ANALYSIS OF THE FREQUENCY CHARACTERISTICS OF THE AUTOMATIC CONTROL SYSTEM OF MANUFACTURING PROCESS PARAMETERS

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Abstract: A technique for the synthesis of hardware controls of the manufacturing processes parameters with specified accuracy and capacity has been developed on basis of the frequency characteristics method. The mathematical model of a computing unit in the form of an aperiodic circuit with delay is proposed for the first time. An information technology for analyzing the frequency characteristics of a control system with using the capabilities of the Matlab has been developed. An amplitude frequency characteristics, the Nyquist hodograph, transfer and pulse response transfer characteristics were obtained by computer simulations. Moreover the stability margin of the system by the amplitude was determined. It is proved that the application of developed techniques increases the productivity of the process of hardware synthesis. 

Key words: frequency-response method, automatic control system, hardware, capacity, mathematics and computer modeling.

1. INTRODUCTION

By the adoption of the modern information and energy-saving technologies into engineering industry of Ukraine distributed computer systems of coordinate control software for machine tools have found application [1-4]. The expediency of using these systems is ensured by decreasing of the processing time and obtaining a specified roughness of the workpiece surface. In turn, the accuracy and performance of hardware determine the quality indicators of software control systems by the parameters of manufacturing processes.

Modern computer mathematics offers integrated software (Eureka, Gauss, Derive, MathCAD, Mathematical, Matlab etc.) and application packages that make it possible to significantly simplify mathematical calculations in the synthesis of hardware with specified metrological characteristics and processing of input information [5-8]. In addition, the computational approach for using artificial neural networks with the related software is proposed in paper [9-10]. On basis of the use of matrix operations, these software environments also provide the solution to the problems of studying the dynamics of automatic control systems (ACS) by the parameters of manufacturing processes.

1.1 Analysis of the current state of knowledge and the formulation of the problem

There are known methods for analyzing and synthesizing hardware with specified accuracy and processing efficiency of incoming information are described in the work [11]. The development of these methods is based on the capabilities provided by the Matlab software with the extension Simulink. The method of frequency characteristics in the synthesis of hardware ACS uses the representation of the input signal as a Fourier transform [12-15]. With this representation, the calculation of the output signal of the ACS is rather complicated, therefore this computational procedure is used in the analysis of simple expressions for transfer functions. Increase the productivity of the process of calculating the output signal of ACS is possible in principle by using the graphical method of integration [11]. The technique of analysis of ACS and the synthesis of signal
processing hardware, specified accuracy and productivity, solves such tasks [16]:
- mathematical modeling of components with taking into account the uncertainty factor;
- equivalent transformation of the ACS structural scheme;
- setting of zeros and poles of the transfer function;
- determination of the stability reserve of the ACS and the corresponding adjustment of its transfer function;
- construction and analysis of frequency characteristics of ACS;
- synthesis under the conditions of random interference of the input signal processing device on the basis of minimization of the quadratic quality criterion using the reference ACS model;
- constructing a computer simulation scheme for the process of transformation of information;
- establishing the quality, accuracy and speed of ACS hardware.

Practical implementation of this technique involves the use of a sufficiently large amount of manual labour, which significantly reduces the productivity of the process of synthesizing hardware for program control of the parameters of manufacturing processes of machining of parts.

1.1 The aim and tasks of research

The aim of the research is to increase the productivity of the hardware synthesis process with the corresponding metrological characteristics and productivity based on the using of Matlab capabilities.

To achieve the aim it is necessary to solve the following tasks:
- obtaining a mathematical model of the lathe actuator;
- plotting a structural diagram of the ACS with longitudinal feed of the lathe;
- carrying out mathematical modeling of the components of the ACS taking into account the uncertainty factor caused by random interference and measurement errors;
- development a technique for analyzing the frequency characteristics of the ACS by an longitudinal feed of a lathe in a Matlab.

2. RESEARCH MATERIALS AND METHODS OF FREQUENCY CHARACTERISTICS OF ACS

2.1 Mathematical modeling of components

The object of regulation of the system for controlling the parameters of the technological process of machining a workpiece is a direct current (DC) motor or alternating current (AC) motor. The instantaneous rotation speed of the shaft is its output signal. The behavior of the DC motor in the transient states is described by the following system of integro-differential equations [17]:

\[
\begin{align*}
U & = c_1 \Omega + RI + L \frac{dI}{dt} \\
J \frac{d\Omega}{dt} & = M_1 - M_2 \\
\phi & = \int \Omega dt
\end{align*}
\]  

where 
- \(c_1\) – counterelectromotive force coefficient;
- \(U\) – control voltage;
- \(L, R, I\) – respectively inductance, winding resistance and current strength of anchor;
- \(\Omega, \phi\) – respectively instantaneous speed and angle of rotation of engine;
- \(M_1, M_2\) – the rotational and load moment, respectively.

In the operator form under zero initial conditions, the equations (1) of the motion of the DC motor acquire this view

\[
\begin{align*}
U & = c_1 \Omega + I (R + pL) \\
Jp\Omega & = c_2 I - M_2 \\
p\phi & = \Omega
\end{align*}
\]  

where \(c_2\) is the coefficient of moment.

In the system of equations (2) we take into account the fact that

\[
M_1 = c_2 I
\]

The solution of the system of equations (2) gives
\[
U = \frac{JRL}{c_1c_2} p^2\Omega + \frac{JR}{c_2} p\Omega + \frac{R}{c_2} \left(1 + \frac{L}{R} p\right) M_2
\]

or
\[
kU = (T_1 T_2 p^2 + T_1 + p + 1) \Omega - \frac{M_2}{F} (T_1 + p + 1) \tag{4}
\]

where for the engine type SL281: \(k\) – conversion coefficient, \(k = k_1 k_2 = 0.87 \cdot 25.39 = 22.1\); \(T_1\) – electromagnetic time constant, \(T_1 = L/R = 0.00043 C\); \(T_2\) – mechanical time constant, \(T_2 = \frac{JR}{c_2} = 0.011 C\); \(F\) – damping Coefficient, \(F = c_1 c_2 / R = 0.0018\).

Taking into account that \(M_2 = 0\), the transfer function of the DC motor with control action is obtained from expression (4)

\[
W_1(p) = \frac{k}{U} \frac{T_1 T_2 p^2 + T_1 + p + 1}{T_1 T_2 p^2 + T_1 + p + 1}. \tag{5}
\]

Or in the form of a sequential connection of elementary chains

\[
W_1(p) = \frac{k_1 k_2}{(T_1 p + 1)(T_2 p + 1)}. \tag{6}
\]

Taking into account that \(U = 0\), the transfer function of a DC motor with perturbing influence is obtained from expression (4)

\[
W_2(p) = \frac{T_1 p + 1}{F(T_1 T_2 p^2 + T_1 + p + 1)}. \tag{7}
\]

Or in the form of a sequential connection of elementary chains

\[
W_2(p) = \frac{1}{F(T_2 p + 1)} \tag{8}
\]

Expressions (6) and (8) were used by the authors for the development of the structural scheme of the ACS of the lathe longitudinal feed (fig. 1). The foundation of its construction is based on the principle of deviation control. This scheme is made up from the following elements: comparison node (CN), actuator (A), sensor of law of variation of longitudinal feed (SL); computing unit (CU) based on the microcontroller STM32F745; speed sensor (SS); measuring transducer of rotation speed (MT); \(\xi\) – perturbing influence, which acts on the output signal of the ACS. In order to obtain an expression for the transfer function, the development of mathematical models of components and analysis of the structural scheme of the ACS was performed.

\[
\xi \rightarrow \text{CU} \rightarrow \text{A} \rightarrow \omega(t)
\]

\[
\text{CU performs a comparison of the instantaneous rotation speed with the optimal value that the block SL sets. It also forms a control action for the corresponding A. Computing procedures take some time, the duration of which is calculated on the basis of the number of commands of the control program and the clock frequency of the system generator. Therefore, during the execution of these computational procedures, we submit the CU in the form of a proportional chain with a delay. The output signal CU is the control action for the DC motor. Thus, taking into account expression (6), the representation of the transfer function OB has the following form}
\]

\[
W_3(p) = \frac{k_1 k_2 e^{-\tau_3 p}}{T_1 p + 1}, \tag{9}
\]

where \(\tau_3\) – delay of CU, \(\tau_3 = 0.0001 C\); \(k_3\) – conversion coefficient.

Taking into account that \(e^{\tau_3 p} \approx 1 + p\tau_3\) after mathematical transformations the transfer function CU acquires the following form

\[
W_3(p) = \frac{k_1 k_2}{(p\tau_3 + 1)(p T_1 + 1)}. \tag{10}
\]
The conversion coefficient $CU$ is determined on basis of following normalization condition

$$k_1^2k_2k_3 = 1.$$ \hspace{1cm} (11)

Information on the optimum values of the longitudinal feed of the tool is located in the microcontroller database. Therefore, the block $SL$ is shown in the form of a proportional chain with such a transfer function

$$W_4(p) = 1.$$ \hspace{1cm} (12)

The specificity of the operation of the SS and MT blocks in the ACS of the lathe longitudinal feed makes it possible to present them in the form of a proportional chain with a delay. The transfer function of a such chain has the following form

$$W_5(p) = k_5 e^{-p\tau_5},$$ \hspace{1cm} (13)

where $k_5 = 27.669$, $\tau_5 = 0.0002$ s.

The transfer function of the SS and MT blocks, taking into account the dead band $\Delta_1$ around their nominal conversion characteristics, takes the following form

$$W_5(p) = \Delta_1 k_5 e^{-p\tau_5}. \hspace{1cm} (14)$$

The error in the processing of the input signal by SS and MT blocks is quantitatively manifested in changes in the source code. Its quantitative estimation was obtained on basis of the statistical manipulation of research data

$$\Delta_1 = \sigma e^{-\sigma} = 0.036142.$$ \hspace{1cm} (15)

The transfer function of the SS and MT blocks after mathematical transformations takes the following form

$$W_5(p) = \frac{\Delta_1 k_5}{1 + p\tau_5} \hspace{1cm} (16)$$

As a result of the analysis of the structural scheme and mathematical transformations, the transfer function of the ACS of the lathe longitudinal feed was obtained in this view

$$W_6(p) = \frac{W_1(p)W_3(p)}{1 + W_1(p)W_3(p)W_5(p)} = \frac{\tau_4 p + 1}{\sum_{i=0}^{\infty} A_i p} \hspace{1cm} (17)$$

where

$$A_0 = 1, \quad A_1 = 2T_1 + T_2 + \tau_3 + \tau_s,$$
$$A_2 = (2T_1 + T_2)(\tau_3 + \tau_s) + T_1^2 + 2T_1T_2 + \tau_s \tau_5,$$
$$A_3 = T_1^2T_2 + \tau_3 \tau_5(2T_1 + T_2) + (\tau_3 + \tau_s)(T_1^2 + 2T_1T_2),$$
$$A_4 = T_1^2T_2(\tau_3 + \tau_s) + \tau_3 \tau_5(2T_1^2 + 2T_1T_2),$$
$$A_5 = T_1^2T_2\tau_3 \tau_5.$$

After the transfer function of the ACS of the lathe longitudinal feed has been established, the development of information technology for analyzing its frequency characteristics in the Matlab is made.

2.2 Information technology of analysis of frequency characteristics

The corresponding transfer functions of the components of the ACS of the lathe longitudinal feed (expressions 6, 10 and 16) are the initial data for the Matlab software. They are presented in this forms

- $W_1 = \text{tf([k1],[T1*T2 T2 1])};$
- $W_3 = \text{tf([k1*k2],[\tau3*T1 \tau3+T1 1])};$
- $W_5 = \text{tf([\Delta1*k5],[\tau5 1])};$

The calculation of the frequency characteristics of the ACS is performed in command mode. The Matlab software functionalities allow to directly determine the transfer function of the ACS of lathe. To calculate it the following expression was used

$$W_6 = W_1*W_3/(1+W_1*W_3*W_5).$$

Information technology for calculation in the Matlab software of the frequency response characteristics of the ACS of the lathe longitudinal feed consists of the following computational procedures:

- the search for zeros and poles of the transfer function is performed using the commands zero(W6) and pole(W6);
- logarithmic amplitude and phase frequency characteristics (LAFC and LPFC) are constructed using the command bode(W6);
- using the nyquist(W6) command, the Nyquist hodograph is been defining;
- the stability margin of the ACS in amplitude is calculated with the command margin(W6);
- the transient characteristic of the ACS of the lathe longitudinal feed is built with the command step(W6);
- the impulse response of the ACS is built using the command impulse(W6);
- similar results were obtained using the ltiview(W6) command, with the corresponding settings in the menu "Plot Configuration".

For the further synthesis of signal processing devices the LAFC of the mathematical model of the ACS of the lathe longitudinal feed is used. The synthesis procedure consists of such computational procedures:
- its linearization is carried out;
- the desired characteristic is constructed;
- the difference between them gives the LAFC of the regulator;
- the transfer function of the regulator is determined;
- the transfer function ACS with the regulator is calculated;
- the scheme of computer modeling of the process of input information transformation is constructed.

3. THE RESULTS OF RESEARCH OF FREQUENCY CHARACTERISTICS

The results of searching for zeros and poles of the transfer function of the mathematical model of the ACS of the lathe longitudinal feed are given in table 1. Analysis of this table data allows concluding the following:
- all the roots of the characteristic equation are obeyed to the stability condition of the ACS;
- root eight affects the start of the transition process because it's small;
- the roots of the numerator and denominator of the transfer function, which roughly coincide in magnitude, are reduced;

\[ W_{9}(p) = \frac{1}{(T_4p+1)(T_6p+1)(T_7p+1)(T_8p+1)} \]  

(18)

ACS of the lathe longitudinal feed without significant loss of precision, can be given such a transfer function

\[ W_{9}(p) = \frac{1}{\sum_{i=0}^{4} a_i p^i} \]  

(19)

where

* \( a_0 = 1 \)
* \( a_1 = T_4 + T_6 + T_7 + T_8 \)
* \( a_2 = (T_4 + T_6)(T_7 + T_8) + T_4T_6 + T_4T_7 + T_4T_8 \)
* \( a_3 = T_4T_6(T_7 + T_8) + T_4T_8(T_4 + T_6) \)
* \( a_4 = T_4T_6T_7T_8 \)

The initial data for solving using the Matlab software were formed based on the expression (19). Its capabilities make it possible to correct the coefficients of power polynomials of the transfer function of the ACS. The construction of the ACS of the lathe longitudinal feed with the transfer function \( W_9 \), which has the necessary LAFC and LPFC, is performed by changing the values of the coefficients of the power polynomial of the characteristic equation. The results of the LAFC ACS comparison are obtained in the form of graphs (Fig. 2) using the following command.

Table 1

<table>
<thead>
<tr>
<th>Zero</th>
<th>Pole</th>
</tr>
</thead>
<tbody>
<tr>
<td>ans = 1.0e+04 *</td>
<td>ans = 1.0e+04 *</td>
</tr>
<tr>
<td>-1.0000</td>
<td>-1.0008</td>
</tr>
<tr>
<td>-0.5000</td>
<td>-1.0000</td>
</tr>
<tr>
<td>-0.2326</td>
<td>-0.4848</td>
</tr>
<tr>
<td>-0.2231</td>
<td>-0.3064</td>
</tr>
<tr>
<td>-0.0095</td>
<td>-0.2326</td>
</tr>
<tr>
<td></td>
<td>-0.2231</td>
</tr>
<tr>
<td></td>
<td>-0.1512</td>
</tr>
<tr>
<td></td>
<td>-0.0218</td>
</tr>
<tr>
<td></td>
<td>-0.0095</td>
</tr>
</tbody>
</table>
Similarly, the Nyquist hodograph, transitional and impulse transition characteristics of the ACS of the lathe longitudinal feed can be studied.

Calculation of the stability margin for the amplitude of the ACS of the lathe longitudinal feed is shown in Fig. 3.

The results of the LAFC calculations, the Nyquist hodograph, the transitional and impulse transition characteristics of the ACS were obtained using the ltiview (W) command, with the corresponding settings in the "Plot Configuration" menu, and are shown in Fig. 4.
As a result of the analysis of the transitional characteristic of the ASC with longitudinal feed of the lathe, it is established that under the influence of a signal as a unit function at its input the transitional characteristic is aperiodic.

Therefore, there is no overshoot of the output signal of the ACS. A comparison of the Bode diagrams of a closed ACS (See Fig. 3) and its simplified model (See Fig. 4) makes possible to conclude that this computational procedure is correct.

Thus, the authors propose a technique for simplifying expressions for the transfer functions of the ACS by eliminating the roots of the numerator and the denominator, and also in rejecting the roots of the second order infinitesimal.

The method of frequency characteristics in calculating the parameters of the regulator provides for the use of the transfer function of an open ACS by longitudinal feed of a lathe. It was obtained as a result of an analysis of the structural scheme and mathematical transformations in the next form

\[
W_{10}(p) = W_1(p)W_3(p)W_4(p) = \frac{1}{\sum_{i=0}^{3} b_ip^i} \tag{20}
\]

where

\( b_0 = 1 \), \( b_1 = 2T_1 + T_2 + \tau_3 \),

\( b_2 = (T_1 + T_2)(T_1 + \tau_3) + T_1T_2 + T_1\tau_3 \),

\( b_3 = T_1T_2(T_1 + \tau_3) + T_1\tau_3(T_1 + T_2) \),

\( b_4 = T_1^2T_2\tau_3 \).

The Bode diagram of an open ACS by an longitudinal feed of a lathe is shown in Fig. 5.

The transition to the frequency domain \( (p=j\omega) \) after mathematical transformations gives such a transfer function of the closed ACS with the longitudinal feed of the lathe

\[
W_{11}(j\omega) = \frac{1}{a_0 - a_2\omega^2 + a_4\omega^4 + j(a_1\omega - a_3\omega^3)} \tag{21}
\]

where amplitude and phase-frequency characteristics are defined as

\[
|W_{11}(\omega)| = \frac{1}{\sqrt{a_0 - a_2\omega^2 + a_4\omega^4}^2 + (a_1\omega - a_3\omega^3)^2} \tag{22}
\]

\[
\phi(\omega) = -\frac{a_1\omega - a_3\omega^3}{a_0 - a_2\omega^2 + a_4\omega^4} \tag{23}
\]

To determine the LAFC of a closed ACS, the following expression can be used

\[
20\log|W_{11}(\omega)| = -10\log\left[\frac{(a_0 - a_2\omega^2 + a_4\omega^4)^2 + (a_1\omega - a_3\omega^3)^2}{a_0 - a_2\omega^2 + a_4\omega^4} \right] \tag{24}
\]

The transfer function of a closed ACS by the longitudinal feed of a lathe is four series connected periodic chains. Therefore, its asymptotic amplitude characteristic has four salient points. Knee frequencies are

\[
\omega_1 = \frac{1}{T_4}, \quad \omega_2 = \frac{1}{T_6},
\]

\[
\omega_3 = \frac{1}{T_7}, \quad \omega_4 = \frac{1}{T_8}. \tag{25}
\]
The phase characteristic of the automatic control system is obtained by adding the corresponding characteristics of the chains

\[ \phi(\omega) = -\arctg \omega T_4 - \arctg \omega T_6 - \arctg \omega T_7 - \arctg \omega T_8. \] (26)

From Table 1 it can be seen that the transfer function of a closed ACS by a longitudinal feed of a lathe has one dominating pole. The circular frequency of this pole is 1369.04 rad/sec. The gain and the phase delay of the ACS were determined with a Bode diagram (Fig. 6). Similarly, it is possible to investigate the knee frequencies of the logarithmic amplitude and phase-frequency characteristics of the transfer function, as well as the limiting frequency of a closed ACS by lathe longitudinal feed.

4. DISCUSSION OF THE RESEARCH RESULTS

As a result of theoretical studies, equations (6) and (8) were obtained which establish information links between control signals and disturbances influencing and instantaneous rotation speed of the actuator shaft. On the basis of these expressions, a structural scheme of the ACS of the lathe longitudinal feed has been developed and a transfer function has been obtained. The information technology of the analysis of frequency characteristics is constructed taking into account the possibilities of Matlab software for appropriate calculations. A technique for studying and adjusting the frequency characteristics of the ACS based on simplification of the expression for its transfer function by cancellation the roots of the numerator and denominator with coincided or close numerical values are proposed. Confirmation of the high productivity of the computational procedure for analyzing the frequency characteristics of the ACS of the longitudinal feed of the lathe was received.

5. CONCLUSION

1. An ACS of the longitudinal feed of a lathe on the basis of the mathematical model of a DC motor was developed.

2. The mathematical model of a computing unit in the form of an aperiodic circuit with delay is proposed for the first time. The parameters of the circuit are set by the load of the corresponding communication line, and the delay is calculated as a result of summing the execution time of the commands of the control program.

3. Mathematical models of components are constructed. The error in measuring the rotational speed of the executing mechanism was determined as a result of statistical processing of experimental data using an information approach.

4. Based on the capabilities of the Matlab software, a methodology and information technology for analyzing the frequency characteristics of a control system for the parameters of technological processes was developed. A technique for studying and adjusting the frequency characteristics of the ACS based on simplification of the expression for its transfer function by cancellation the roots of the numerator and denominator with coincided or close numerical values are proposed. The correctness of the proposed technique was established by comparison of the Bode diagrams of a closed system and its simplified mathematical model.

5. It is established that the using of methodology and information technology improves the efficiency of the hardware synthesis process with specified metrological characteristics and productivity.

6. REFERENCES

[17] Yenikieiev OF, Subotin OV. Osnovi sintezu i proektvuvannya slidkuyuchih sistem verstativ ta promislovih robotiv
Analiza caracteristicilor de frecvenţă ale sistemului automat de control al parametrilor procesului de producţie

Rezumat: O tehnică pentru sinteza controalelor hardware ale parametrilor proceselor de fabricaţie cu precizie şi capacitate specificată a fost dezvoltată pe baza metodei caracteristicilor de frecvenţă. Modelul matematic al unei unităţi de calcul sub forma unui circuit aperiodic cu întârziere este propus pentru prima dată. A fost dezvoltată o tehnologie de informare pentru analiza caracteristicilor de frecvenţă ale unui sistem de control cu utilizarea capacităţilor Matlab. O caracteristică a frecvenţei de amplitudine, caracteristicile de transfer ale spectrului de frecvenţă Nyquist, transfer şi răspunsul la impulsuri au fost obţinute prin simulări pe calculator. Mai mult decât atât, marja de stabilitate a sistemului prin amplitudine a fost determinată. Se demonstrează că aplicarea tehnicilor dezvoltate sporeşte productivitatea procesului de sinteză hardware.

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