



TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering  
Vol. 62, Issue III, September, 2019

## INFLUENCE OF THE NUMBER OF FINITE ELEMENTS ON DETERMINATION THE MODAL RESPONSE IN THE ANALYSIS OF MULTIBODY SYSTEMS WITH ELASTIC ELEMENTS

Eliza CHIRCAN, Maria Luminița SCUTARU, Cristian SIMIONESCU, Sorin VLASE

***Abstract:** The use of the finite element method for dynamic analysis of multicorp systems with elastic elements implies the solving of numerous problems related to the computational techniques and the particularity of such a study in comparison with the techniques applied to the classic finite elements. For such systems, special features arise from the non-linearity of the matrix coefficients appearing in the differential equation system that describe the mechanical system response and the occurrence of additional terms within these equations. In the paper a comparison is made between the finite element results of the modal response considering the third degree polynomial shape functions and the fifth degree polynomial shape functions. It is also analyzed how the number of finite elements considered for the analysis of a beam can influence the obtained results.*

***Key words:** finite element method, multibody system, eigenvalues, one-dimensional finite element*

### 1. INTRODUCTION

In engineering, more and more often, applications that involve high-speed operation or high-strength systems appear. This leads to the occurrence of undesirable phenomena such as vibration and loss of stability. For the study of the multibody mechanical systems with elastic elements in these cases theoretical studies were made. The complexity of the obtained equations does not allow us to obtain analytical solutions for these cases. For this reason, the numerical study of these equations has been carried out and the Finite Element Method (FEM) has proven to be the most appropriate method for studying these cases.

First, planar mechanisms with a single elastic element were considered. The method was then extended to planar mechanisms with several elastic elements and then was applied to general systems with two or three-dimensional motion and increasingly complex finite elements were used [1], [2], [3], [6], [13]. Some studies offer the possibility to develop the method to mechanical system with different constitutive laws [4], [5]. In this paper we studied how the

number of finite elements used can influence the results obtained in case when the system is modeled with one-dimensional finite elements [14], [17], [18], [22]. We studied the case of a rotating rod around an end.

Two cases were analyzed, the first using third degree interpolation functions and the second using the fifth degree interpolation functions. Using a large number of finite elements can lead to a sometimes dramatic increase in computing time. As a result, it is necessary to use a large enough number to ensure that an accurate result is obtained within a reasonable computational time. To determine this, several numerical experiments have been done within the work.

### 2. MOTION EQUATIONS

The first step in finite element analysis of such a system is to obtain the motion equations for a single finite element. The problem has been studied by many researchers, including [19], [21]. For the sake of understanding, we summarize the main results obtained in the above-mentioned works.

Only one finite beam type is considered. A point M can make a displacement  $f(u, v, w)$  becoming the M' point. If the shape functions are found in the matrix  $N$  and the vectors  $\delta_1$  and  $\delta_2$  are the displacement vectors of the ends, one can write:

$$f = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = N \delta_e = N \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} \quad (1)$$

and the torsion angle of the cross-section:

$$\alpha = N_1 \alpha_1 + N_2 \alpha_2 = N_{(\alpha)}^* \delta_e \quad (2)$$

We denote:

$$\delta_e = \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix}; \quad (3)$$

the vector defining the displacements (slopes, rotations) of the selected finite element.

The matrix  $N$  can be written as:

$$N = \begin{bmatrix} N_{(u)} \\ N_{(v)} \\ N_{(w)} \end{bmatrix} \quad (4)$$

where the rows  $N_{(u)}$ ,  $N_{(v)}$  and  $N_{(w)}$  correspond to the displacement  $u$ ,  $v$  and  $w$ :

The slopes  $\beta$  and  $\gamma$  as [22] :

$$\beta = -\frac{dw}{dx} \quad \text{and} \quad \gamma = \frac{dv}{dx}. \quad (5)$$

The angular speed and the angular acceleration are:

$$\omega = \begin{Bmatrix} 0 \\ 0 \\ \omega \end{Bmatrix}; \quad \varepsilon = \begin{Bmatrix} 0 \\ 0 \\ \varepsilon \end{Bmatrix}. \quad (9)$$

The bending, traction-compression and torsion lead to an internal energy of form:

$$E_{pt} = \delta_e^T (k_i + k_a + k_t) \delta_e = \delta_e^T k_e \delta_e \quad (10)$$

It is noted

$$k_e = k_i + k_a + k_t. \quad (11)$$

The axial force in beam due to the rotation is:

$$E_a = \frac{1}{2} \delta_e^T k_e^G \delta_e \quad (12)$$

The total internal energy for one single finite element is:

$$E_p = \delta_e^T (k_e + k_e^G) \delta_e. \quad (13)$$

The Lagrangian for the one single finite element is:

$$L = E_c - E_p + W + W^c. \quad (14)$$

where  $E_c$  represents the kinetic energy of the element,  $E_p$  internal energy and  $W + W^c$  the work of the concentrated and distributed forces acting on the element. Applying the Lagrange equations [7], [11], [15], [16]:

$$\frac{d}{dt} \left\{ \frac{\partial L}{\partial \dot{\delta}_e} \right\} - \left\{ \frac{\partial L}{\partial \delta_e} \right\} = 0. \quad (15)$$

the motion equations are [3], [12] (written in a local coordinates system):

$$\begin{aligned} m_e \ddot{\delta}_e + 2c_e(\omega) \dot{\delta}_e + (k_e + k_e(\varepsilon) + k_e(\omega^2) + \\ k_e^G) \delta_e = &= q_e + q_e^* - q_e^i(\varepsilon) - q_e^i(\omega^2) - \\ m_{eL}^i I \varepsilon_L - m_{oe}^i R^T \ddot{r}_{oG} \end{aligned} \quad (16)$$

These equations will be used in the following in the study of a beam being in a rotation around one end. It is now possible to determine the matrix coefficients of the system of the differential equations. These coefficients are nonlinear through the angular speed and angular acceleration.

### 3. EIGENVALUES OF A ROTATING BEAM

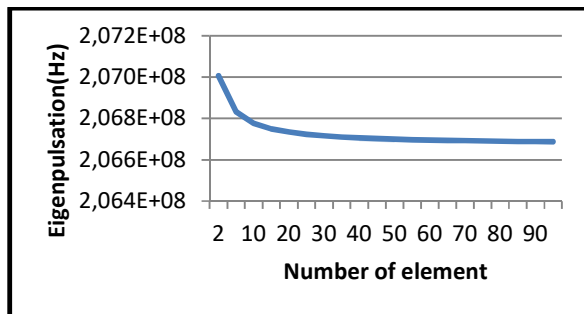
#### 3.1 Shape function of third degree

Shape functions have been chosen third degree polynomials. In the following, we calculated eigenvalues for a beam that rotates around an end with an angular velocity of 1000, 2000 and 10,000 [1/s]. The calculus was made considering different discretization of a beam having 1m length and 1 cm in diameter. The results are mentioned below. Only the first two eigenvalues were presented, things happening the same for the other eigenvalues. The error was also defined as the ratio of the difference between two successive eigenvalues and the eigenvalue. It is noted that if we use more 20 finite elements error falls below 1 E-04 and if the number of elements is greater than 60 then the error falls below 1 E-05. Things happen similarly for the three values of the angular speed chosen.

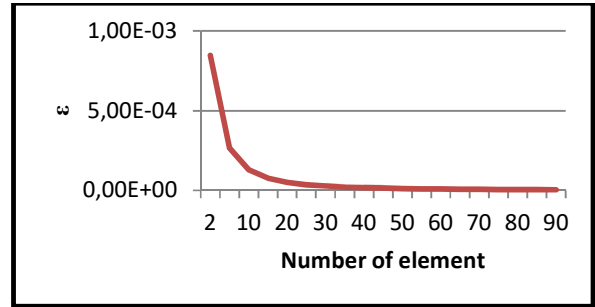
*Table 1*  
**The first two eigenpulsation for the angular speed**  
 $\omega=1000(1/s)$

Nr. of el.	Eigenpulsation p <sub>1</sub>	$\epsilon$	Eigenpulsation p <sub>2</sub>	$\epsilon$
	Hz	--		--
5	2,07E+08	0,000847	8,177E+09	1,08E-03
10	2,07E+08	0,000265	8,169E+09	1,03E-04
15	2,07E+08	0,00013	8,168E+09	3,37E-05
20	2,07E+08	7,71E-05	8,167E+09	1,72E-05
25	2,07E+08	5,11E-05	8,167E+09	1,07E-05
30	2,07E+08	3,63E-05	8,167E+09	7,38E-06
35	2,07E+08	2,72E-05	8,167E+09	5,43E-06
40	2,07E+08	2,11E-05	8,167E+09	4,17E-06
45	2,07E+08	1,68E-05	8,167E+09	3,31E-06
50	2,07E+08	1,37E-05	8,167E+09	2,70E-06
55	2,07E+08	1,14E-05	8,167E+09	2,24E-06
60	2,07E+08	9,7E-06	8,167E+09	1,89E-06
65	2,07E+08	8,29E-06	8,167E+09	1,62E-06
70	2,07E+08	7,12E-06	8,167E+09	1,40E-06
75	2,07E+08	6,3E-06	8,167E+09	1,22E-06
80	2,07E+08	5,47E-06	8,167E+09	1,08E-06
85	2,07E+08	4,78E-06	8,167E+09	9,57E-07
90	2,07E+08	4,67E-06	8,167E+09	8,61E-07

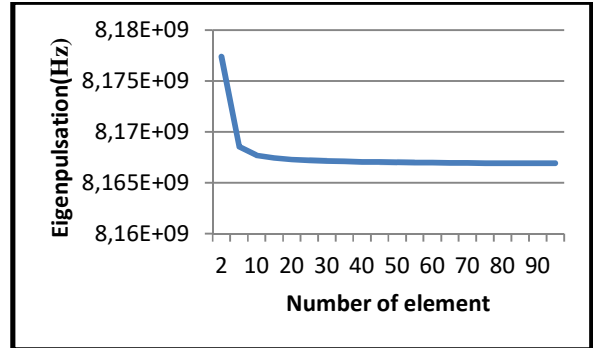
In Fig.1 și 3 present the eigenpulsations p<sub>1</sub> and p<sub>2</sub> for different number of elements. In Fig.2 and 4 are presented the diagram of the error if were chosen different number of elements. In Fig. 5,6,7 and 8 are presented the same chart if the beam has a rotation around one end.



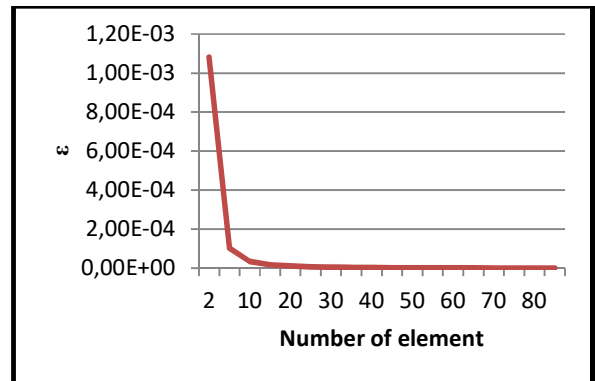
**Fig. 1.** Eigenpulsation p<sub>1</sub>(Hz)



**Fig.2.** The diagram of  $\epsilon$  for eigenpulsation p<sub>1</sub>



**Fig. 3.** Eigenpulsation p<sub>2</sub>(Hz)



**Fig.4.** The diagram of  $\epsilon$  for eigenpulsation p<sub>2</sub>

*Table 2*  
**The first two eigenpulsation for the angular speed**  
 $\omega=2000(1/s)$

Nr. of el.	Eigenpulsation p <sub>1</sub>	$\epsilon$	Eigenpulsation p <sub>2</sub>	$\epsilon$
	Hz	--		--
5	2,040E+08	3,36E-03	8,18E+09	1,53E-03
10	2,033E+08	1,07E-03	8,168E+09	2,51E-04
15	2,031E+08	5,28E-04	8,166E+09	1,08E-04
20	2,030E+08	3,14E-04	8,165E+09	6,15E-05
25	2,029E+08	2,08E-04	8,165E+09	4,02E-05
30	2,029E+08	1,48E-04	8,164E+09	2,84E-05
35	2,028E+08	1,11E-04	8,164E+09	2,12E-05
40	2,028E+08	8,59E-05	8,164E+09	1,64E-05
45	2,028E+08	6,85E-05	8,164E+09	1,31E-05

50	2.028E+08	5.60E-05	8.164E+09	1.07E-05
55	2.028E+08	4.66E-05	8.164E+09	8.90E-06
60	2.027E+08	3.95E-05	8.164E+09	7.52E-06
65	2.027E+08	3.36E-05	8.163E+09	6.44E-06
70	2.027E+08	2.93E-05	8.163E+09	5.58E-06
75	2.027E+08	2.56E-05	8.163E+09	4.88E-06
80	2.027E+08	2.26E-05	8.163E+09	4.31E-06
85	2.027E+08	1.99E-05	8.163E+09	3.82E-06
90	2.027E+08	1.79E-05	8.163E+09	3.42E-06

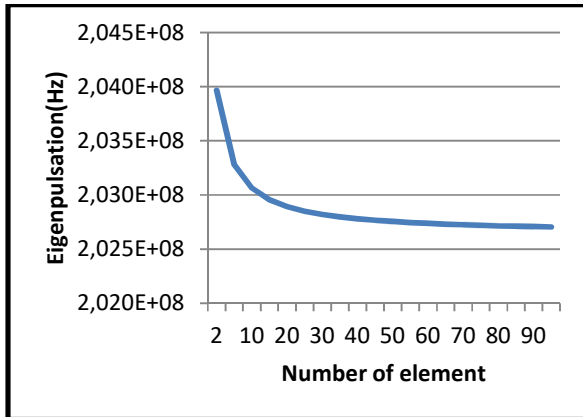


Fig. 5. Eigenpulsation  $p_1$ (Hz)

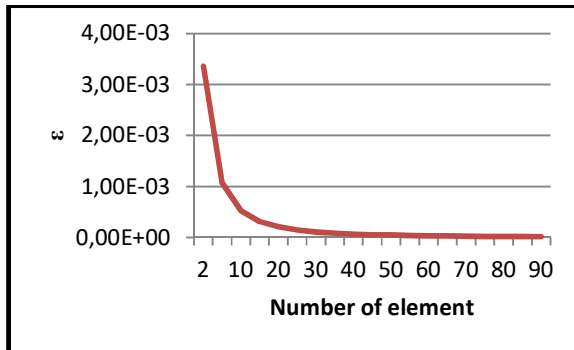


Fig.6. The diagram of  $\epsilon$  for eigenpulsation  $p_1$

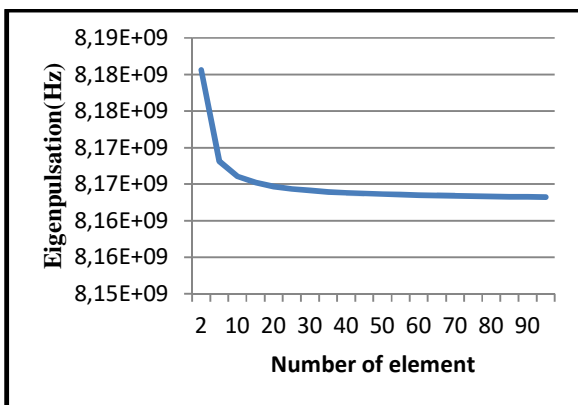


Fig.7. Eigenpulsation  $p_2$ (Hz)

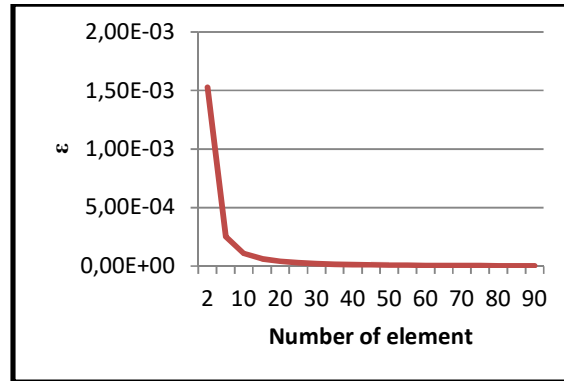


Fig.8. The diagram of  $\epsilon$  for eigenpulsation  $p_2$

Table 3

The first two eigenpulsation for the angular speed  $\omega=10000$ (1/s)

Nr. of el.	Eigenpulsation $p_1$ Hz	$\epsilon$	Eigenpulsation $p_2$	$\epsilon$
5	1.062E+08	1.57E-01	8.284E+09	1.56E-02
10	8.956E+07	6.00E-02	8.155E+09	5.01E-03
15	8.419E+07	3.15E-02	8.114E+09	2.49E-03
20	8.153E+07	1.94E-02	8.094E+09	1.49E-03
25	7.996E+07	1.31E-02	8.082E+09	9.92E-04
30	7.891E+07	9.46E-03	8.074E+09	7.08E-04
35	7.816E+07	7.14E-03	8.068E+09	5.31E-04
40	7.760E+07	5.58E-03	8.064E+09	4.13E-04
45	7.717E+07	4.49E-03	8.06E+09	3.30E-04
50	7.682E+07	3.68E-03	8.058E+09	2.70E-04
55	7.654E+07	3.08E-03	8.055E+09	2.25E-04
60	7.631E+07	2.61E-03	8.054E+09	1.90E-04
65	7.611E+07	2.24E-03	8.052E+09	1.63E-04
70	7.594E+07	1.95E-03	8.051E+09	1.41E-04
75	7.579E+07	1.70E-03	8.05E+09	1.24E-04
80	7.566E+07	1.51E-03	8.049E+09	1.09E-04
85	7.554E+07	1.34E-03	8.048E+09	9.69E-05
90	7.544E+07	1.20E-03	8.047E+09	8.67E-05

For an angular speed 10.000 [1/s] the results are presented in Table 3. The diagram are similar with the previous presented.

A comparison of the eigenpulsations for the three angular speeds is shown in Figure 9 and a comparison of the error in the three cases is presented in Figure 10.

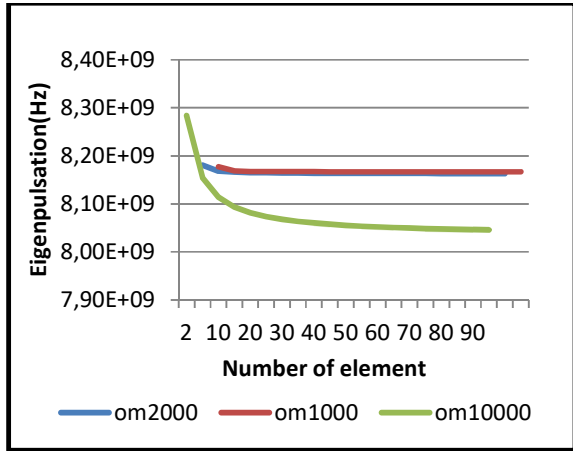


Fig. 9. Comparison between eigenpulsation  $p_2$ (Hz) for each case

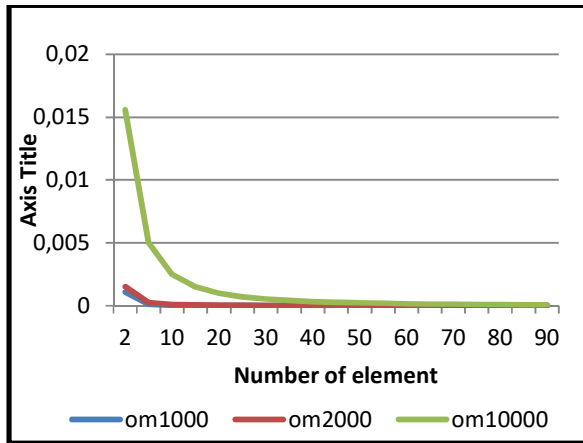


Fig. 10. Comparison between  $\epsilon$  of eigenpulsation  $p_2$ (Hz) for each case

### 3.2 Shape function of fifth degree

Shape functions have been chosen fifth degree polynomials. The same calculus as in 3.1 in mase. We present the results in Table 4, 5 and 6.

30	2.07E+08	4.98E-05	8.168E+09	9.55E-06
35	2.07E+08	3.56E-05	8.168E+09	6.84E-06
40	2.07E+08	2.66E-05	8.167E+09	5.14E-06
45	2.07E+08	2.08E-05	8.167E+09	4.00E-06
50	2.07E+08	1.65E-05	8.167E+09	3.21E-06
55	2.07E+08	1.35E-05	8.167E+09	2.63E-06
60	2.07E+08	1.10E-05	8.167E+09	2.20E-06
65	2.07E+08	9.77E-06	8.167E+09	1.85E-06
70	2.07E+08	8.28E-06	8.167E+09	1.59E-06
75	2.07E+08	6.91E-06	8.167E+09	1.36E-06
80	2.07E+08	5.90E-06	8.167E+09	1.20E-06
85	2.07E+08	6.18E-06	8.167E+09	1.08E-06
90	2.07E+08	4.88E-06	8.167E+09	9.25E-07

Table 5. The first two eigenpulsation for the angular speed  $\omega=2000$ (1/s)

Nr. of el.	Eigenpulsation $p_1$ Hz	$\epsilon$ --	Eigenpulsation $p_2$ Hz	$\epsilon$ --
5	2.071E+08	8.70E-03	8.182E+09	1.10E-03
10	2.053E+08	3.03E-03	8.173E+09	5.35E-04
15	2.046E+08	1.01E-03	8.169E+09	1.87E-04
20	2.044E+08	5.06E-04	8.167E+09	9.46E-05
25	2.043E+08	3.03E-04	8.166E+09	5.71E-05
30	2.043E+08	2.02E-04	8.166E+09	3.82E-05
35	2.042E+08	1.44E-04	8.166E+09	2.74E-05
40	2.042E+08	1.08E-04	8.165E+09	2.06E-05
45	2.042E+08	8.41E-05	8.165E+09	1.60E-05
50	2.042E+08	6.73E-05	8.165E+09	1.28E-05
55	2.041E+08	5.52E-05	8.165E+09	1.05E-05
60	2.041E+08	4.55E-05	8.165E+09	8.76E-06
65	2.041E+08	3.84E-05	8.165E+09	7.41E-06
70	2.041E+08	3.34E-05	8.165E+09	6.36E-06
75	2.041E+08	2.93E-05	8.165E+09	5.53E-06
80	2.041E+08	2.45E-05	8.165E+09	4.84E-06
85	2.041E+08	2.21E-05	8.165E+09	4.25E-06
90	2.041E+08	2.03E-05	8.165E+09	3.79E-06

Table 4. The first two eigenpulsation for the angular speed  $\omega=1000$ (1/s)

Nr. of el.	Eigenpulsation $p_1$ Hz	$\epsilon$ --	Eigenpulsation $p_2$ Hz	$\epsilon$ --
5	2.08E+08	2.17E-03	8.172E+09	2.86E-04
10	2.07E+08	7.52E-04	8.169E+09	1.34E-04
15	2.07E+08	2.50E-04	8.168E+09	4.67E-05
20	2.07E+08	1.25E-04	8.168E+09	2.36E-05
25	2.07E+08	7.48E-05	8.168E+09	1.43E-05

Table 6. The first two eigenpulsation for the angular speed  $\omega=10000$ (1/s)

Nr. of el.	Eigenpulsation $p_1$ Hz	$\epsilon$ --	Eigenpulsation $p_2$ Hz	$\epsilon$ --
5	1.82E+08	2.38E-01	8.52E+09	2.63E-02
10	1.39E+08	1.10E-01	8.29E+09	1.32E-02
15	1.23E+08	4.14E-02	8.18E+09	4.65E-03
20	1.18E+08	2.16E-02	8.15E+09	2.37E-03

25	1.16E+08	1.33E-02	8.13E+09	1.43E-03
30	1.14E+08	8.97E-03	8.12E+09	9.61E-04
35	1.13E+08	6.47E-03	8.11E+09	6.89E-04
40	1.12E+08	4.88E-03	8.10E+09	5.18E-04
45	1.12E+08	3.82E-03	8.10E+09	4.04E-04
50	1.11E+08	3.07E-03	8.09E+09	3.23E-04
55	1.11E+08	2.52E-03	8.09E+09	2.65E-04
60	1.11E+08	2.10E-03	8.09E+09	2.21E-04
65	1.11E+08	1.78E-03	8.09E+09	1.87E-04
70	1.10E+08	1.53E-03	8.09E+09	1.60E-04
75	1.1E+08	1.33E-03	8.08E+09	1.39E-04
80	1.1E+08	1.17E-03	8.08E+09	1.22E-04
85	1.1E+08	1.03E-03	8.08E+09	1.08E-04
90	1.1E+08	9.15E-04	8.08E+09	9.56E-05

A comparison of two eigenpulsations for the three angular speeds is shown in Figure 11 and 13 and a comparison of the error in the three cases is presented in Figure 12 and 14.

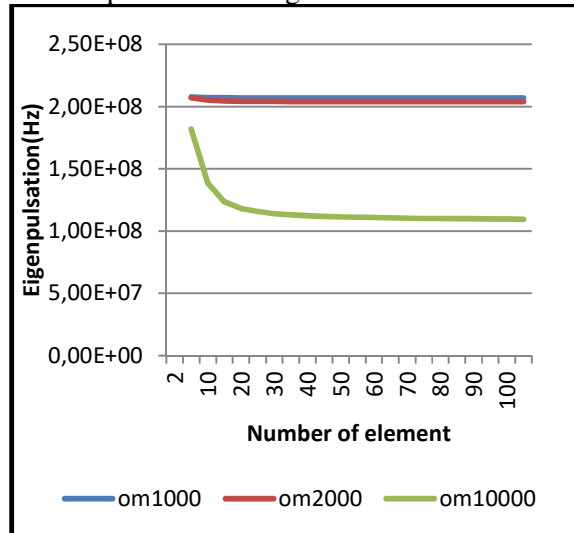


Fig. 11. Comparison between eigenpulsation  $p_1$ (Hz) for each case

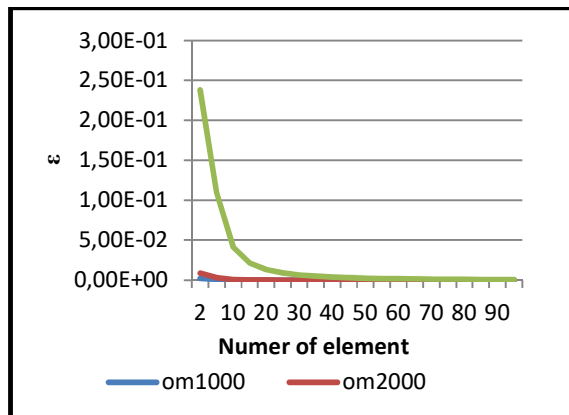


Fig. 12. Comparison between  $\epsilon$  of eigenpulsation  $p_1$ (Hz) for each case

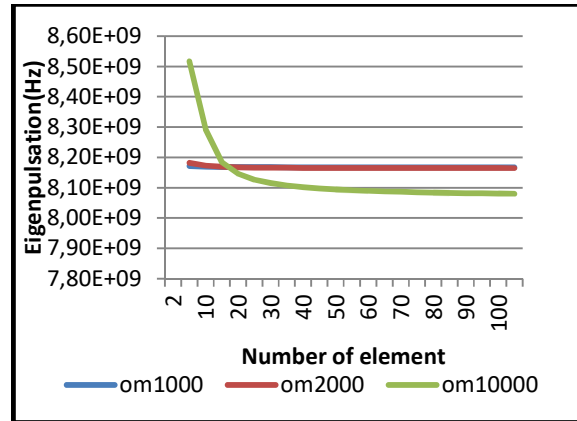


Fig. 13. Comparison between eigenpulsation  $p_2$ (Hz) for each case

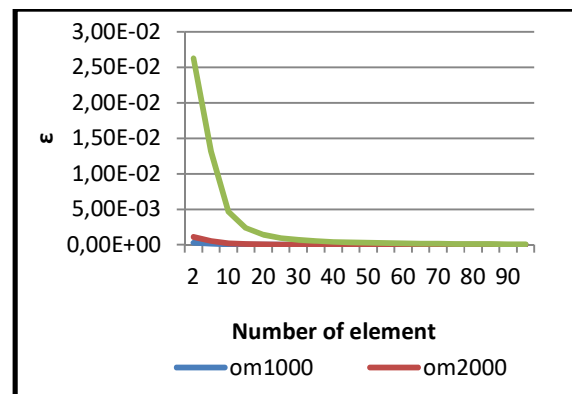


Fig. 14. Comparison between  $\epsilon$  of eigenpulsation  $p_2$ (Hz) for each case

#### 4. CONCLUSIONS

In the paper we analyzed how the number of finite elements chosen for the study of a beam can influence the accuracy of the obtained results and the necessary computational time. For the input data used in the paper it was concluded that a number of several dozen finite elements can provide a satisfactory result in a modal analysis. Increasing the number of finite elements does not lead to a significant increase in the result. If the number of finite elements increases to several hundred, the time required to get the results is a few hours when using the Matlab computing platform, so it is prohibitive.

The conclusion is that an analysis of the discretization, in the case of the dynamic analysis of multicorp systems using the finite element method, is required. A too small number of finite elements used can lead to significant error results, while a large number of finite elements, even if they provide greater precision,

leads to prohibitive computation times. Moreover, this high precision is often not necessary at all.

## 8. REFERENCES

- [1] Fanghella, P., Galletti, C., Torre, G., An explicit independent-coordinate formulation for equations of motion of flexible multibody systems. *Mech. Mach. Theory* 38, 417–437, 2003.
- [2] De Falco, D., Pennestri, E., Vita, L., An investigation of the influence of pseudoinverse matrix calculations on multibody dynamics by means of the Udwadia–Kalaba formulation. *J. Aerosp. Eng.* 22(4), 365–372, 2009.
- [3] Gerstmayr, J., Schberl, J., A 3D finite element method for flexible multibody systems. *Multibody Syst. Dyn.* 15(4), 305–320, 2006.
- [4] Hassan, M., Marin, M., Ellahi, R., Alamri, SZ, Exploration of convective heat transfer and flow characteristics synthesis by Cu–Ag/water hybrid-nanofluids, *Heat Transfer Research*, 49 (18), 1837-1848 (2018)
- [5] Marin, M., Craciun, EM, Uniqueness results for a boundary value problem in dipolar thermoelasticity to model composite materials, *Composites Part B: Engineering* 126, 27-37(2017),
- [6] Mayo, J., Dominguez, J., Geometrically nonlinear formulation of flexible multibody systems in terms of beam elements: geometric stiffness. *Comput. Struct.* 59(6), 1039–1050, 1996.
- [7] Negrean I., New Formulations in Analytical Dynamics of Systems, *Acta Technica Napocensis, Series: Applied Mathematics, Mechanics and Engineering*, Vol. 60, Issue I, pp. 49-56, 2017.
- [8] Negrean I., Mass Distribution in Analytical Dynamics of Systems, *Acta Technica Napocensis, Series: Applied Mathematics, Mechanics and Engineering*, Vol. 60, Issue II, pp. 175-184, 2017.
- [9] Negrean I., Generalized Forces in Analytical Dynamics of Systems, *Acta Technica Napocensis, Series: Applied Mathematics, Mechanics and Engineering*, Vol. 60, Issue III, pp. 357-368, 2017.
- [10] Negrean I., Advanced Notions in Analytical Dynamics of Systems, *Acta Technica Napocensis, Series: Applied Mathematics, Mechanics and Engineering*, Vol. 60, Issue IV, pp. 491-502, 2017.
- [11] Negrean I., Advanced Equations in Analytical Dynamics of Systems, *Acta Technica Napocensis, Series: Applied Mathematics, Mechanics and Engineering*, Vol. 60, Issue IV, pp. 503-514, 2017.
- [12] Neto, M.A., Ambrosio, J.A.C., Leal, R.P., Composite materials in flexible multibody systems. *Comput. Methods Appl. Mech. Eng.* 195(5051), 6860–6873, 2006.
- [13] Piras, G., Cleghorn, W.L., Mills, J.K., Dynamic finite-element analysis of a planar high speed, high-precision parallel manipulator with flexible links. *Mech. Mach. Theory* 40(7), 849–862, 2005.
- [14] Scutaru, ML, Chircan, E., Marin, M., Study of an Elastic Beam, in Centrifugal Field, using Finite Element Method. *Series: Applied Mathematics, Mechanics, and Engineering* Vol. 62, Issue II, June, 2019
- [15] Simeon, B., On Lagrange multipliers in flexible multibody dynamic. *Comput. Methods Appl. Mech. Eng.* 195 (50–51), 6993–7005, 2006.
- [16] Vlase, S., A Method of Eliminating Lagrangian-multipliers from the Equation of Motion of Interconnected Mechanical Systems. *Journal of Applied Mechanics-Transactions of ASME*, 54(1), 235-237,1987.
- [17] Vlase, S., Eigenvalues and Eigenmodes of an Inclined Homogeneous Truss in a Rotational Field. *Vol.59, (7-8)*, 699-715, 2014.
- [18] Vlase, S., Teodorescu, P. P., Itu, C. et al., Elasto-Dynamics of a Solid with a General "Rigid" Motion using FEM Model Part II. Analysis of a Double Cardan Joint. *Romanian Journal of Physics*, Vol. 58, 7-8, pp. 882-892,2013.
- [19] Vlase, S., Marin, M., Öchsner, A. et al. Motion equation for a flexible one-dimensional element used in the dynamical analysis of a multibody system. *Continuum*

- Mech. Thermodyn. 31: 715. <https://doi.org/10.1007/s00161-018-0722-y>, 2019.
- [20] Vlase, S., Dynamical response of a multibody system with flexible element with a general three-dimensional motion. Rom. J. Phys. 57(3–4), 676–693, 2012.
- [21] Vlase, S., Danasel, C., Scutaru, M.L., Mihalcica, M., Finite element analysis of two-dimensional linear elastic systems with a plane rigid motion. Rom. J. Phys. 59(5–6), 476–487, 2014.
- [22] Vlase, S., Itu, C., Vasile, O. et al., Vibration Analysis of a Mechanical System Composed of Two Identical Parts. Romanian Journal of Acoustic and Vibration, Vol. 15, Issue: 1, 58-63, 2018.

### **Influența numărului de elemente finite în determinarea răspunsului modal în analiza sistemelor multicorp cu elemente elastice**

***Rezumat:** Utilizarea metodei elementelor finite pentru analiza dinamică a sistemelor multicorp cu elemente elastic presupune rezolvarea a numeroase probleme, legate de tehnicile de calcul și de particularitățile unui astfel de studiu prin comparație cu tehnicile aplicate la studiul cu elemente finite în cazul classic. Pentru astfel de sisteme apar caracteristici special, determinate de nelinearitatea coeficienților matriceali care apar în sistemul de ecuații diferențiale care descriu răspunsul sistemului mecanic și de apariția unor termeni suplimentari, caracteristi în cadrul acestor ecuații. În lucrare se face o comparație între calculul cu elemente finite considerând funcții de interpolare de gradul trei și funcțiile de interpolare de gradul 5. Este analizat și modul în care numărul de elemente finite considerat pentru discretizarea unei bare poate influența rezultatele obținute.*

**CHIRCAN Eliza**, Ph.D. student, Transylvania University of Brașov, Department of Mechanical Engineering, eliza.chircan@unitbv.ro, Office phone: +40-268-418992, 29, B-dul Eroilor, 500036, Brașov.

**SCUTARU Maria Luminița**, Professor, Dr. hab., Transylvania University of Brașov, Department of Mechanical Engineering, lscutaru@unitbv.ro, Office phone: +40-268-418992, 29, B-dul Eroilor, 500036, Brașov, home phone: +40-723-242735.

**SIMIONESCU Cristian Silviu**, Prof.Dr.Eng., "Dunărea de Jos" University in Galați, the Engineering Faculty in Braila, the Engineering Sciences and Management Department, cristiansimionescu@ugal.ro, Mobile: + 40-745352090, no.29 Calea Călărașilor Street, 810008 Brăila (Romania), Home phone: +40-239673921.

**VLASE Sorin**, Prof. Dr.hab., Head of Department, Transylvania University of Brașov, Department of Mechanical Engineering, svlase@unitbv.ro, Office phone: +40-268-418992, 30, Castelului, 500014, Brașov, home phone: +40-722-643020.