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THE KINEMATIC MODEL OF THE TRTR MODULAR SERIAL ROBOT

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Abstract: *The paper has three parts in its structure: theoretical approach, the kinematic modeling of the TRTR modular serial robot and conclusions, followed by a reference list. After a brief presentation of the iterative method for the kinematic modeling, the mechanical structure of the TRTR robot is mentioned, according to [4]. The iterative method is then applied to the mechanical structure of TRTR robot, based on certain geometrical expressions previously determined. The kinematic parameters are thus obtained, which are the operational angular and linear velocities and accelerations of the gripper with respect to the mobile frame (T_5). Applying some transformation equations, these kinematic parameters are eventually expressed with respect to the fixed frame (T_0) from the robot base.*

Key words: *modular serial robot, kinematic model, iterative method.*

1. THEORETICAL APPROACH

The iterative method is one of the study methods frequently used in the kinematic modeling, according to [1]. This method is based on considering the position vectors, the rotation matrices and it makes use of iterative matrix computation in order to determine the kinematic parameters.

According to [1] and [2], the figure 1 presents the kinematic structure of a robot having (n) degrees of freedom (DOF), consisting in (n) links considered as rigid bodies, which can perform movements determined by the active kinematic joints of the 5th class, considered to be mechanically perfect.

The iterative method consists in crossing the robot kinematic chain from the fixed base 0 to the gripper (n+1) and determining the following kinematic parameters by consecutive iterations:

$$\left\{ \bar{k}_i^i,\ \bar{\omega}_i^i,\ \bar{\varepsilon}_i^i,\ \bar{v}_i^i,\ \bar{a}_i^i,\ i = 1 \div n \right\}. \qquad (1)$$

The mentioned kinematic parameters describe the motion of each link i (i = 1÷n) with respect to the fixed frame (T_0) from the robot base.

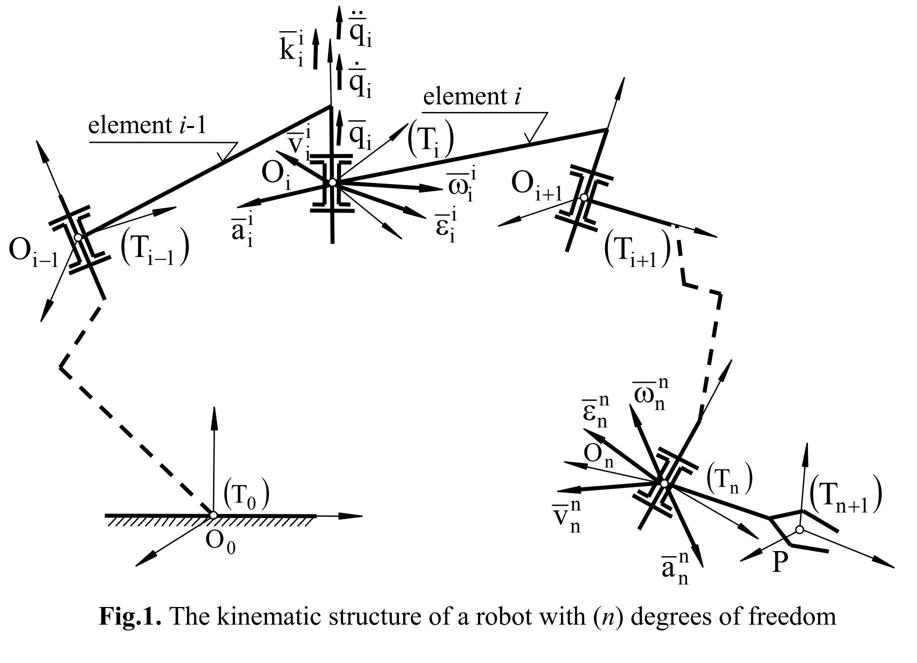

Fig.1. The kinematic structure of a robot with (n) degrees of freedom

These parameters are expressed into the frame (T_i) and they have the following meaning:

$\bar{k}_i^i$ is the i^{th} order axis versor;

$\bar{\omega}_i^i$, $\bar{\varepsilon}_i^i$ represent the angular velocity and acceleration of the rotation of link i around its point O_i, the origin of (T_i) frame, expressed with respect to (T_0);

$\bar{v}_i^i$, $\bar{a}_i^i$ are the linear velocity and acceleration of the point O_i, expressed with respect to the frame (T_0).

The equations (2) and (3) are eventually used for determining the gripper operational kinematic parameters with respect to the fixed frame (T_0), representing the direct kinematic model (DKM) equations, according to [3].

$$\begin{gathered}\left[\dot{X}^n\right]=\left[\left[\bar{v}_n^n\right]^T\ \left[\bar{\omega}_n^n\right]^T\right]^T\\ \left[\ddot{X}^n\right]=\left[\left[\bar{a}_n^n\right]^T\ \left[\bar{\omega}_n^n\right]^T\ \left[\bar{\varepsilon}_n^n\right]^T\right]^T;\end{gathered} \qquad (2)$$

$$\begin{bmatrix}\dot{X}^0\\ --\\ \ddot{X}^0\end{bmatrix}=\begin{bmatrix}[R]^0 & | & [0]\\ ---&|&---\\ [0] & | & [R]^0\end{bmatrix}\cdot\begin{bmatrix}\dot{X}^n\\ --\\ \ddot{X}^n\end{bmatrix}. \qquad (3)$$

The matrix $[R]^0$ from the equation (3) represents a 6×6 square matrix used for transforming the column vectors of the operational velocities and accelerations from the frame (T_n) into the frame (T_0).

The iterative method is also used for determining the kinematic parameters below mentioned:

$$\left\{\bar{k}_i^0,\ \bar{\omega}_i^0,\ \bar{\varepsilon}_i^0,\ \bar{v}_i^0,\ \bar{a}_i^0,\ i=1\div n\right\}. \qquad (4)$$

These parameters express the motion of each element i (i = 1÷n) with respect to the fixed frame (T_0).

2. THE DIRECT KINEMATIC MODEL OF THE TRTR MODULAR SERIAL ROBOT

The direct kinematic model of industrial robots assumes the determination of the operational kinematic parameters of the gripper with respect to the mobile frame (T_n) attached to the gripper and with respect to the frame (T_0) from the robot base, given the robot's constructive mechanical parameters and the instantaneous values of the generalized coordinates, velocities and accelerations from the robot's joint.

After the direct geometric model (DGM) of the robot from the figure 2, the column vector of the operational coordinates was obtained, according to [4].

Using the rotation matrices obtained in [4] and the position vectors also mentioned in [4], the following homogeneous transformation

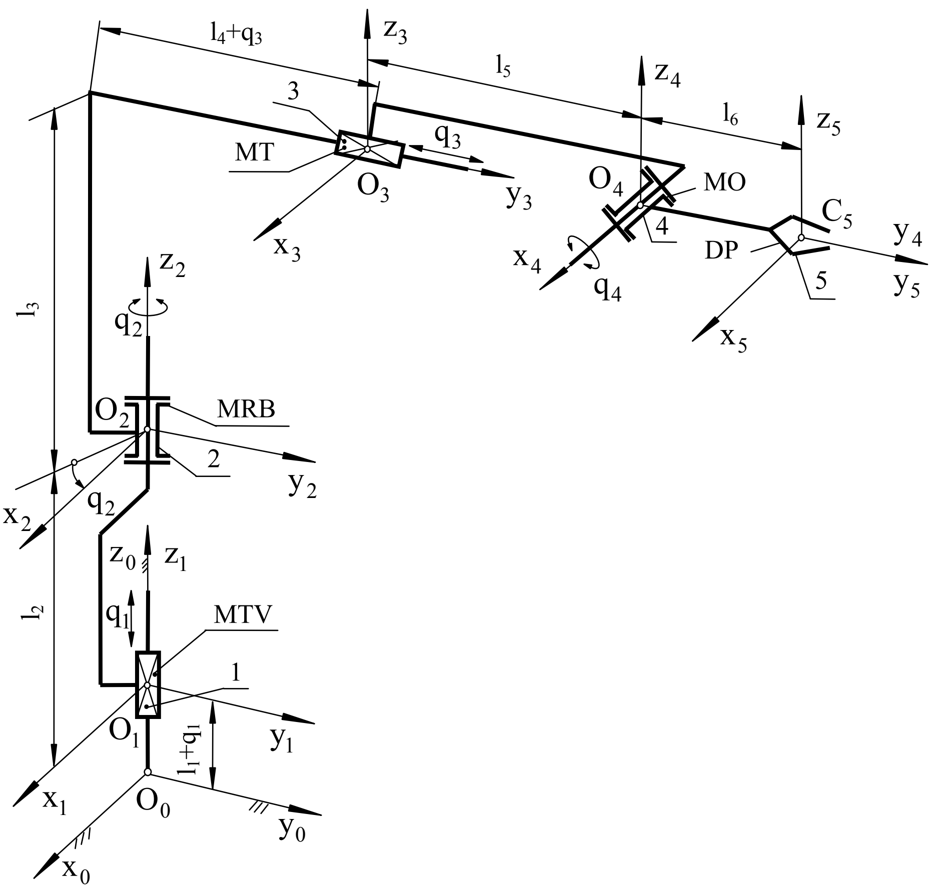

Fig.2. The kinematic structure of the TRTR modular serial industrial robot

matrices can be determined:

$$[T]_1^0(t)=\left[\begin{array}{ccc|c}1&0&0&0\\0&1&0&0\\0&0&1&l_1+q_1\\\hline 0&0&0&1\end{array}\right];\ [T]_2^1(t)=\left[\begin{array}{ccc|c}cq_2&-sq_2&0&0\\sq_2&cq_2&0&0\\0&0&1&l_2\\\hline 0&0&0&1\end{array}\right];\qquad(5)$$

$$[T]_3^2(t)=\left[\begin{array}{ccc|c}1&0&0&0\\0&1&0&l_4+q_3\\0&0&1&l_3\\\hline 0&0&0&1\end{array}\right];\ [T]_4^3(t)=\left[\begin{array}{ccc|c}1&0&0&0\\0&cq_4&-sq_4&l_5\\0&sq_4&cq_4&0\\\hline 0&0&0&1\end{array}\right];\ [T]_5^4(t)=\left[\begin{array}{ccc|c}1&0&0&0\\0&1&0&l_6\\0&0&1&0\\\hline 0&0&0&1\end{array}\right],\qquad(6)$$

and

$$[T]_2^0(t)=\left[\begin{array}{ccc|c}cq_2&-sq_2&0&0\\sq_2&cq_2&0&0\\0&0&1&l_1+l_2+q_1\\\hline 0&0&0&1\end{array}\right];\ [T]_3^0(t)=\left[\begin{array}{ccc|c}cq_2&-sq_2&0&-(l_4+q_3)sq_2\\sq_2&cq_2&0&(l_4+q_3)cq_2\\0&0&1&l_1+l_2+l_3+q_1\\\hline 0&0&0&1\end{array}\right];\qquad(7)$$

$$[T]_4^0(t)=\left[\begin{array}{ccc|c}cq_2&-sq_2cq_4&sq_2sq_4&-(l_4+l_5+q_3)sq_2\\sq_2&cq_2cq_4&-cq_2sq_4&(l_4+l_5+q_3)cq_2\\0&sq_4&cq_4&l_1+l_2+l_3+q_1\\\hline 0&0&0&1\end{array}\right];\qquad(8)$$

$$[T]_5^0(t)=\left[\begin{array}{ccc|c}cq_2&-sq_2cq_4&sq_2sq_4&-(l_4+l_5+q_3+l_6cq_4)sq_2\\sq_2&cq_2cq_4&-cq_2sq_4&(l_4+l_5+q_3+l_6cq_4)cq_2\\0&sq_4&cq_4&l_1+l_2+l_3+q_1+l_6sq_4\\\hline 0&0&0&1\end{array}\right],\qquad(9)$$

respectively.

The homogeneous transformation matrices established in the geometric model are used for determining the kinematic parameters $\overline{\omega}_i^i, \overline{v}_i^i, \overline{\varepsilon}_i^i, \overline{a}_i^i$. Therefore, using the relation below, the inverses of the rotation matrices can be determined:

$$[R]_{i-1}^i=\left[R_i^{i-1}\right]^{-1}=\left[R_i^{i-1}\right]^T.\qquad(10)$$

It is established that:

$$[R]_0^1=[I_3]=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix};\qquad(11)$$

$$[R]_1^2=\begin{bmatrix}cq_2&sq_2&0\\-sq_2&cq_2&0\\0&0&1\end{bmatrix};\qquad(12)$$

$$[R]_2^3=[I_3]=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix};\qquad(13)$$

$$[R]_3^4=\begin{bmatrix}1&0&0\\0&cq_4&sq_4\\0&-sq_4&cq_4\end{bmatrix};\qquad(14)$$

$$[R]_4^5 = [I_3] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \qquad (15)$$

The versors of the kinematic axes and the versor of the axis O_5y_5 (fig. 2) are expressed as the following matrices:

$$[\bar{k}]_1^1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \quad [\bar{k}]_2^2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \quad [\bar{j}]_3^3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}; \qquad (16)$$

$$[\bar{i}]_4^4 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}; \quad [\bar{j}]_5^5 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}. \qquad (17)$$

The kinematic parameters corresponding to the robot base have the expressions:

$$[\overline{\omega}]_0^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \quad [\overline{\mathrm{v}}]_0^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};$$

$$[\overline{\varepsilon}]_0^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \quad [\overline{\mathrm{a}}]_0^0 = \begin{bmatrix} 0 \\ 0 \\ \mathrm{g} \end{bmatrix}. \qquad (18)$$

According to the results from [4] and to the relations (11) - (18), the operational angular velocities can be determined, according also to [3], as:

$$[\overline{\omega}]_1^1 = [\mathrm{R}]_0^1 \cdot [\overline{\omega}]_0^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};. \qquad (19)$$

$$[\overline{\omega}]_2^2 = [R]_1^2 \cdot [\overline{\omega}]_1^1 + \dot{q}_2 \cdot [\bar{k}]_2^2 =$$

$$= \begin{bmatrix} cq_2 & sq_2 & 0 \\ -sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \dot{q}_2 \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_2 \end{bmatrix}; \qquad (20)$$

$$[\overline{\omega}]_3^3 = [\mathrm{R}]_2^3 \cdot [\overline{\omega}]_2^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \dot{\mathrm{q}}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\mathrm{q}}_2 \end{bmatrix}; \qquad (21)$$

$$[\overline{\omega}]_4^4 = [R]_3^4 \cdot [\overline{\omega}]_3^3 + \dot{q}_4 \cdot [\bar{i}]_4^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_4 & sq_4 \\ 0 & -sq_4 & cq_4 \end{bmatrix} \cdot$$

$$\cdot \begin{bmatrix} 0 \\ 0 \\ \dot{q}_2 \end{bmatrix} + \dot{q}_4 \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{q}_4 \\ \dot{q}_2 sq_4 \\ \dot{q}_2 cq_4 \end{bmatrix}; \qquad (22)$$

$$[\overline{\omega}]_5^5 = [R]_4^5 \cdot [\overline{\omega}]_4^4 =$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \dot{q}_4 \\ \dot{q}_2 sq_4 \\ \dot{q}_2 cq_4 \end{bmatrix} = \begin{bmatrix} \dot{q}_4 \\ \dot{q}_2 sq_4 \\ \dot{q}_2 cq_4 \end{bmatrix}. \qquad (23)$$

According to [5], [6] and [7], the 3×3 skew matrix $\{\overline{\omega}\times\}$, where $\overline{\omega}$ is the angular velocity vector, can be written as:

$$\{\overline{\omega}\times\} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}. \qquad (24)$$

In the following matrix relations, the cross products and double cross products are expressed according to (24). According to [4], [5] and [8], the expressions of the operational linear velocities are:

$$[\bar{v}]_1^1 = [R]_0^1 \cdot \{\bar{v}_0^0 + \overline{\omega}_0^0 \times \bar{r}_1^0\} + \dot{q}_1 \cdot [\bar{k}]_1^1;$$

$$[\bar{v}]_1^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ l_1 + q_1 \end{bmatrix} \right\} + \dot{q}_1 \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix}; \qquad (25)$$

$$[\bar{v}]_2^2 = [R]_1^2 \cdot \{\bar{v}_1^1 + \bar{\omega}_1^1 \times \bar{r}_2^1\};$$

(26)

$$[\bar{v}]_2^2 = \begin{bmatrix} cq_2 & sq_2 & 0 \\ -sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \left\{ \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ l_2 \end{bmatrix} \right\} = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix};$$

$$[\bar{v}]_3^3 = [R]_2^3 \cdot \{\bar{v}_2^2 + \bar{\omega}_2^2 \times \bar{r}_3^2\} + \dot{q}_3 \cdot [\bar{j}]_3^3;$$

(27)

$$[\bar{v}]_3^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \left\{ \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix} + \begin{bmatrix} 0 & -\dot{q}_2 & 0 \\ \dot{q}_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ l_4 + q_3 \\ l_3 \end{bmatrix} \right\} + \dot{q}_3 \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\dot{q}_2(l_4 + q_3) \\ \dot{q}_3 \\ \dot{q}_1 \end{bmatrix};$$

$$[\bar{v}]_4^4 = [R]_3^4 \cdot \{\bar{v}_3^3 + \bar{\omega}_3^3 \times \bar{r}_4^3\};$$

(28)

$$[\bar{v}]_4^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_4 & sq_4 \\ 0 & -sq_4 & cq_4 \end{bmatrix} \cdot \left\{ \begin{bmatrix} -\dot{q}_2(l_4 + q_3) \\ \dot{q}_3 \\ \dot{q}_1 \end{bmatrix} + \begin{bmatrix} 0 & -\dot{q}_2 & 0 \\ \dot{q}_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ l_5 \\ 0 \end{bmatrix} \right\} = \begin{bmatrix} -\dot{q}_2(l_4 + l_5 + q_3) \\ \dot{q}_1 sq_4 + \dot{q}_3 cq_4 \\ \dot{q}_1 cq_4 - \dot{q}_3 sq_4 \end{bmatrix};$$

$$[\bar{v}]_5^5 = [R]_4^5 \cdot \{\bar{v}_4^4 + \bar{\omega}_4^4 \times \bar{r}_5^4\};$$

$$[\bar{v}]_5^5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \left\{ \begin{bmatrix} -\dot{q}_2(l_4 + l_5 + q_3) \\ \dot{q}_1 sq_4 + \dot{q}_3 cq_4 \\ \dot{q}_1 cq_4 - \dot{q}_3 sq_4 \end{bmatrix} + \begin{bmatrix} 0 & -\dot{q}_2 cq_4 & \dot{q}_2 sq_4 \\ \dot{q}_2 cq_4 & 0 & -\dot{q}_4 \\ -\dot{q}_2 sq_4 & \dot{q}_4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ l_6 \\ 0 \end{bmatrix} \right\} = \quad (29)$$

$$= \begin{bmatrix} -\dot{q}_2(l_4 + l_5 + q_3 + l_6 cq_4) \\ \dot{q}_1 sq_4 + \dot{q}_3 cq_4 \\ \dot{q}_1 cq_4 - \dot{q}_3 sq_4 + \dot{q}_4 l_6 \end{bmatrix}.$$

The operational angular accelerations, according to [4], can be expressed by the following matrix relations:

$$[\bar{\varepsilon}]_1^1 = [R]_0^1 \cdot [\bar{\varepsilon}]_0^0; \quad [\bar{\varepsilon}]_1^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \quad (30)$$

$$[\bar{\varepsilon}]_2^2 = [R]_1^2 \cdot [\bar{\varepsilon}]_1^1 + \{[R]_1^2 \cdot \bar{\omega}_1^1 \times \dot{q}_2 \cdot \bar{k}_2^2 + \ddot{q}_2 \cdot \bar{k}_2^2\};$$

(31)

$$[\bar{\varepsilon}]_2^2 = \begin{bmatrix} cq_2 & sq_2 & 0 \\ -sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \left\{ \begin{bmatrix} cq_2 & sq_2 & 0 \\ -sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \dot{q}_2 \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \ddot{q}_2 \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_2 \end{bmatrix};$$

$$[\varepsilon]_3^3 = [R]_2^3 \cdot [\varepsilon]_2^2\,; \quad [\varepsilon]_3^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_2 \end{bmatrix}; \tag{32}$$

$$[\varepsilon]_4^4 = [R]_3^4 \cdot [\varepsilon]_3^3 + \left\{[R]_3^4 \cdot \overline{\omega}_3^3 \times \dot{q}_4 \cdot \bar{i}_4^4 + \ddot{q}_4 \cdot \bar{i}_4^4\right\};$$

$$[\varepsilon]_4^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_4 & sq_4 \\ 0 & -sq_4 & cq_4 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_2 \end{bmatrix} + \left\{\begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_4 & sq_4 \\ 0 & -sq_4 & cq_4 \end{bmatrix} \cdot \begin{bmatrix} 0 & -\dot{q}_2 & 0 \\ \dot{q}_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{q}_4 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \ddot{q}_4 \\ 0 \\ 0 \end{bmatrix}\right\} = \tag{33}$$

$$= \begin{bmatrix} \ddot{q}_4 \\ \ddot{q}_2 sq_4 + \dot{q}_2\dot{q}_4 cq_4 \\ \ddot{q}_2 cq_4 - \dot{q}_2\dot{q}_4 sq_4 \end{bmatrix};$$

$$[\varepsilon]_5^5 = [R]_4^5 \cdot [\varepsilon]_4^4\,; \quad [\varepsilon]_5^5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \ddot{q}_4 \\ \ddot{q}_2 sq_4 + \dot{q}_2\dot{q}_4 cq_4 \\ \ddot{q}_2 cq_4 - \dot{q}_2\dot{q}_4 sq_4 \end{bmatrix} = \begin{bmatrix} \ddot{q}_4 \\ \ddot{q}_2 sq_4 + \dot{q}_2\dot{q}_4 cq_4 \\ \ddot{q}_2 cq_4 - \dot{q}_2\dot{q}_4 sq_4 \end{bmatrix}. \tag{34}$$

Considering [4] and [8], the operational linear accelerations can be determined as the following matrices:

$$[\bar{a}]_1^1 = [R]_0^1 \cdot \left\{\bar{a}_0^0 + \varepsilon_0^0 \times \bar{r}_1^0 + \overline{\omega}_0^0 \times \left(\overline{\omega}_0^0 \times \bar{r}_1^0\right)\right\} + \left\{2\overline{\omega}_1^1 \times \dot{q}_1 \cdot \bar{k}_1^1 + \ddot{q}_1 \cdot \bar{k}_1^1\right\};$$

$$[\bar{a}]_1^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \left\{\begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ l_1 + q_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ l_1 + q_1 \end{bmatrix}\right\} + \tag{35}$$

$$+ \left\{2 \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \dot{q}_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \ddot{q}_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right\} = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1 + g \end{bmatrix};$$

$$[\bar{a}]_2^2 = [R]_1^2 \cdot \left\{\bar{a}_1^1 + \bar{\varepsilon}_1^1 \times \bar{r}_2^1 + \overline{\omega}_1^1 \times \left(\overline{\omega}_1^1 \times \bar{r}_2^1\right)\right\};$$

$$[\bar{a}]_2^2 = \begin{bmatrix} cq_2 & sq_2 & 0 \\ -sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \left\{\begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1 + g \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ l_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ l_2 \end{bmatrix}\right\} = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1 + g \end{bmatrix}; \tag{36}$$

$$[\bar{a}]_3^3 = [R]_2^3 \cdot \left\{\bar{a}_2^2 + \bar{\varepsilon}_2^2 \times \bar{r}_3^2 + \overline{\omega}_2^2 \times \left(\overline{\omega}_2^2 \times \bar{r}_3^2\right)\right\} + \left\{2\overline{\omega}_3^3 \times \dot{q}_3 \cdot \bar{j}_3^3 + \ddot{q}_3 \cdot \bar{j}_3^3\right\};$$

$$[\bar{a}]_3^3=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\cdot\left\{\begin{bmatrix}0\\0\\\ddot{q}_1+g\end{bmatrix}+\begin{bmatrix}0&-\ddot{q}_2&0\\\ddot{q}_2&0&0\\0&0&0\end{bmatrix}\cdot\begin{bmatrix}0\\l_4+q_3\\l_3\end{bmatrix}+\begin{bmatrix}0&-\dot{q}_2&0\\\dot{q}_2&0&0\\0&0&0\end{bmatrix}\cdot\begin{bmatrix}0&-\dot{q}_2&0\\\dot{q}_2&0&0\\0&0&0\end{bmatrix}\cdot\right.$$
$$\left.\cdot\begin{bmatrix}0\\l_4+q_3\\l_3\end{bmatrix}\right\}+\left\{2\cdot\begin{bmatrix}0&-\dot{q}_2&0\\\dot{q}_2&0&0\\0&0&0\end{bmatrix}\cdot\begin{bmatrix}0\\\dot{q}_3\\0\end{bmatrix}+\begin{bmatrix}0\\\ddot{q}_3\\0\end{bmatrix}\right\}=\begin{bmatrix}-\ddot{q}_2(l_4+q_3)-2\dot{q}_2\dot{q}_3\\-\dot{q}_2^2(l_4+q_3)+\ddot{q}_3\\\ddot{q}_1+g\end{bmatrix}; \tag{37}$$

$$[\bar{a}]_4^4=[R]_3^4\cdot\left\{\bar{a}_3^3+\bar{\varepsilon}_3^3\times\bar{r}_4^3+\bar{\omega}_3^3\times\left(\bar{\omega}_3^3\times\bar{r}_4^3\right)\right\};$$

$$[\bar{a}]_4^4=\begin{bmatrix}1&0&0\\0&cq_4&sq_4\\0&-sq_4&cq_4\end{bmatrix}\cdot\left\{\begin{bmatrix}-\ddot{q}_2(l_4+q_3)-2\dot{q}_2\dot{q}_3\\-\dot{q}_2^2(l_4+q_3)+\ddot{q}_3\\\ddot{q}_1+g\end{bmatrix}+\begin{bmatrix}0&-\ddot{q}_2&0\\\ddot{q}_2&0&0\\0&0&0\end{bmatrix}\cdot\begin{bmatrix}0\\l_5\\0\end{bmatrix}+\begin{bmatrix}0&-\dot{q}_2&0\\\dot{q}_2&0&0\\0&0&0\end{bmatrix}\cdot\right. \tag{38}$$
$$\left.\cdot\begin{bmatrix}0&-\dot{q}_2&0\\\dot{q}_2&0&0\\0&0&0\end{bmatrix}\cdot\begin{bmatrix}0\\l_5\\0\end{bmatrix}\right\}=\begin{bmatrix}-\ddot{q}_2(l_4+l_5+q_3)-2\dot{q}_2\dot{q}_3\\-\dot{q}_2^2(l_4+l_5+q_3)cq_4+\ddot{q}_1sq_4+\ddot{q}_3cq_4+gsq_4\\\dot{q}_2^2(l_4+l_5+q_3)sq_4+\ddot{q}_1cq_4-\ddot{q}_3sq_4+gcq_4\end{bmatrix};$$

$$[\bar{a}]_5^5=[R]_4^5\cdot\left\{\bar{a}_4^4+\bar{\varepsilon}_4^4\times\bar{r}_5^4+\bar{\omega}_4^4\times\left(\bar{\omega}_4^4\times\bar{r}_5^4\right)\right\};$$

$$[\bar{a}]_5^5=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\cdot\left\{\begin{bmatrix}-\ddot{q}_2(l_4+l_5+q_3)-2\dot{q}_2\dot{q}_3\\-\dot{q}_2^2(l_4+l_5+q_3)cq_4+\ddot{q}_1sq_4+\ddot{q}_3cq_4+gsq_4\\\dot{q}_2^2(l_4+l_5+q_3)sq_4+\ddot{q}_1cq_4-\ddot{q}_3sq_4+gcq_4\end{bmatrix}+\right.$$
$$+\begin{bmatrix}0&-\ddot{q}_2cq_4+\dot{q}_2\dot{q}_4sq_4&\ddot{q}_2sq_4+\dot{q}_2\dot{q}_4cq_4\\\ddot{q}_2cq_4-\dot{q}_2\dot{q}_4sq_4&0&-\ddot{q}_4\\-\ddot{q}_2sq_4-\dot{q}_2\dot{q}_4cq_4&\ddot{q}_4&0\end{bmatrix}\cdot\begin{bmatrix}0\\l_6\\0\end{bmatrix}+$$
$$\left.+\begin{bmatrix}0&-\dot{q}_2cq_4&\dot{q}_2sq_4\\\dot{q}_2cq_4&0&-\dot{q}_4\\-\dot{q}_2sq_4&\dot{q}_4&0\end{bmatrix}\cdot\begin{bmatrix}0&-\dot{q}_2cq_4&\dot{q}_2sq_4\\\dot{q}_2cq_4&0&-\dot{q}_4\\-\dot{q}_2sq_4&\dot{q}_4&0\end{bmatrix}\cdot\begin{bmatrix}0\\l_6\\0\end{bmatrix}\right\}=$$
$$=\begin{bmatrix}-\ddot{q}_2(l_4+l_5+q_3+l_6cq_4)-\dot{q}_2\dot{q}_3(2-l_6sq_4)+l_6\dot{q}_2\dot{q}_4sq_4\\-\dot{q}_2^2(l_4+l_5+q_3+l_6cq_4)cq_4-l_6\dot{q}_4^2+\ddot{q}_1sq_4+\ddot{q}_3cq_4+gsq_4\\\dot{q}_2^2(l_4+l_5+q_3-l_6cq_4)sq_4+\ddot{q}_1cq_4-\ddot{q}_3sq_4+l_6\ddot{q}_4+gcq_4\end{bmatrix}. \tag{39}$$

The operational kinematic parameters from the frame $O_5x_5y_5z_5$ can be written, according to [4], as:

$$\left[\dot{\bar{X}}\right]^5=\begin{bmatrix}-\dot{q}_2(l_4+l_5+q_3+l_6cq_4)\\\dot{q}_1sq_4+\dot{q}_3cq_4\\\dot{q}_1cq_4-\dot{q}_3sq_4+\dot{q}_4l_6\\\dot{q}_4\\\dot{q}_2sq_4\\\dot{q}_2cq_4\end{bmatrix}; \tag{40}$$

$$\left[\ddot{\bar{X}}\right]_5^5 = \begin{bmatrix} -\ddot{q}_2(l_4+l_5+q_3+l_6cq_4)-\dot{q}_2\dot{q}_3(2-l_6sq_4)+l_6\dot{q}_2\dot{q}_4sq_4 \\ -\dot{q}_2^2(l_4+l_5+q_3+l_6cq_4)cq_4-l_6\dot{q}_4^2+\ddot{q}_1sq_4+\ddot{q}_3cq_4+gsq_4 \\ \dot{q}_2^2(l_4+l_5+q_3-l_6cq_4)sq_4+\ddot{q}_1cq_4-\ddot{q}_3sq_4+l_6\ddot{q}_4+gcq_4 \\ \ddot{q}_4 \\ \ddot{q}_2sq_4+\dot{q}_2\dot{q}_4cq_4 \\ \ddot{q}_2cq_4-\dot{q}_2\dot{q}_4sq_4 \end{bmatrix}. \tag{41}$$

Using the transformation relations presented in [4], the operational kinematic parameters can be written in the fixed frame $O_0x_0y_0z_0$ from the robot base, [3]. These relations are the following:

$$[\bar{v}]_5^0 = [R]_5^0 \cdot [\bar{v}]_5^5,$$

$$[\bar{v}]_5^0 = \begin{bmatrix} cq_2 & -sq_2cq_4 & sq_2sq_4 \\ sq_2 & cq_2cq_4 & -cq_2sq_4 \\ 0 & sq_4 & cq_4 \end{bmatrix} \cdot \begin{bmatrix} -\dot{q}_2(l_4+l_5+q_3+l_6cq_4) \\ \dot{q}_1sq_4+\dot{q}_3cq_4 \\ \dot{q}_1cq_4-\dot{q}_3sq_4+\dot{q}_4l_6 \end{bmatrix} = \tag{42}$$

$$= \begin{bmatrix} -\dot{q}_2(l_4+l_5+q_3+l_6cq_4)cq_2-\dot{q}_3sq_2+\dot{q}_4l_6sq_2sq_4 \\ -\dot{q}_2(l_4+l_5+q_3+l_6cq_4)sq_2+\dot{q}_3cq_2-\dot{q}_4l_6cq_2sq_4 \\ \dot{q}_1+\dot{q}_4l_6cq_4 \end{bmatrix};$$

$$[\bar{\omega}]_5^0 = [R]_5^0 \cdot [\bar{\omega}]_5^5,$$

$$[\bar{\omega}]_5^0 = \begin{bmatrix} cq_2 & -sq_2cq_4 & sq_2sq_4 \\ sq_2 & cq_2cq_4 & -cq_2sq_4 \\ 0 & sq_4 & cq_4 \end{bmatrix} \cdot \begin{bmatrix} \dot{q}_4 \\ \dot{q}_2sq_4 \\ \dot{q}_2cq_4 \end{bmatrix} = \begin{bmatrix} \dot{q}_4cq_2 \\ \dot{q}_4sq_2 \\ \dot{q}_2 \end{bmatrix}; \tag{43}$$

$$[\bar{a}]_5^0 = [R]_5^0 \cdot [\bar{a}]_5^5,$$

$$[\bar{a}]_5^0 = \begin{bmatrix} cq_2 & -sq_2cq_4 & sq_2sq_4 \\ sq_2 & cq_2cq_4 & -cq_2sq_4 \\ 0 & sq_4 & cq_4 \end{bmatrix} \cdot \begin{bmatrix} -\ddot{q}_2(l_4+l_5+q_3+l_6cq_4)-\dot{q}_2\dot{q}_3(2-l_6sq_4)+l_6\dot{q}_2\dot{q}_4sq_4 \\ -\dot{q}_2^2(l_4+l_5+q_3+l_6cq_4)cq_4-l_6\dot{q}_4^2+\ddot{q}_1sq_4+\ddot{q}_3cq_4+gsq_4 \\ \dot{q}_2^2(l_4+l_5+q_3-l_6cq_4)sq_4+\ddot{q}_1cq_4-\ddot{q}_3sq_4+l_6\ddot{q}_4+gcq_4 \end{bmatrix} =$$

$$= \begin{bmatrix} -\ddot{q}_2(l_4+l_5+q_3+l_6cq_4)cq_2-\dot{q}_2\dot{q}_3(2-l_6sq_4)cq_2+l_6\dot{q}_2\dot{q}_4sq_4cq_2-\ddot{q}_3sq_2+l_6\dot{q}_4^2sq_2cq_4+ \\ +l_6\ddot{q}_4sq_2sq_4+\dot{q}_2^2(l_4+l_5+q_3+l_6cq_4c^2q_4)sq_2 \\ -\ddot{q}_2(l_4+l_5+q_3+l_6cq_4)sq_2-\dot{q}_2\dot{q}_3(2-l_6sq_4)sq_2+l_6\dot{q}_2\dot{q}_4sq_2sq_4+\ddot{q}_3cq_2-l_6\dot{q}_4^2cq_2cq_4- \\ -l_6\ddot{q}_4cq_2sq_4-\dot{q}_2^2(l_4+l_5+q_3+l_6cq_4c^2q_4)cq_2 \\ -2\dot{q}_2^2l_6sq_4c^2q_4+\ddot{q}_1+g-l_6\dot{q}_4^2sq_4+l_6\ddot{q}_4cq_4 \end{bmatrix}; \tag{44}$$

$$[\overline{\varepsilon}]_5^0 = [R]_5^0 \cdot [\overline{\varepsilon}]_5^5,$$

$$[\overline{\varepsilon}]_5^0 = \begin{bmatrix} cq_2 & -sq_2cq_4 & sq_2sq_4 \\ sq_2 & cq_2cq_4 & -cq_2sq_4 \\ 0 & sq_4 & cq_4 \end{bmatrix} \cdot \begin{bmatrix} \ddot{q}_4 \\ \ddot{q}_2sq_4 + \dot{q}_2\dot{q}_4cq_4 \\ \ddot{q}_2cq_4 - \dot{q}_2\dot{q}_4sq_4 \end{bmatrix} = \begin{bmatrix} \ddot{q}_4cq_2 - \dot{q}_2\dot{q}_4sq_2 \\ \ddot{q}_4sq_2 + \dot{q}_2\dot{q}_4cq_2 \\ \ddot{q}_2 \end{bmatrix}. \quad (45)$$

According to [4] and considering (42), (43), (44) and (45), the operational velocities and acceleration from the fixed frame $O_0x_0y_0z_0$ can be expressed as:

$$\left[\dot{\overline{X}}\right]^0 = \begin{bmatrix} -\dot{q}_2(l_4 + l_5 + q_3 + l_6cq_4)cq_2 - \dot{q}_3sq_2 + \dot{q}_4l_6sq_2sq_4 \\ -\dot{q}_2(l_4 + l_5 + q_3 + l_6cq_4)sq_2 + \dot{q}_3cq_2 - \dot{q}_4l_6cq_2sq_4 \\ \dot{q}_1 + \dot{q}_4l_6cq_4 \\ \dot{q}_4cq_2 \\ \dot{q}_4sq_2 \\ \dot{q}_2 \end{bmatrix}; \quad (46)$$

$$\left[\ddot{\overline{X}}\right]^0 = \begin{bmatrix} -\ddot{q}_2(l_4 + l_5 + q_3 + l_6cq_4)cq_2 - \dot{q}_2\dot{q}_3(2 - l_6sq_4)cq_2 + l_6\dot{q}_2\dot{q}_4sq_4cq_2 - \ddot{q}_3sq_2 + \\ + l_6\dot{q}_4^2sq_2cq_4 + l_6\ddot{q}_4sq_2sq_4 + \dot{q}_2^2(l_4 + l_5 + q_3 + l_6cq_4c^2q_4)sq_2 \\ -\ddot{q}_2(l_4 + l_5 + q_3 + l_6cq_4)sq_2 - \dot{q}_2\dot{q}_3(2 - l_6sq_4)sq_2 + l_6\dot{q}_2\dot{q}_4sq_2sq_4 + \ddot{q}_3cq_2 - \\ - l_6\dot{q}_4^2cq_2cq_4 - l_6\ddot{q}_4cq_2sq_4 - \dot{q}_2^2(l_4 + l_5 + q_3 + l_6cq_4c^2q_4)cq_2 \\ -2\dot{q}_2^2l_6sq_4c^2q_4 + \ddot{q}_1 + g - l_6\dot{q}_4^2sq_4 + l_6\ddot{q}_4cq_4 \\ \ddot{q}_4cq_2 - \dot{q}_2\dot{q}_4sq_2 \\ \ddot{q}_4sq_2 + \dot{q}_2\dot{q}_4cq_2 \\ \ddot{q}_2 \end{bmatrix}. \quad (47)$$

The equations (40), (41), (46) and (47) are the equations of the direct kinematic model of the TRTR robot. They are used for determining the operational kinematic parameters of the gripper with respect to the frames (T_5) and (T_0).

3. CONCLUSIONS

The paper has the following structure: theoretical approach, the direct kinematic model of the TRTR modular serial robot, conclusions and references.

The iterative method of robots kinematic study is mentioned from the start. The direct kinematic model of the TRTR robot is then presented, establishing eventually the operational kinematic parameters of the gripper with respect to the frames (T_5) and (T_0).

By kinematic modeling, the disadvantages of the geometric model are eliminated, determined by the non-linearity of the equations $\overline{X}^0 = f(\overline{q})$ and by the lack of control upon the velocities and accelerations on the motion path.

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Modelarea cinematică a robotului serial modular TRTR

Rezumat:. Lucrarea este structurată pe trei părţi: consideraţii teoretice, modelarea cinematică a robotului serial modular TRTR şi concluzii, succedate de o listă bibliografică. După o prezentare succintă a metodei iterative de modelare cinematică, este menţionată structura mecanică a robotului TRTR, conform lucrării [4]. Asupra structurii mecanice a robotului TRTR, se aplică apoi metoda iterativă, bazată pe unele expresii geometrice determinate anterior. Se obţin astfel parametrii cinematici, adică vitezele şi acceleraţiile operaţionale, liniare şi unghiulare ale dispozitivului de prehensiune, în raport cu sistemul mobil (T_5). Prin aplicarea ecuaţiilor de transformare între sisteme de referinţă, aceşti parametri cinematici sunt exprimaţi, în cele din urmă, în raport cu sistemul de referinţă fix (T_0), de la baza robotului.

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