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THE ROLE OF THE PERT METHOD AND MONTE CARLO SIMULATION IN THE PROGRAMMING OF THE UNIQUE PRODUCTION

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Abstract: *Not finalizing the unique production in the proper time (whether it is the production of a large complex metal piece, a luxury car or a mansion) will lead to substantial additional costs which will take the form of penalties provided in the delivery contracts of products. Because in practice, the duration of the activities from which the project is composed in order to achieve that product can not always be accurately determined and respected (due to unpredictable events that may occur along the way), it is desirable to study the likelihood that the project will be completed up to a certain point in time before making a delivery contract.*
Key words: *unique production, network planning, probability, PERT Method, Monte Carlo Simulation.*

1. INTRODUCTION

This type of production is becoming more and more popular, because nowadays the consumer demand has become very diversified.[2]

In order to meet the expectations of the consumers the small companies but also premium companies from all over the world uses this type of production.[19]

Unique production is generally characterized by a very large product nomenclature, a small production volume of each type of product (even one copy), the degree of specialization of the working places is universal (the equipment from the endowment is universal, and the staff that uses it is highly qualified) and the form of movement of the products is individual. In this case, the manufacture of some products may be repeated at indefinite intervals or may never be repeated.[2]

If the product is of a very large size, it will be placed on a fixed location, and for product processing the teams of workers will move from one product to another and in the order of the technological flow. Often, flexible manufacturing systems can be used to produce small series and unique production.

In order to manage the production activity in good conditions it is recommended to use methods such as:

1. Linear programming - used to optimize resource allocation;
2. The PERT Method - useful "in the case of the unique production that involves pieces very complex and of great importance, in which the successive operations must be accomplished in conformity with the priority restrictions and deadlines";[15]
3. "Just in Time" Method - used to achieve superior organization of production.

2. PERT METHOD

The method helps to estimate the probability of which a project will be achieved at a given time (specified time interval).[17]

In order to achieve the probabilistic characterization of the duration of activities, the PERT method introduces the following elements:[1]

- **optimistic duration** (o) is the shortest possible duration of activity, performed under the most favorable conditions;
- **pessimistic duration** (p) is the longest possible duration of activity, performed under the most unfavorable conditions;

- **most probable duration** (m) is the duration with the highest chance of achieving under normal conditions.

The mean value of the random variable described by the Beta distribution is a linear combination of the three durations defined above:

$$\mu = \frac{o + 4m + p}{6} \quad (1)$$

The root-mean-square deviation will be:

$$\sigma = \frac{p - o}{6} \quad (2)$$

A path through the network is a chain of activities; consequently, the duration of a path, as a sum of uncertain durations, is itself a random variable. As a result, the sum will be calculated with the relation:

$$S=d_1 + d_2 + \ldots + d_m, \quad (3)$$

where:

- d_1, d_2, …,d_m are independent random variables, with the mean values and dispersions:
- the mean value of S is the sum of the mean values:

$$\mu_S=\mu_{d1}+\mu_{d2}+\ldots+\mu_{dm}, \quad (4)$$

- the dispersion of the sum is the sum of the individual dispersions:

$$\sigma_S^2 = \sigma_{d1}^2 + \sigma_{d2}^2 + \ldots + \sigma_{dm}^2 \quad (5)$$

The Gaussian distribution density for the critical path allows to make probabilistic assertions about the chances of completion the project represented by the network at different times (see Figures 3 and 4 and their explanations from the case study).

Therefore, to determine the probability of the project to be made at a moment of time "t", the table with values of the standard normal distribution function[20] and formula[17] will be used:

$$z = \frac{t - T}{\sigma} \quad (6)$$

where:

z = number of standard deviations;

t = time when the project is intended to be achieved;

T = estimated time of project realization (will be equal to the sum of the estimated average times for the activities from the critical path).

The graph of the Normal distribution (Gaussian curve) is a bell-shaped curve, known as the "Gauss bell", which is symmetrical relative to the vertical line m = μ.

This distribution depends on two parameters:

- mean – m;
- standard deviation – σ;

and has the following density of probability:[14]

$$f(x) = \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} \quad (7)$$

where:

x = finite random variable.

3. MONTE CARLO SIMULATION

Since the probabilistic assessments are not sufficient based only on the critical path determined by the average duration of the activities, it is proposed to use the Monte Carlo Simulation.

This involves determining, repeatedly and in a very large number, the critical path through the network, each time using other durations of the activities chosen randomly from the associated probability distributions. Based on these, the mean and the criticality index for each activity are calculated, index defined as the ratio between the number of simulations in which that activity was critical and the total number of simulations. The index estimates the probability that, when executing the project, the activity in question will be on the critical path. The higher the criticality index of an activity is, the more attention needs to be given to the conduct of that activity.[1]

4. CASE STUDY

For a company which aims the rebuild one of its workshops that consists in building a furnace, the following data from Table 1 is known and can be used to represent the network from Figure 1.

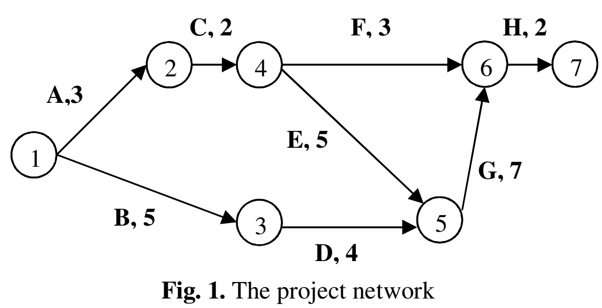

Fig. 1. The project network

Table 1.

Analysis of activities

Activity symbol	Activity description	Predecessors	The probable duration
A	Construction of internal components	---	3
B	Changing the roof and the floor	---	5
C	Construction of the furnace's tub	A	2
D	Installing the frame and casting the concrete	B	4
E	Construction of the high temperature burner	C	5
F	Installation of the control system	C	3
G	Installation of the air purification device	D,E	7
H	Inspection and testing	F,G	2

Hereinafter three estimated time values from Table 2 will be used to represent the project activities in the form of the PERT network, and with the help of 6 simulations, the project duration according to the PERT Method will be determined.

Thus, for each activity of the network in Figure 1, the three times required by the PERT Method are estimated, after which the means and dispersions of the durations are calculated and recorded on the network.

Therefore, it starts from Table 2, where the estimated times for each activity of the project are presented.

Table 2.

The system of the three time proposed by the PERT Method

Activity symbol	Predecessors	Optimistic duration [o]	Most probable duration [m]	Pessimistic duration [p]
A	---	2	3	5
B	---	3	5	6
C	A	1	2	4
D	B	3	4	6
E	C	2	5	7
F	C	1	3	6
G	D,E	5	7	12
H	F,G	1	2	4

By entering the data from Table 2 in the formulas 1 and 2, the values in Table 3 are obtained.

Table 3.

Centralizer with the mean values of the random variables, root-mean-square deviations and of dispersions

Activity	A	B	C	D
μ	3,17	4,83	2,17	4,17
σ	0,5	0,5	0,5	0,5
σ^2	0,25	0,25	0,25	0,25
Activity	**E**	**F**	**G**	**H**
μ	4,83	3,17	7,5	2,17
σ	0,83	0,83	1,17	0,5
σ^2	0,69	0,69	1,37	0,25

The values obtained in Table 3 are inserted in the figure, therefore resulting Figure 2.

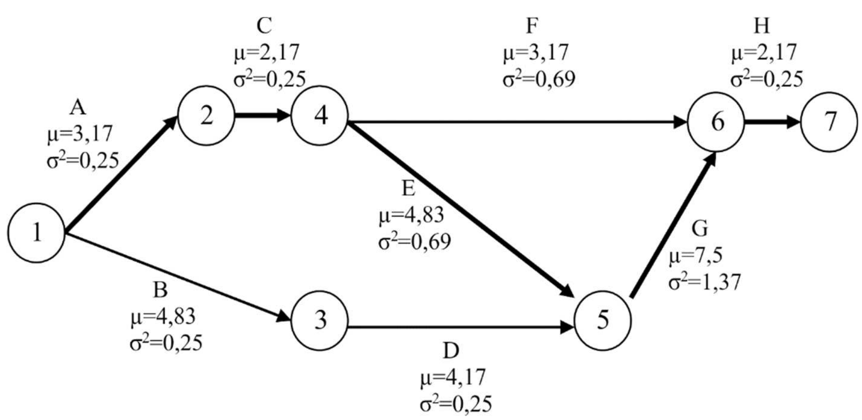

Fig. 2. PERT estimation of the duration of activities and their dispersions

For critical path are calculated:

- average duration:

$$S = \mu_S = \mu_A + \mu_C + \mu_E + \mu_G + \mu_H = 3{,}17+2{,}17+4{,}83+7{,}5+2{,}17 = 19{,}84$$

- dispersion of duration:

$$\sigma_S^2 = \sigma_A^2 + \sigma_C^2 + \sigma_E^2 + \sigma_G^2 + \sigma_H^2 = 0{,}25+0{,}25+0{,}69+1{,}37+0{,}25 = 2{,}81$$

- root-mean-square deviation:

$$\sigma_S = 1{,}68$$

The Gaussian distribution density for the critical path allows to make probabilistic assertions about the chances of completion the project represented in Figure 2 at different moments of time.

In order to achieve the Gaussian curve from the Figures 3 and 4, the formula for density of probability, meaning formula 7, was used. The average duration of the critical path is 19.84 days according to the calculation (the duration of the project is equal to the duration of the critical road).

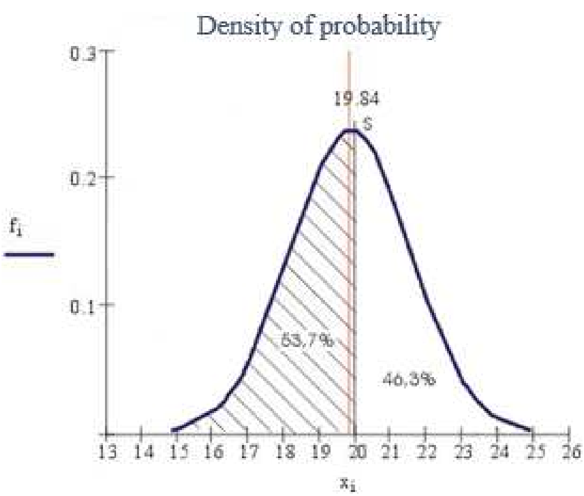

Fig. 3. Approximation after the normal distribution of the length of path A-C-E-G-H, when the planned project completion time is 20 days

In Figure 3 the option to complete the project within 20 days is considered, illustrated in the figure with the S symbol, with a 0,16 days deviation from the average duration of the critical path, previously calculated.

Further, the table with values of the standard normal distribution function and formula 6 were used in order to determine the probability of the project to be made in 20 days.

$$z = \frac{t - \mu_S}{\sigma} = \frac{20 - 19{,}84}{1{,}68} = 0{,}095 \quad \rightarrow$$

probability of 53,7%

Therefore, if the planned project completion time is 20 days, as can be seen in Figure 3, there is a 53,7% chance of success, or there is a chance of 46,3% to complete the project even in 19,68 days or faster.

In Figure 4 the option to complete the project within 21 days is considered, meaning a 1,16 days deviation from the average duration of the critical path.

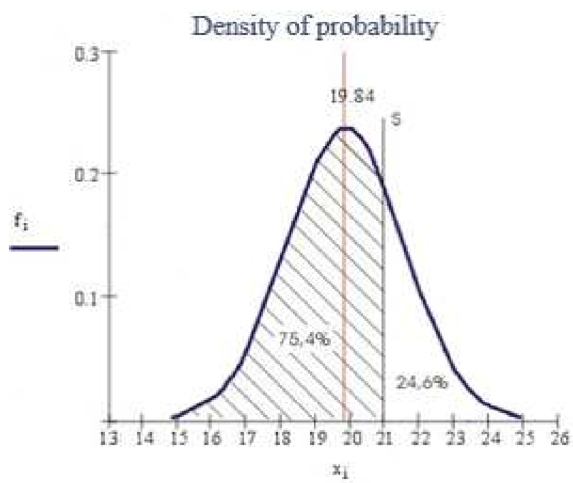

Fig. 4. Approximation after the normal distribution of the length of path A-C-E-G-H, when the planned project completion time is 21 days

$$z = \frac{t - \mu_S}{\sigma} = \frac{21 - 19{,}84}{1{,}68} = 0{,}690 \quad \rightarrow$$

probability of 75,4%

Thus, if the planned project completion time is 21 days, there is a 75,4% chance of success, or there is a chance of 24,6% to complete the project even in 18,68 days or faster.

Table 4.
Monte Carlo Simulation and the resulting critical paths

Number of simulations		**1**	**2**	**3**
Random values of the duration of activities	A	2,108	2,866	2,050
	B	5,322	3,663	3,535
	C	1,537	1,239	3,054
	D	4,976	4,377	3,239
	E	5,861	6,100	2,217
	F	1,117	3,144	2,846
	G	9,994	9,689	8,464
	H	2,027	2,831	3,339
The length of the path	A-C-F-H	6,789	10,08	11,289
	A-C-E-G-H	21,527	**22,725**	**19,124**
	B-D-G-H	**22,319**	20,56	18,577
Critical path		B-D-G-H	A-C-E-G-H	A-C-E-G-H

By using the Monte Carlo Simulation, the following results which are given in Table 4 are obtained.

Table 4.
Monte Carlo Simulation and the resulting critical paths (continuation)

Number of simulations		**4**	**5**	**6**
Random values of the duration of activities	A	3,179	2,403	3,213
	B	5,785	5,263	4,923
	C	2,892	3,162	3,294
	D	4,771	3,999	5,210
	E	6,591	6,810	2,271
	F	4,151	2,458	5,096
	G	10,344	6,229	7,771
	H	1,873	1,014	1,862
The length of the path	A-C-F-H	12,095	9,037	13,465
	A-C-E-G-H	**24,879**	**19,618**	18,411
	B-D-G-H	22,773	16,505	**19,766**
Critical path		A-C-E-G-H	A-C-E-G-H	B-D-G-H

Table 5.
Degree of activity criticality

Activity	**A**	**B**	**C**	**D**
Criticality index	4/6	2/6	4/6	2/6
Activity	**E**	**F**	**G**	**H**
Criticality index	4/6	0/6	6/6	6/6

5. CONCLUSIONS

In practice in network planning, the duration of activities is often not determined, and can only be characterized with a certain probability. In such situations, calculations are first made to program the project activities on the basis of the average values of the durations, after which it is necessary to specify how much trust can be given to these results obtained by applying a deterministic method to a probabilistic problem. That is why it is proposed to use the Monte Carlo Simulation.

Analyzing the planned project completion time of 20 days proposed in the case study, it is noted that the project was completed in time only in three from the six simulations, meaning a percentage of 50%, very similar to Figure 3 (53,7%). It can also be noticed that following these simulations, the critical path is A-C-E-G-H in four out of six simulations.

If this result is compared with the result achieved through the CPM Method, method by which the duration of activities is considered to be accurately known (therefore completed in time), the project is completed within 19 days, but according to the PERT Method, there is a 50% chance that the project will be completed within 20 days, in both methods, the critical path that ensures completion of the project is composed of A-C-E-G-H activities.

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Rolul metodei PERT și al simulării Monte Carlo în programarea producției de unicate

Rezumat: Nefinalizarea la termen a producției de unicate (indiferent că este vorba de realizarea unor piese complexe de dimensiuni mari, a unei mașini de lux sau a unei vile) va conduce la apariția unor costuri suplimentare substanțiale ce vor lua forma unor penalizări prevăzute în contractele de livrare/predare ale produselor. Cum în practică, duratele activităților din care se compune proiectul pentru realizarea produsului respectiv nu pot fi întotdeauna determinate cu exactitate și respectate (datorită unor evenimente imprevizile ce pot să apară pe parcurs) este de dorit ca înainte de întocmirea unui contract de livrare/predare să se studieze și probabilitatea ca proiectul să fie realizat până la un anumit moment de timp.

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