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THE ZENER RHEOLOGICAL VISCOELASTIC MODELLING OF THE DYNAMIC COMPACTION OF THE ECOLOGICALLY STABILIZED SOILS

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Abstract: *This paper treats the concept of Zener linear viscoelastic modulus applied in the vibration compaction process of the soils stabilized with ecological liquid substances brought into atomization state at significant pressures and flows. In this case, by treating soils with ecological stabilizers in the process of milling, mixing and making the road layers, a suitable mixture is obtained for road layers with superior performance to the natural soil. The dynamic compaction effect with vibrating roller cylinders is determined by the stiffness of the land and the structural damping influenced dynamically by the excitation frequency and by the parametric inertial values in static mode. Thus, taking into account experimental results obtained in the laboratory and "in situ", this research highlights the behavior of soil stabilized according to the Zener model. Thus, the parametric values of the model as well as their variation are determined according to the excitation angular frequency are determined for several experimental cases obtained in the testing area.*
Keywords: *Zener rheological viscoelastic modelling; dynamic compaction; stabilized soils*

1. INTRODUCTION

It is presented the concept of the Hooke-Maxwell composite model subjected to the external action of a harmonic force so that based on the dynamic analysis of the response, the physical measures for dynamic stiffness and for the damping of the structural system of the stabilized soil may be determined [1]-[3],[6].

In this context, it is tackled first of all, the dynamic action given by any force $F(t)$ with impulsive features and then by a harmonic force $F_0 \sin \omega t$.

The second case is the basic analysis that faithfully shapes the interaction of the vibratory roller-stabilized soil.

Finally, there emerge and are presented the analytical relations for stiffness and the structural damping of the soil layer stabilized and compacted soil layer with harmonic forces applied in technological regime.

2. ANALYSIS OF BEHAVIOR IN THE DYNAMIC MODEL

2.1. External impulsive dynamic action

Consider the Zener model in Figure 1 with the elastic characteristics k_1, k_2 and the viscous characteristic c with mixed bonds so that the Hooke and the Maxwell branches may assure the parallel connection of the entire linear-viscoelastic model [4],[5],[7].

For the external force $F(t)$ applied over a time $\Delta\tau$, any instantaneous displacement is obtained where the effect of the "memory" of the "hereditary" material is significantly highlighted [8],[9].

In this case we take into account that the hereditary model consists of the simple Hooke model with elastic constant k_1 and the rheological model of the "standard linear deformable solid".

Exterior force F is balanced in any moment by forces k_1x and $c(\dot{x}_1 - \dot{x}_2)$, that is:

$$F = k_1 x + c(\dot{x}_1 - \dot{x}_2), \quad (1)$$

and for *Maxwell* model (the branch on the right k_2, c) we have, when in balance, the relation

$$c(\dot{x}_1 - \dot{x}_2) = k_2 x_2. \quad (2)$$

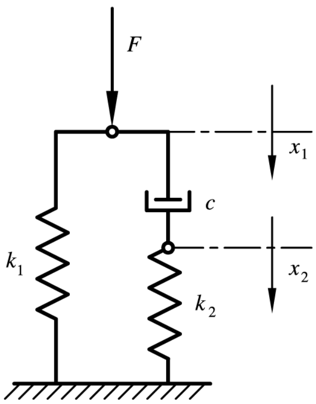

Figure 1. The Zener rheological model

Relation (1) may be described as follows:

$$c\dot{x}_2 + k_2 x_2 = c\dot{x}_1,$$

or

$$\dot{x}_2 + \frac{k_2}{c} x_2 = \dot{x}_1, \quad (3)$$

first degree differential equation x_2, non-homogeneous (1).

The solution of equation (3) along with equation (1) entails the settlement of force F, considering that at $t = 0$ the system is undistorted:

$$F = k_1 x_1 + \int_0^{\tau} f(t-\tau)\dot{x}_1(\tau)\, d\tau, \quad (4)$$

where

$$f(t-\tau) = k_2 e^{-\frac{k_2}{c}t}. \quad (5)$$

It is given that force $F = F(t)$ depends on the dissipative term according to the "*history*" of the distortion speed in which case the damping is called "*hereditary*".

For $F = F(t)$ known on the basis of the solutions to relations (1) and (2), we have

$$x_1(t) = \frac{F(t)}{k_1 + k_2} + \frac{1}{c}\left(\frac{k_2}{k_1 + k_2}\right)^2 \int e^{-\tau/\tau_2} F(t-\tau)\, d\tau, \quad (6)$$

where

$$\tau_2 = \frac{c}{k} = c\frac{k_1 + k_2}{k_1 k_2} \quad (7)$$

In relation (6) the first term $\frac{F(t)}{k_1 + k_2}$ describes the instantaneous response of the viscous-elastic system specific to the linear viscoelastic materials with "memory" [10], [11], [14].

2.2. Exterior harmonic dynamic action.

We consider the elastic harmonic displacements as

$$x_1 = A_1 e^{j\omega t}, \; x_2 = A_2 e^{j\omega t},$$

with their speeds given by the following relations

$$\dot{x}_1 = j\omega A_1 e^{j\omega t}, \dot{x}_2 = j\omega A_2 e^{j\omega t},$$

which, inserted in (1) and (2), lead to

$$\begin{cases} F = k_1 A_1 e^{j\omega t} + k_2 A_2 e^{j\omega t} \\ jc(A_1 - A_2)\,\omega e^{j\omega t} = k_2 A_2 e^{j\omega t}. \end{cases} \quad (8)$$

From the second equation of relation (8) we have

$$A_1 - A_2 = \frac{k_2}{jc\omega} A_2,,$$

from where

$$A_1 = A_2\left(1 + \frac{k_2}{jc\omega}\right),,$$

or

$$A_2 = A_1 \frac{jc\omega}{k_2 + jc\omega},$$

Thus, we obtain:

$$\tilde{F} = k_1 A_1 e^{j\omega t} + A_1 \frac{jk_2 c\omega}{k_2 + jc\omega} e^{j\omega t},,$$

or

$$\tilde{F} = A_1 e^{j\omega t}\left[k_1 + j\frac{k_2 c\omega}{k_2 + jc\omega}\right],,$$

and, in the end, we have

$$\tilde{F} = A_1 e^{j\omega t}\left[\frac{k_1 k_2^2 + (k_1+k_2)\,c^2\omega^2}{k_2^2 + c^2\omega^2} + j\frac{k_2^2 c\omega}{k_2 + c^2\omega^2}\right] \quad (9)$$

Relation (9) may also be written as

$$\tilde{F} = e^{j\omega t} A_1 \tilde{K}(\omega) = x_1 \tilde{K}(\omega), , \quad (10)$$

where: $\tilde{K}(\omega)$ is the complex stiffness coefficient or the *complex stiffness* of the entire hereditary linear viscoelastic system; $x_1 = A_1 e^{j\omega t}$ – instantaneous harmonic displacement of the application point of force $\tilde{F} = \tilde{F}(t)$.

Thus, the complex stiffness of (9) may be set as

$$\tilde{K}(\omega) = K_1(\omega) + jK_2(\omega), , \quad (11)$$

where

$$K_1(\omega) = \frac{k_1 k_2^2 + (k_1+k_2)\,c^2\omega^2}{k_2 + c^2\omega^2}, \quad (12)$$

is the stiffness coefficient which expresses the predominantly elastic behaviour of the system, and

$$K_2 = \frac{k_2^2 c\omega}{k_2 + c^2\omega^2}, \quad (13)$$

is the energy internal loss coefficient (dissipation).

Relation (11) may be written as

$$\tilde{K}(\omega) = K_1(\omega)\left[1 + j\Delta(\omega)\right], , \quad (14)$$

where $\Delta(\omega) = \frac{K_2(\omega)}{K_1(\omega)}$ is the energy *loss* (dissipation) *internal facto*r (angle).

Thus, in the end, the analytical expression is obtained as

$$\Delta(\omega) = \frac{K_2(\omega)}{K_1(\omega)} = \frac{k_2^2 c\omega}{k_1 k_2^2 + (k_1+k_2)\,c^2\omega^2} \quad (15)$$

,

Taking into account relation (15) we have

$$\Delta(\omega) = \frac{ck_2^2\omega}{k_2^2 + c^2\omega^2}\cdot\frac{k_2^2 + c^2\omega^2}{k_1 k_2^2 + (k_1+k_2)\,c^2\omega^2}$$

,

or

$$\Delta(\omega) = \frac{ck_2^2\omega}{k_2^2 + c^2\omega^2}\cdot\frac{1}{K_1(\omega)},$$

Knowing that the internal loss angle for the viscous-elastic system is $\delta = \omega c/k$, we can similarly write the expression as

$$\Delta(\omega) = \frac{\omega C}{K_1(\omega)}, , \quad (16)$$

where C is the viscous damping coefficient equivalent with *Voigt-Kelvin* system dependent on angular frequency ω. Thus, we have

$$C = \underset{sistem}{C} = C(\omega) = c\frac{k_2^2}{k_2^2 + c^2\omega^2}, \quad (17)$$

- *The graphical representation of the stiffness coefficient $K_1(\omega)$ of the hereditary model* according to the excitatory angular frequency ω is achieved taking into account the expressions of the first $K_1'(\omega)$ and second $K_1''(\omega)$, derivates as well as the values of ω it is annulled for.

Thus, we have the following significant functions:

$$K_1(\omega) = \frac{1}{k_2^2 + c^2\omega^2}\left[k_1 k_2^2 + (k_1+k_2)c^2\omega^2\right]; , \quad (18)$$

$$K_1'(\omega) = \frac{dK_1}{d\omega} = \frac{2c^2\omega}{(k_2^2 + c^2\omega^2)^2}\left[(k_1+k_2)k_2^2 - k_1 k_2^2\right]; \quad (19)$$

$$K_1''(\omega) = \frac{d^2K_1}{d\omega^2} = \frac{2c^2k_2^3(k_2^2 + c^2\omega^2)}{(k_2^2 + c^2\omega^2)^4}\left[k_2^2 - 3\omega^2c^2\right]., \quad (20)$$

From condition $\omega \equiv 0$ it emerges $K_1(0) = k_1$, which defines the coordinates of point A $(0, k_1)$ on figure 2, a. For $\omega \to \infty$, when $\lim_{\omega\to\infty} K_1(\omega) = k_1 + k_2$, curve K_1 – ω asymptotically tends to the horizontal right $K_1(\omega) = k_1 + k_2$ =const.

The point of inflexion of curve K_1 – ω noted with $I_1\left(\omega_1, K_1^I\right)$ has the coordinates ω_1 and respectively $K_1^I = K_1(\omega_1)$. Thus:

- condition $K_1''(\omega) = 0$ leads to solution

$$\omega_1 = \frac{1}{\sqrt{3}}\cdot\frac{k_2}{c} = \frac{\sqrt{3}}{3}\cdot\frac{k_2}{c}; , \quad (21)$$

- ordinate $K_1^I(\omega_1)$ is obtained from condition $K_1^I(\omega_1) = K_1(\omega)$, that is we have

$$K_1^I(\omega_1) = \frac{1}{k_2^2 + \frac{1}{3}k_2^2}\left[k_1 k_2^2 + (k_1 + k_2)\frac{k_2^2}{3}\right],$$

or

$$K_1^I(\omega) = k_1 + \frac{1}{4}k_2., \qquad (22)$$

The coordinates of the inflexion point of curve $K_1 - \omega$ are:

$$(I_1)\begin{cases} \omega_1 = \frac{\sqrt{3}}{3}\cdot\frac{k_2}{c} \cong 0{,}577\frac{k_2}{c} \\ K_1^I(\omega_1) = k_1 + \frac{1}{4}k_2. \end{cases},$$

The variation curve of function $K_1(\omega_1)$ in relation to the current variable ω is presented in figure 2.a.

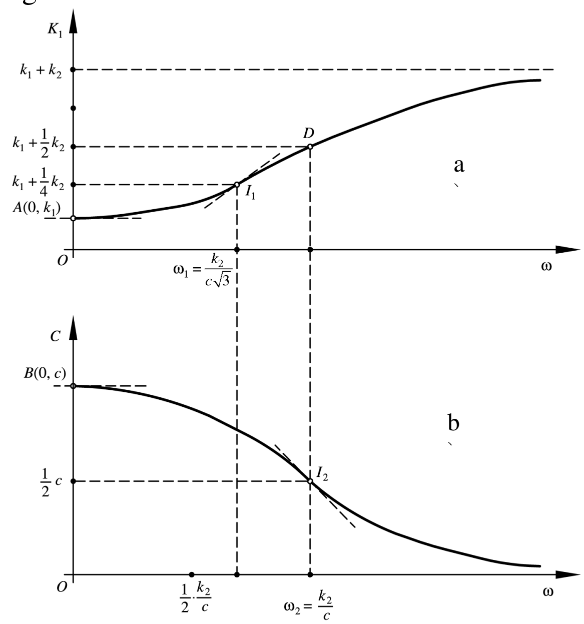

Figure 2. Variation curves of K_1 and C in relation with angular frequency ω

a) Variation of $K_1(\omega)$; **b)** Variation of C(ω)

•• *Graphical representation of the dissipation coefficient (*damping*) of the hereditary system,* equivalent with the *Voigt-Kelvin* dissipation mechanism may be achieved in relation to the variation of the excitatory angular frequency ω as follows:

- Function $C = C(\omega)$ is

$$C(\omega) = c\frac{k_2^2}{k_2^2 + c^2\omega^2},, \qquad (23)$$

which tends to zero for ω→∞ and has value c for $\omega \equiv 0$, that is point B has coordinates (0, c).

- Function $C'(\omega) = \frac{dC}{d\omega}$ is

$$C'(\omega) = \frac{dC}{d\omega} = -2k_2^2c^3\frac{\omega}{(k_2^2 + c^2\omega^2)^2} < 0, \qquad (24)$$

,

that is derivative $C'(\omega) < 0$ for any positive value of ω, which means that function $C(\omega)$ is monotonously decreasing.

For $C'(\omega) = 0$, we have $\omega \equiv 0$, that is in point $B(0, c)$ there is a maximum of function $C(\omega)$.

- Function $C''(\omega) = \frac{d^2C}{d\omega^2}$ is as

$$C''(\omega) = \frac{dC'}{d\omega} = -2k_2^2c^3\frac{k_2^2 - c^2\omega^2}{(k_2^2 + c^2\omega^2)^3}., \qquad (25)$$

From condition $C''(\omega) = 0$, we have the abscissa of the inflexion point $I_2(\omega_2, C_2^I)$ that is

$$\omega_2 = \frac{k_2}{c}, \qquad (26)$$

and ordinate C_2^I of point I_2 is

$$C_2^I(\omega_2) = c\frac{k_2^2}{k_2^2 + c^2\frac{k_2^2}{c^2}} = \frac{1}{2}c., \qquad (27)$$

The coordinates of the inflexion point I_2 are:

$$(I_2)\begin{cases} \omega_2 = \frac{k_2}{c} \\ C_2^I(\omega_2) = \frac{1}{2}c. \end{cases},$$

For $\omega_2 = k_2/c$ we have point D, on curve $K_1 - \omega$ of figure 2, a, with the ordinate

$$K_1^D(\omega_2) = k_1 + \frac{1}{2}k_2.,$$

The variation curve of function $C(\omega)$ in relation to the excitatory angular frequency variation ω is presented in figure 2. b.

3. PARAMETRIC ASSESSMENT BASED ON EXPERIMENTAL DATA

The trials were conducted in a trial polygon with several categories of stabilized land [12],[13].

For sandy clay, the physic-mechanical characteristics have been established in the laboratory. Within the experimental pilot, based on the methods of resonance instrumental analysis and a vibratory compactor with mass m=10^4 ^{kg}, maximum disturbance force F_{max} = 985 KN at 314 rad / s angular frequency with the static moment of the 10 kgm eccentric masses. For successive layers of ecologically stabilized clay, the elastic and damping parameters were determined as static mediation of experimentally obtained numerical values series, as follows:

- stiffness k_1=10^8 N/m
- stiffness k_2=$4{\cdot}10^8$ N/m
- stiffness c=$8{\cdot}10^8$ Ns/m

Based on the experimentally determined values, the values of the inflection points I_1 and I_2 were calculated in the specific coordinates, after which it was possible to identify the calculated values with a deviation of maximum 3% by scaling the frequency from zero to 100 Hz. Thus, the parametric values of the stiffness and damping for points I_1 and I_2 are as follows:

- for point I_1 at ω_1=280 rad/s corresponds the dynamic stiffness k_1=$2{\cdot}10^8$N/m
- for point I2 at ω_2 = 500 rad/s corresponds the damping c_2 =$4{\cdot}10^5$Ns/m

The vibratory compactor with amplitude A=1mm angular frequency ω=280 rad/s, frequency f=44 Hz , by successive passes on the same layer with an equivalent length of 100 m and the speed of 1m/s achieves a compaction energy equivalent to the dissipated energy. Thus, the energy dissipated per cycle is $W_d=\pi c\,\omega A^2$, that is $W_d=\pi{\cdot}5{,}3{\cdot}10^5{\cdot}280{\cdot}10^{-6}$=466 J/cycle.

The total dissipated energy W_t in the stabilized soil layer is W_t=NW_d, where $N=f\Delta t$ = 44 Hz ·100 s = 4400 cycles, so that we have W_t =4400·466=2050 kJ.

Thus, the efficiency of compaction can be assumed by adopting as accurately as possible the Zener hereditary rheological model.

4. CONCLUSIONS

The modelling of soil through rheological schemes as accurate as possible, in relation to the conditions of treatment and use of ecological stabilizers made it possible to solve the following specific problems on the basis of the phenomenological and instrumental analysis:

a) adopting the Zener hereditary model as a result of laboratory tests on specimens taken from the field;

b) assessment of the static and dynamic behaviour of the sample for the identification of the response curves;

c) establishing the global response characteristic in dynamic stiffness depending on variation by continuous sweeping of the excitation angular frequency;

d) defining and identifying the overall damping characteristic curve according to the continuous variation of the excitation angular frequency by sweeping;

e) identifying the remarkable points I_1 and I_2 for the two characteristic curves;

f) assessment of the compaction energy based on the analytically and experimentally obtained data.

Considering the above, this paper is a realistic basis for assessing the dynamic compaction capacity based on adopting and verifying the characteristics of the adopted Zener model.

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Modelarea viscoelastică Zener reologică
a compactarea dinamică a solurilor stabile din punct de vedere ecologic

Rezumat: *Această lucrare tratează conceptul de modulul liniar vascoelastice Zener aplicat în procesul de compactare a vibrațiilor solurilor stabilizate cu substanțe lichide ecologice introduse în stare de atomizare la presiuni și fluxuri semnificative. În acest caz, prin tratarea solurilor cu stabilizatori ecologici în procesul de frezare, amestecare și luare a straturilor rutiere, se obține un amestec adecvat pentru straturile rutiere cu performanțe superioare la solul natural. Efectul de compactare dinamic cu cilindrii cu role vibratoare este determinat de rigiditatea terenului și de amortizare structurală influențată dinamic de frecvența excitației și de valorile inerțiale parametrice în modul static. Astfel, luând în considerare rezultatele experimentale obținute în laborator și "in situ", această cercetare evidențiază comportamentul solului stabilizat în funcție de modelul Zener. Astfel, valorile parametrice ale modelului, precum și variația acestora sunt determinate în funcție de frecvența unghiulară de excitație sunt determinate pentru mai multe cazuri experimentale obținute în zona de testare.*

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