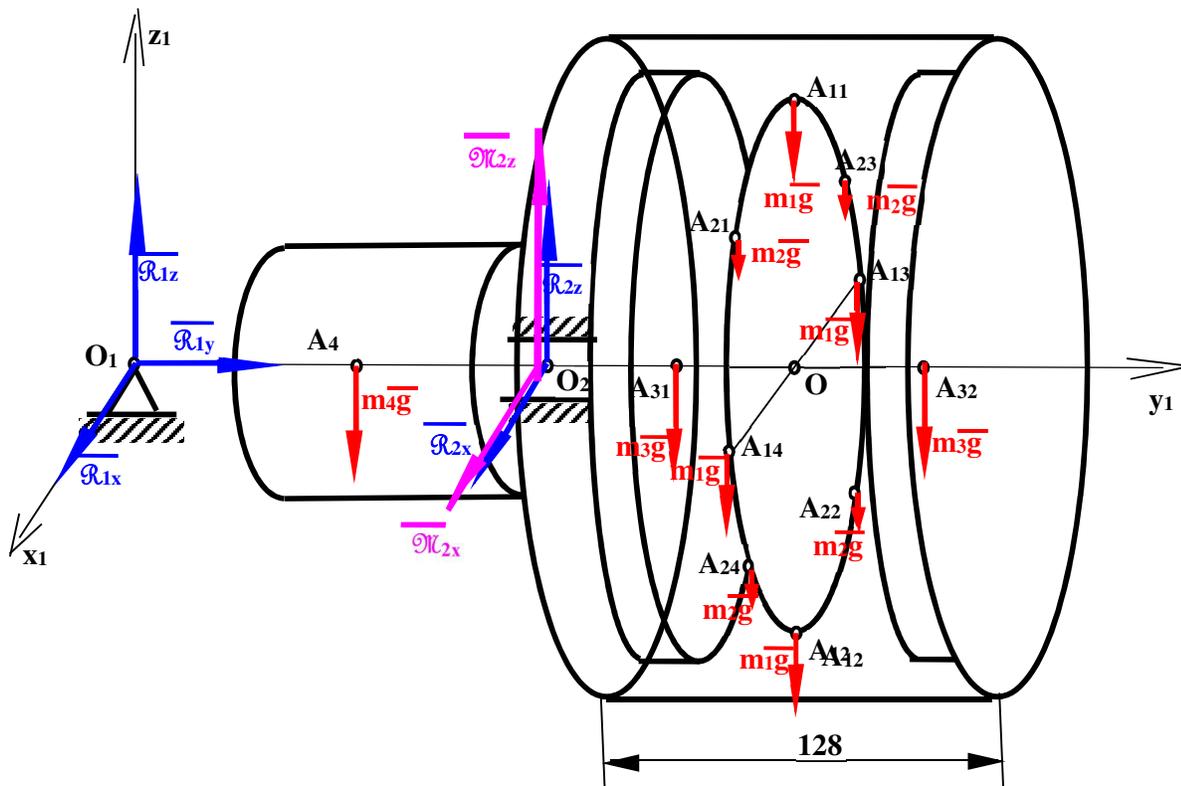


Fig. 1. Mechanical Scheme of the Mill with Horizontal Axis:

red are for the active forces; blue are the linkage forces; lilah are the linkage moments

Observation: In the figure, the linear and the reactions are not present at the same time [21]. They will be considered in successive images.

In Figure 1 the meaning of the note is the following:

- O_1 is the center of spherical articulation and represents the origin of the fixed reference system, to which the dynamics of the horizontal shaft mill is reported;

- O_2 is located on the axis of symmetry of the cylindrical joint;
- O is the symmetry center of the mill rotor to the empty operation;
- m_{1i} , $i = 1, \dots, 4$ represents the masses of the hammer ensemble;
- m_{2i} , $i = 1, \dots, 4$ are the masses of stopper stacks for the hammer assembly;
- m_{31} – is the mass of the rotor ring;
- m_{32} – is the mass of the impeller flange;

- m_4 – is the mass of the electric motor rotor with its shaft, which is supported on the two joints, allowing the rotation movement.

2.2. The Hammer Assembly in Static Balance

Before training this mill is found in static equilibrium, for which the hammer assembly is reported to an Oxyz reference system, which has its origin on the axis of symmetry of the mill to empty operation (Fig. 1), and the xOz plan is the symmetry plan of assembly (Fig. 2).

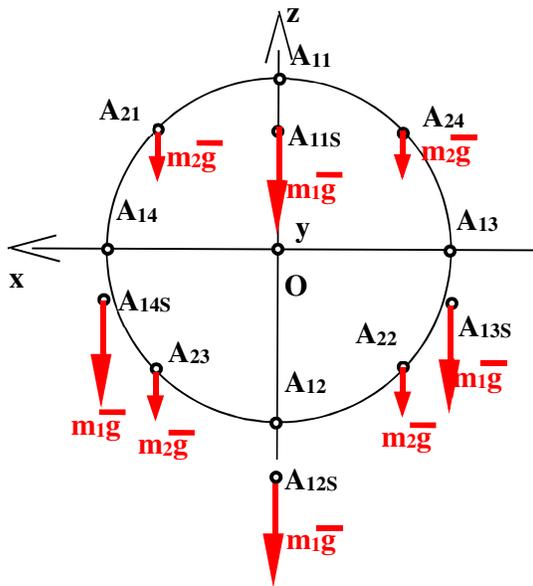


Fig. 2. Position of the mass center for the hammer assembly in static balance

In Figure 2 are made representations for the center of the hammer masses towards the adopted reference system, and each hammer is articulated in a cylindrical articulation.

Compared to the reference system adopted in Figure 2, the center of the masses of each mechanical system shall be presented centrally in table 2. The scoring of points is made with the same succession of characters to the index, to which the letter "s" is added, which signipose the static position.

Table 2.

Mass Center Position of the Hammers Systems

CR No.	Notation point	Conc. Mass [kg]	Coordinate x [m]	Coordinate z [m]
1.	A _{11s}	0,84	0,06	0,0
2.	A _{12s}	0,84	-0,16	0,0
3.	A _{13s}	0,84	0,0	-0,05
4.	A _{14s}	0,84	0,0	-0,05
5.	A _{21s}	0,085	0,11*0.705	0,11*0.705
6.	A _{22s}	0,085	- 0,11*0.705	- 0,11*0.705
7.	A _{23s}	0,085	0,11*0.705	- 0,11*0.705
8.	A _{24s}	0,085	- 0,11*0.705	0,11*0.705

2.3. Center of the Material System Masses Assimilated Horizontal Shaft Mill

The entire material system is reported to the reference system in Figure 1 and differentiates the electric motor from which the mass of the rotor and the mass of the shaft is taken into account. The quotas are the result of the dimensions presented in the engine catalogue [22] and in the user manual of the horizontal shaft mill [23]. The figure that highlights all these elements is figure 3.

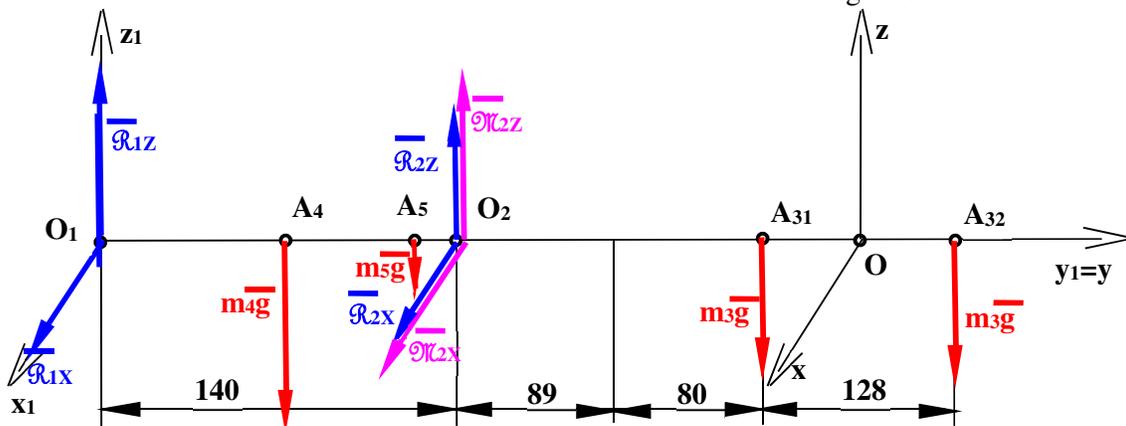


Fig. 3. Position of the active forces, linkage forces and linkage moments for the mass center of the system

The data of table 2, coupled with the reporting of the material system to the reference system in Figure 3, will be centralised in table 3,

in order to establish the position of the center of the mechanical system masses assimilated to the mill.

Table 3.

Mass Center of the Mechanical System Assimilated to th Mill with Horizontal Axis

Crt. No.	Point name as the local center of the masses	Mass [kg]	Coordinates in the reference system $O_1x_1y_1z_1$		
			x_1 [m]	y_1 [m]	z_1 [m]
1.	A ₁₁	0,84	0,0	0,373	0,06
2.	A ₁₂	0,84	0,0	0,373	- 0,16
3.	A ₁₃	0,84	0,11	0,373	- 0,05
4.	A ₁₄	0,84	- 0,11	0,373	- 0,05
5.	A ₂₁	0,085	0,07755	0,373	0,07755
6.	A ₂₂	0,085	- 0,07755	0,373	- 0,07755
7.	A ₂₃	0,085	0,07755	0,373	- 0,07755
8.	A ₂₄	0,085	- 0,07755	0,373	0,07755
9.	A ₃₁	16,8	0,0	0,309	0,0
10.	A ₃₂	16,8	0,0	0,437	0,0
11.	A ₄	18,0	0,0	0,070	0,0
12.	A ₅	2,75	0,0	0,1545	0,0

Centralised data is applied to the formulas for calculating the position of the mass of the entire material system.

$$\left\{ \begin{array}{l} x_{1C} = \frac{\sum_{i=1}^{12} m_i x_{1i}}{\sum_{i=1}^{12} m_i} \\ y_{1C} = \frac{\sum_{i=1}^{12} m_i y_{1i}}{\sum_{i=1}^{12} m_i} \\ z_{1C} = \frac{\sum_{i=1}^{12} m_i z_{1i}}{\sum_{i=1}^{12} m_i} \end{array} \right. \quad (1)$$

Such:

$x_{1C} = 0 \text{ m};$

$y_{1C} = 0,2686955211 \text{ m} \approx 0,269 \text{ m};$

$z_{1C} = 0,0028940568 \text{ m} \approx 0,003 \text{ m}.$

This means, as the center of the masses is found under the axis of symmetry of the system at 3 mm from it, located between the two supports [12], [13], [16].

After starting the electric motor, the system centers and operates without disruptive. Vibrations are produced due to the collision of the hammers with the grains of the fodder mixture and due to their end.

2.4. Moments of Mechanical Axial and Centrifugal Inertion of the Mechanical System

The moments of mechanical axial and centrifugal inertion are calculated for each component piece, which performs rotation movement.

The parts are considered prismatic or cylindrical, to which known formulas are applied. In the first phase of calculation is reported to a reference system positioned in the center of the masses of each piece, after which the Steiner theorem is applied to parallel axes or planes, calculating the moments of axial and centrifugal mechanical inertion towards the system of Reference $O_1x_1y_1z_1$, which has the axis O_1y_1 routed along the rotation axis [17]. The centralized situation is presented as in the table that follows the text.

Table 4.

Axial Moments of Inertia

Crt. No.	Component Name	Moment of mechanical inertion axial towards its own system [kgm ²]	Moment of mechanical inertion axial towards the rotation axis [kgm ²]
1.	Hammer Assembly	0.0022446687	0.0496346748
2.	Hammer Stoppers	0.00000544	0.00413576
3.	Rotator Ring	0.189	0.189
4.	Flange	0.189	0.189
5.	Shaft of the impeller motor	0.0218810625	0.0218810625
6.	Electric motor impeller	0.015	0.015
7.	Mill + electric motor impeller at idle (rounded to 5 decimal places)		0.46867

The timing of mechanical centrifugal inertial is established only in relation to the Cartesian reference system $O_1x_1y_1z_1$, because each component part reference system is symmetry and that is why the moments of centrifugal mechanical inertial are null [18].

The reference system is evaluated only the moment of centrifugal mechanical inertion, which will be used in the dynamics of the mill with horizontal shaft. It has the value:

$$J_{y_1z_1} = J_{z_1y_1} = -0.062664 \text{ kgm}^2 \quad (2)$$

3. EQUATION SETTING DIFFERENTIALS, WHICH GOVERNEES DYNAMICS HORIZONTAL SHAFT MILLS

The dynamics of the horizontal shaft mill are classically studied, with the help of the fundamental theorem of dynamics:

- Impulse theorem in relation to the center of the masses;
- Kinetic moment theorem in relation to O_1 pole.

3.1. Impulse Theorem in Relation to The Center of the Masses

For the grain mill with horizontal shaft the theorem of the kinetic moment in relation to the center of the masses has the vector expression:

$$M\bar{a}_C = \bar{H} = \bar{\mathfrak{R}}_{O_1} + \bar{\mathfrak{R}}_{O_2} + \bar{R} \quad (3)$$

In the Relationship (3) the significance of the note is the following:

- ✓ $\bar{\mathfrak{R}}_{O_1}$ is the resultant of the connecting forces in O_1 pole;

- ✓ $\bar{\mathfrak{R}}_{O_2}$ is the resultant of the connecting forces in O_2 pole

- ✓ \bar{R} is the active forces resultant.

The entire material system has the expression of the acceleration of the mass center, in the form:

$$\bar{a}_C = \bar{\varepsilon} \times \bar{r}_C + \bar{\omega} \times (\bar{\omega} \times \bar{r}_C) \quad (4)$$

Report to the reference system adopted vectors are:

$$\bar{\varepsilon} = \varepsilon_{y_1} \bar{j}_1 = \varepsilon \bar{j}_1 \quad (5)$$

$$\bar{\omega} = \omega_{y_1} \bar{j}_1 = \omega \bar{j}_1 \quad (6)$$

$$\bar{r}_C = y_{1C} \bar{j}_1 + z_{1C} \bar{k}_1 \quad (7)$$

Relationships (5), (6) and (7) are introduced in the relationship (4). The obtained result is introduced in the relationship (3), which becomes:

$$M(\varepsilon z_{1C} \bar{l}_1 - \omega^2 z_{1C} \bar{k}_1) = \bar{\mathfrak{R}}_{O_1} + \bar{\mathfrak{R}}_{O_2} + \bar{R} \quad (8)$$

This form of presentation of the vector expression is because the center of the masses is found in plan $y_1O_1z_1$, in the position of static equilibrium, which correspond to the moment of start, when unbalancing is the maximum of the material system [19], [20].

3.2. Kinetic Moment Theorem in Relation to Pole O_1

Report rigid solids to the reference system (Fig. 1) originating in the center of the spherical joint, to which the theorem of the kinetic moment has the expression:

$$\bar{K}_{O_1} = \bar{M}_{O_1} + \bar{\mathcal{M}}_{O_1} + \bar{M}_m \quad (8)$$

The significance of the relationship (8) is:

- \bar{M}_{O1} is the resultant moment of the active forces system, in relation to the pole O_1 ;
- \bar{M}_{O1} is the resultant moment of the link in relation to the pole O_1 ;
- \bar{M}_m is the moment of the electric motor.

In general form the kinetic moment of the rigid joint in rotating motion around a fixed shaft, passing through the pole O_1 , is expressed by the relationship:

$$\bar{K}_{O1} = (J_{x1} \omega_{x1} - J_{x1y1} \omega_{y1} - J_{x1z1} \omega_{z1}) \bar{i}_1 + (J_{y1} \omega_{y1} - J_{y1z1} \omega_{z1} - J_{y1x1} \omega_{x1}) \bar{j}_1 + (J_{z1} \omega_{z1} - J_{z1x1} \omega_{x1} - J_{z1y1} \omega_{y1}) \bar{k}_1 \quad (9)$$

Compared to the adopted reference system, vector relationships (5), (6), (7) are valid, so the vector relationship (9) turns into:

$$\bar{K}_{O1} = -J_{x1y1} \omega_{y1} \bar{i}_1 + J_{y1} \omega_{y1} \bar{j}_1 - J_{z1y1} \omega_{y1} \bar{k}_1 \quad (10)$$

The vector Relationship (10) is derived to apply the relationship (8), considering that the angular velocity derivative is the angle acceleration and is obtained:

$$\dot{\bar{K}}_{O1} = -\bar{i}_1 (J_{x1y1} \varepsilon + J_{z1y1} \omega^2) - J_{y1} \varepsilon \bar{j}_1 + \bar{k}_1 (J_{z1y1} \varepsilon + J_{z1y1} \varepsilon + J_{x1y1} \omega^2) \quad (11)$$

The vector relationship (11) is introduced in the vector relationship (8) and the vector differential equation of the movement of the mill in rotation around the horizontal axle is obtained.

$$-\bar{i}_1 (J_{x1y1} \varepsilon + J_{z1y1} \omega^2) - J_{y1} \varepsilon \bar{j}_1 + \bar{k}_1 (J_{z1y1} \varepsilon + J_{z1y1} \varepsilon + J_{x1y1} \omega^2) = \bar{M}_{O1} + \bar{M}_{O1} + \bar{M}_m \quad (12)$$

3.3. The Torsor of Active and Linkage Forces

The active and connecting force system for the fodder grain Mill is explained by the fixed system of reference $a_1x_1y_1z_1$ and the results are centered on the tabs, in the table 5. The study shall be initiated for the material system in stable static equilibrium [8].

Table 5.

Components of the Active and Connecting Forces Torsor

Crt. No.	Vector name	Component	
		Symbol	Value

1.	The resultant vector of the active forces	R_{x1}	0.0
		R_{y1}	0.0
		R_{z1}	569.47 N
2.	The resultant vector of the connecting forces	\mathcal{R}_{x1}	0.0
		\mathcal{R}_{x2}	0.0
		\mathcal{R}_{y1}	0.0
3.	The resultant moment vector of the active forces	M_{x1}	0.0
		M_{y1}	1.64808 Nm
		M_{z1}	0.0
4.	The resultant moment vector of the linkage forces	\mathcal{M}_{Oy1}	-1.64808 Nm
5.	The dynamic moment due to the inertial forces of the pecans	$M_{Odynamic}$	0.07265 ε Nm

Obsevation: The compensations of the system of forces and moments of connection, which are not specified in the table, will be some of the unknown system of differential equations.

3.4. Differential Equation System

The differential equation system that governs the dynamics of the mill for fodder grain with horizontal shaft, is obtained from the two vectorial differential relationships (8) and (12) on the axes of the reference fixed Cartesian system $O_1x_1y_1z_1$. They have the scalar components explained in table 5. The differential equation system is presented as follows:

$$\begin{cases} M \varepsilon z_{1C} = 0 \\ -M \omega^2 z_{1C} = \mathcal{R}_{O1z} + \mathcal{R}_{O2z} + R_z \\ -\omega^2 J_{y1z1} = M_{O1x} + \mathcal{M}_{O1x} \\ -\varepsilon J_{y1} - 0.07625 \varepsilon = M_m \\ \varepsilon J_{z1y1} = \mathcal{M}_{O1z} \end{cases} \quad (13)$$

The system (13) of the differential equations, contains five unknowns, which are given by:

- Kinematic unknown ω and ε ;
- Unknown due to links \mathcal{R}_{O1z} , \mathcal{R}_{O2z} , \mathcal{M}_{O1z}

Integrating the system of differential equations, with the data determined from the dynamic study carried out, it is possible.

4. CONCLUSIONS

The work is part of a three-piece cycle, which studies the dynamics of the feed grain mill with horizontal shaft.

This is considered the second part of this cycle of works, in which the dynamics of the mill with horizontal shaft are studied, which is adapted to the real situation of an experimental mill, used in laboratory works.

The paper contains a study of the dynamics of the mill, which has not been approached in this manner [6], [7]. The study is necessary in the thesis at the UTCN, which refers to the action of vibrations on the human operator, serving the mill in operation and acting as a vibrating source.

The dynamics of the horizontal shaft mill was approached in view of the theoretical foundation of the thesis, in succession:

1. The establishment of the mechanical scheme corresponding to the actual system, consisting of the mill driven by an electric motor, which has two bearings made by a spherical joint and a cylindrical joint, which determines the rotation movement of the mill;
2. Establishing the center of the masses of the entire material system, in static equilibrium, corresponding to the start of the mill function;
3. Determination of mechanical characteristics, reported to the Cartesian reference system, which originated in the center of the spherical joint;
4. Using the fundamental theorems of dynamics:
 - a. the impulse theorem in relation to the center of the masses and
 - b. kinetic moment theorem in relation to the reference pole;
5. Elaboration of differential equations that govern the dynamics of the mill with horizontal shaft.

The work can be considered self-contained in the dynamics of the mill with horizontal shaft. The system of differential equations (13), which governs the dynamics of the mill is a system of five equations with five unknowns, which is compatible and can be solved by the

approximative numerical method Runge – Kutta of the order 4.5 with variable pitch, which will be achieved in the third part of this cycle of works.

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Contribuții la studiul experimental al dinamicii morii cu ax orizontal.

Partea II-a: Sistemul ecuațiilor diferențiale

Rezumat Lucrarea face parte dintr-un grupaj de trei lucrari, in care se studiaza dinamica unei mori de cereale furajere, prevazuta cu ax orizontal. Arborele morii este sudat direct pe arborele motorului de antrenare. Lucrarea de fata reprezinta cea de a doua parte a acestei lucrari, prin care se intocmeste sistemul mecanic corespunzator morii, se stabilesc caracteristicile mecanice, cu care se realizeaza sistemul ecuatiilor diferentiale, ce guverneaza dinamica morii.

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