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PARAMETRIC ASSESSMENT OF ANTI-SEISMIC DEVICES ACCORDING TO THE NATURE OF KINEMATIC EXCITATION

Polidor BRATU

effects. The most realistic estimation of the dissipation is due to the rigidity k, the exciting pulse ω, together with the viscous damping coefficient c, to the kinematic excitations defined by periodic functions in relation to time. The linear viscoelastic behavior corresponds to the Voigt-Kelvin model without attached mass, so as to reproduce a real test stand, through a scheme of type (c, k).

Key words: *antiseismic devices, kinematic excitation, linear viscoelastic behavior, Voigt-Kelvin model*

1. INTRODUCTION

This study highlights the fact that the amortization of the insulator from the elastomer, as a mass-free system (*c, k*), is determined on experimental bases by drawing the hysterical loop. Thus, a ζ_{eq} is defined which can be correlated with that of a viscoelastic system with mass a mass (*m*, *c, k*).

In this case, the applied kinematic excitation must be capable of a dynamic response in direct correspondence with the viscoelastic or hysteretic force *Q(t)*.

The modeling of the viscoelastic system, without mass, is described by a first-order differential equation, of the form

$$c\dot{x} + kx = Q(t) \tag{1}$$

In the differential equation (1) the instantaneous deformation $x = x(t)$ was introduced that coincides with the instantaneous displacement as the applied kinematic excitation parameter (displacement control).

For each case, depending on the nature and laws of excitation, the following energies will be determined as follows:

- ΔW_d energy dissipated for a complete cycle of 2π or T period;
- W_{el}^{max} maximum elastic energy that corresponds to the time moment $t = \frac{1}{4}T$, for the periodic movement is harmonic as $x = x(t) = A\sin\omega t$.

The equivalent amortization for either a viscoelastic system without mass (*c, k*) marked ζ_{eq}, either for a system with hysterical behavior marked $\eta = \eta_0 = \frac{c\omega}{k}$, is assessed on the basis of the definition relationship $\eta = \frac{\Delta W_d}{2\pi W_{el}^{max}} = 2\zeta_{eq}$ or

$$\zeta_{eq} = \frac{\Delta W}{4\pi W_{el}^{max}} \tag{2}$$

2. VARIETIES OF HARMONIC KINEMATIC EXCITATIONS

The harmonic cinematic excitement will be modeled by circular trigonometric functions, defined by the amplitude of displacement A and the excitation pulse ω, where these are constant measures of a stable stationary regime.

2.1. Harmonic excitation with symmetric alternant cycles.

We consider the instantaneous displacement function such as

$$x(t) = A\sin\omega t \tag{3}$$

which determines the instantaneous speed given by the relation

$$\dot{x}(t) = \omega A\cos\omega t \tag{4}$$

In this case, the energetic measures ΔW_d, W_{el}^{max} may be written as

$$\begin{cases} \Delta W_d = \pi c \omega A^2 \\ W_{el}^{max} = \frac{1}{2} k A^2 \end{cases} \tag{5}$$

and the parametric measures of dissipation may be expressed as such

$$\begin{cases} \eta = \frac{\Delta W_d}{2\pi W_{el}^{max}} = \frac{c\omega}{k} = \eta_0 \\ \zeta_{eq} = \frac{1}{2}\eta_0 = 0{,}50\eta_0 \end{cases} \tag{6}$$

Noting $\eta = \eta_0 = \frac{c\omega}{k}$, we have $\zeta_{eq} = \frac{1}{2}\eta_0$ which constitutes the defining parameters for harmonization in ***harmonic excitation*** regime.

2.2. Periodic excitation with null pulsing cycles

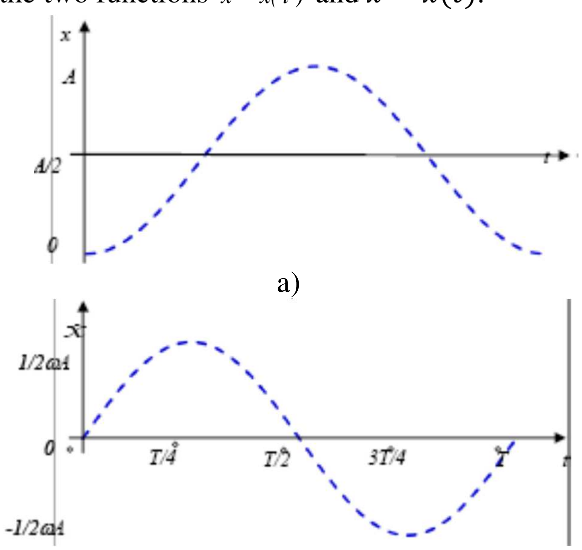
Instantaneous displacement defined by the haversin trigonometric function is as

$$x(t) = \frac{1}{2} A(1 - \cos \omega t) = hav \sin \omega t \tag{7}$$

with speed expressed by the relation

$$\dot{x}(t) = \frac{1}{2} \omega A \sin \omega t \tag{8}$$

Figure 1 presents the variation over time of the two functions $x = x(t)$ and $\dot{x} = \dot{x}(t)$.



a)

b)

Fig. 1. Excitation functions versus time

a) Displacement function

$$x(t) = \frac{1}{2} A(1 - \cos \omega t) = haver \sin us \omega t$$

b) Speed function

$$\dot{x}(t) = \frac{1}{2} \omega A \sin \omega t$$

Dissipated energy ΔW_d may be determined based on the harmonic variation of speed $\dot{x}$ as follows

$$\Delta W_d = \int_0^T c\dot{x}\, dx = \int_0^T c\dot{x}^2\, dt \tag{9}$$

where $\dot{x}(t) = \frac{1}{2} \omega A \sin \omega t$

In this case, we have

$$\Delta W_d = \frac{1}{4} c\omega^2 A^2 \int_0^T \sin^2 \omega t dt \tag{10}$$

or

$$\Delta W_d = \frac{1}{4} c\omega^2 A^2 \frac{1}{2}\left[\int_0^T dt - \int_0^T \cos 2\omega t dt\right] \tag{11}$$

from where it emerges

$$\Delta W_d = \frac{1}{4} \pi c \omega^2 A^2 \tag{12}$$

Maximum elastic energy W_{el}^{max} is determined based on the integral relation, as follows

$$W_{el}^{max} = \int kx dx \tag{13}$$

where $x(t) = \frac{1}{2} A(1 - \cos \omega t)$, and $dx = \frac{1}{2} A \omega \sin \omega t dt$.

In this case, we obtain the relation

$$W_{el} = \frac{1}{4} k\omega\ A^2 \int (1 - \cos \omega t) \sin \omega t dt \tag{14}$$

in which we use the trigonometric transformations as

$$1 - \cos \omega t = 2 \sin^2 \frac{\omega t}{2}$$

$$\sin \omega t = 2 \sin \frac{\omega t}{2} \cos \frac{\omega t}{2}$$

Consequently, the relation (14) may be written as follows

$$W_{el} = \frac{1}{4} k\omega\ A^2 \int 4 \sin^3 \frac{\omega t}{2} \cos \frac{\omega t}{2} dt \tag{15}$$

We note $u = \sin \frac{\omega t}{2}$ and $du = \frac{\omega t}{2} \cos \frac{\omega t}{2} dt$, so that we have $W_{el} = 2k\omega\ A^2 \int \frac{1}{\omega} u^3 du = 2kA^2 \frac{u^4}{4}$ or

$$W_{el} = \frac{1}{2} kA^2 \sin^4 \frac{\omega t}{2}$$

The maximum value for $\omega t = \pi$, it emerges as

$$W_{el}^{max} = \frac{1}{2} kA^2 \tag{16}$$

For a complete cycle that is $\omega t = 2\pi$ it emerges $W_{el} \equiv 0$, that is the elastic system is non-dissipative only with **restoration** feature for the periodic movement.

The equivalent amortization ζ_{eq} at **haversine** excitation emerges as $\zeta_{eq} = \frac{\Delta W_d}{4\pi W_{el}^{max}}$ or

$$\zeta_{eq}^h = \frac{\frac{1}{2}\pi c\omega A^2}{4\pi\frac{1}{2}kA^2} = \frac{1}{8}\frac{c\omega}{k} \tag{17}$$

Considering $\eta_0 = \frac{c\omega}{k}$ the relation (17) becomes

$$\zeta_{eq}^h = \frac{1}{8}\eta_0 = 0{,}25\zeta_{eq} \tag{18}$$

It is found that in case of excitation with function $x = x(t) = hav\, sin\, \omega t$, the equivalent amortization ζ_{eq}^h represents 0.25 of the equivalent amortization corresponding to the harmonic excitation ζ_{eq}.

3. KINEMATIC EXCITATIONS DEFINED BY TRIANGULAR PERIODIC FUNCTIONS

In the testing technique, two types of triangular periodic excitations are used, namely: symmetrical alternating and null pulsing, applied to a (*k, c*) Voigt-Kelvin rheologic mass-free system. Usually symmetrical alternating triangular cycles are used.

3.1. Cinematic excitation with symmetrical alternating triangular cycles.

Image 2 presents the excitation function, $x = x(t)$, speed $\dot{x}(t)$, acceleration $\ddot{x}(t)$ as well as the reaction force $Q(t) = c\dot{x} + kx$ as response to the given excitation. Thus, these functions are defined as:

a) Displacement function

$$x(t) = \begin{cases} \frac{4A}{T}\left(t + \frac{T}{4}\right) - A \;\; pentru \; -\frac{T}{4} \le t \le +\frac{T}{4} \\ -\frac{4A}{T}\left(t - \frac{T}{4}\right) + A \;\; pentru \; +\frac{T}{4} \le t \le \frac{3T}{4} \\ \frac{4A}{T}(t - T) \qquad pentru \; \frac{3T}{4} \le t \le T \end{cases} \tag{19}$$

b) Speed function

$$\dot{x}(t) = \begin{cases} \frac{4A}{T} = \frac{2A\omega}{\pi} \text{pentru} -\frac{T}{4} \le t \le +\frac{T}{4} \\ -\frac{4A}{T} = -\frac{2A\omega}{\pi} \text{pentru} +\frac{T}{4} \le t \le \frac{3T}{4} \\ \frac{4A}{T} \text{pentru} \frac{3T}{4} \le t \le T \end{cases} \tag{20}$$

c) Acceleration function – null identical $\ddot{x} = 0$

d) Reaction force function

$$Q(t) = \begin{cases} \frac{4A}{T}kt + \frac{2}{\pi}A\omega c \;\; pentru \; -\frac{T}{4} \le t \le +\frac{T}{4} \\ -\frac{4A}{T}kt - \frac{2}{\pi}A\omega c + 2Ak \;\; pentru \; +\frac{T}{4} \le t \le \frac{3T}{4} \\ \frac{4A}{T}kt + \frac{2}{\pi}A\omega c - 4Ak \;\; pentru \; +\frac{3T}{4} \le t \le +T \end{cases} \tag{21}$$

Figure 2 presents in temporal concordance the functions $x, \dot{x}, \ddot{x}$ and Q, with the particularity the instantaneous acceleration $\ddot{x} \equiv 0$ for any value of $t \in (0, +\infty)$.

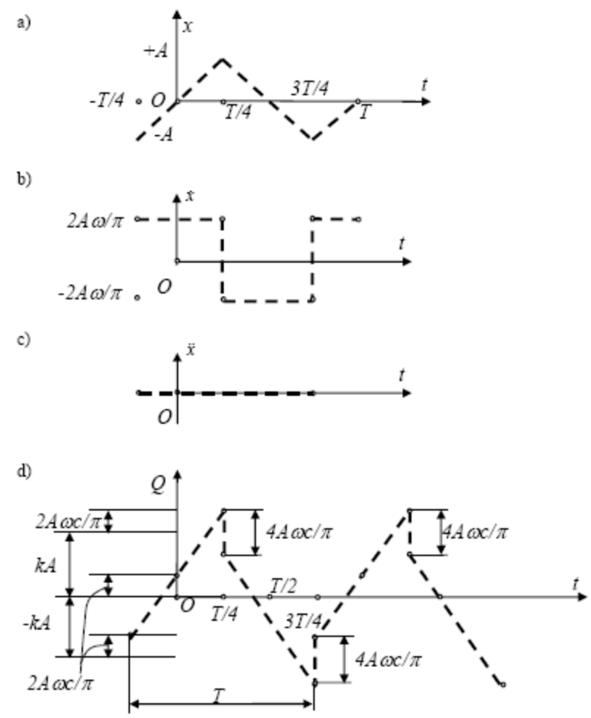


Fig. 2. Response *Q(t)* versus time to the symmetrical triangular cinematic excitation $x = x(t)$

a) variation of instantaneous displacement $x(t)$;

b) variation of instantaneous speed $\dot{x}(t)$;

c) null instantaneous acceleration;

d) variation of reactive force *Q(t)*.

In Fourier harmonic expression, the functions $x(t)$ and $\dot{x}(t)$ may be expressed as follows:

$$x(t) = \frac{8A}{\pi^2}\sum \frac{-1^{\frac{j-1}{2}}}{j^2} sin\, j\omega t; \; j = 1,3,5,7.... \tag{22}$$

$$\dot{x}(t) = \frac{8A\omega}{\pi^2}\sum\frac{-1^{\frac{j-1}{2}}}{j^2}\cos j\,\omega t;\quad j = 1,3,5,7\ldots. \tag{23}$$

Maximum values of functions x and $\dot{x}$ emerge from the relations (22) and (23) as follows:

- for maximum displacement we have:

$$x\left(-\frac{T}{4}\right) = x\left(\frac{T}{4}\right) = \mu\frac{8A}{\pi^2}\left[\sin\frac{\pi}{2}+\frac{1}{3^2}\sin\left(+\frac{3\pi}{2}\right)+\frac{1}{5^2}\sin\left(+\frac{5\pi}{2}\right)+\ldots\right]$$

or

$$x_{max}\left(-\frac{T}{4}\right) = x_{max}\left(\frac{T}{4}\right) = \mu\frac{8A}{\pi^2}\left[1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots\right] = \mu\frac{8A}{\pi^2}\frac{\pi^2}{8} = \mu A \tag{24}$$

- for maximum speed, it is obtained:

$$\dot{x}_{max}(0) = \mp\frac{8A\omega}{\pi^2}\left[\cos 0 - \frac{1}{3}\cos 0 + \frac{1}{5}\cos 0 - \ldots\right] \tag{25}$$

or

$$\dot{x}_{max}(0) = \mp\frac{8A\omega}{\pi^2}\left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}\ldots\right] = \frac{8A\omega}{\pi^2}\frac{\pi}{4} = \mp 2\frac{A\omega}{\pi} \tag{26}$$

but taking into account $\omega = \frac{2\pi}{T}$ we have

$$\dot{x}_{max}(0) = v_{max} = \frac{4A}{T}$$

3.2. Dissipative characteristic of the dynamic system

The dynamic system is characterized by structural parametric measures *c, k* and by excitation parametric measures, expressed by ω, *A*.

Dissipated energy ΔW_d may be expressed on temporal intervals in a complete cycle, as follows

$$\Delta W_d = \int_0^{T/4} c\dot{x}_I dx_1 + \int_{T/4}^{3T/4} c\dot{x}_{II} dx_2 + \int_{3T/4}^{T} c\dot{x}_I dx_3 \tag{27}$$

where $\dot{x}_I = \frac{2}{\pi}A\omega$ and $\dot{x}_{II} = -\frac{2}{\pi}A\omega$ so that it may be obtained:

$$dx_1 = \frac{4A}{T}dt;\ dx_2 = -\frac{4A}{T}dt;\ dx_3 = \frac{4A}{T}dt;$$

which inserted in (27) emerges

$$\Delta W_d = \frac{2}{\pi}A\omega c\int_0^{T/4}\frac{4A}{T}dt - \frac{2}{\pi}A\omega c\int_{T/4}^{3T/4}-\left(\frac{4A}{T}\right)dt + \frac{2}{\pi}A\omega c\int_{3T/4}^{T}\frac{4A}{T}dt$$

or

$$\Delta W_d = \frac{2}{\pi}A\omega c\left[\frac{4A}{T}\frac{T}{4}+\frac{4A}{T}\frac{2T}{4}+\frac{4A}{T}\frac{T}{4}\right]$$

from where we have the expression of dissipated energy such as

$$\Delta W_d = \frac{8}{\pi}\omega c A^2 \tag{28}$$

Maximum elastic energy W_{el}^{max} is determined based on formulating the expression W_{el}, as follows

$$W_{el} = \int kx dx = k\int \dot{x}x dt$$

where $\dot{x} = \frac{2}{\pi}A\omega = v = \text{const}$

Elastic energy on cycle may be written down as:

$$W_{el}^{ciclu} = kv\frac{4A}{T}\int_0^T t dt + k(-v)\left[\left(-\frac{4A}{T}\right)\int_{T/4}^{3T/4} t dt + 2\int_{T/4}^{3T/4} dt\right] + kv\left[\frac{4A}{T}\int_{T/4}^{3T/4} t dt - 4\int_{T/4}^{3T/4} dt\right] \tag{29}$$

from where

$$W_{el}^{ciclu} = kv\left[\frac{1}{8}AT - (-AT + AT) + \left(\frac{7}{8}AT - \frac{4}{4}AT\right)\right] = 0 \tag{30}$$

Maximum energy W_{el}^{max} corresponds to t=T/4, that is it emerges

$$W_{el}^{max} = kv\frac{4A}{T}\int_0^T t dt = k\frac{4A}{T}\frac{4A}{T}\frac{t^2}{2}\Big|_0^{T/4}$$

or

$$W_{el}^{max} = k\frac{16A^2}{T^2}\frac{T^2}{16}\frac{1}{2} = k\frac{A^2}{2} \tag{31}$$

The equivalent amortization for symmetrical triangular cycles ζ_{eq}^t is as

$$\zeta_{eq}^t = \frac{\Delta W}{4\pi W_{el}^{max}} \tag{32}$$

or

$$\zeta_{eq}^t = \frac{8c\omega A^2}{4\pi\frac{1}{2}kA^2} = \frac{4}{\pi^2}\frac{c\omega}{k} \tag{33}$$

where we take into account $\eta_0 = \frac{c\omega}{k}$ and thus we have:

$$\zeta_{eq}^t = \frac{4}{\pi^2}\eta_0 = 0{,}8\zeta_{eq} \tag{34}$$

where $\eta_0 = 2\zeta_{eq}$ is the structural amortization for a harmonically excited dissipative system.

4. CONCLUSIONS

The qualification of the antiseismic elastomeric insulators requires specially designed stands. According to the size of the reaction forces, the amplitude of the movement of the driving actuator, the size of the driving masses in the symmetrical alternating movement, the constructive and functional variety involves distinct approaches. For example, at symmetrical harmonic excitations or null pulsing (haversine), the influence of the mass of the moving equipment may introduce significant influences at the assessment of the reaction force strictly individualized for the elastomeric element.

In case of reduction of the inertial forces with significant values, actuators are used which give symmetrical alternating triangular movement excitations; in which case speed is linear on time intervals and acceleration is null. In this case, there are some disadvantages related to the command and control system. Thus, by computer and automation methods, the command system must correct the **singularities** and **discontinuities** in **the critical points of the excitation function graphs.**

a) For the cinematic excitation by applied instantaneous displacement such as $x(t) = A_0 \sin \omega t$ of a dynamic system (*c, k*), generates a linear reaction force of $c\dot{x} + kx = Q(t)$. Based on the hysteretic loop and on the maximum elastic energy, there may experimentally be determined, the critical amortization report

$$\zeta_{eq} = \frac{\Delta W_d}{4\pi W_{el}^{max}} = \frac{c\omega}{2k};$$

b) For the pulsing-null cinematic excitation by the instantaneous displacement such as $x(t) = \frac{1}{2} A_0 (1 - \cos \omega t) = hav \sin \omega t$, a hysteretic loop has smaller area than in the previous case. Thus, amortization ζ_{eq}^h of the system (*c, k),* excited with function $x = hav \sin \omega t$ is given by $\zeta_{eq}^h = 0{,}25\zeta_{eq}$

c) For the cinematic excitation in the instantaneous triangular displacement it is characteristic the fact that the deformation speed is steady on temporal parts and acceleration is null which enables the influence of the inertial force to be neglected. In this case, the amortization of system (*c,k*), is $\zeta_{eq}^t = 0{,}8\zeta_{eq}$.

In view of the above, it is found that the amortization for mass-free systems of type (*c,k*) can be experimentally assessed and measured only under well-specified structural conditions, of actuator driving that generate instantaneous controlled displacements based on defined excitation functions. Essentially, the amortization defined by $\eta_0 = 2\zeta_{eq}$ is different from one driving system to another being dependent on the excitation function. In this case, the hysteretic loop of force *Q(x)* in relation to the instantaneous displacement *x = x(t)* depends on the excitation function.

The triangular excitement is also used in the case of heat action tests for fluid dissipators. In this case, the period of the heat movement is very high, of (4 ... 20) hours, following the variation of force *Q*, according to EN 15129.

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Evaluarea parametrică a dispozitivelor anti-seismice potrivite la natura excitației cinematice

Rezumat: *Materialele elastomerice sunt caracterizate în regim dinamic, atât prin efecte elastice, cât şi pe cele disipative. Estimarea cea mai realistă a disipării se datorează rigiditatea k, pulsul excitant ω, împreună cu coeficientul de amortizare vâscous c, la excitațiile cinematice definite de funcțiile periodice în raport cu timpul. Comportamentul vâscoelastic liniar corespunde modelului Voigt-Kelvin fără masă atașată, astfel încât să se reproducă un stand de testare real, printr-un sistem de tip (c, k).*

BRATU Polidor, Emeritus University Professor PhD Engineer**,** Research Institute for Construction Equipment and Technology – ICECON SA, Bucharest, Romania, icecon@icecon.ro, +4021 202 55 00