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ABOUT THE STUDY OF FRAMES BY TRANSFER-MATRIX METHOD (TMM) - SIMILARITY OF DENTAL BRIDGES WITH FRAMES

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Abstract: *Similarity with orthodontic problems, makes the study of frames a very important problem, with practical applications in life domains, in the field of health, especially in dentistry. This work presents an analytical calculus for a frame, using the Transfer-Matrix Method (TMM). The frame has two vertical poles, embedded at inferior ends and with a vertical concentrated load, in the middle of horizontal part of frame. The two pole teeth can be assimilated with the two vertical parts of frame and the aggregation elements, together with the bridge body, can be assimilated with the horizontal part of frame. We can determine the displacements aggregation elements. The results will allow, in practice construction of dental bridges to ensure the best resistance. We can solve this problem very easy on an appropriate soft.*

Key words: *dental bridge, frame, aggregation elements, bridge body, pole teeth, Transfer-Matrix Method (TMM).*

1. INTRODUCTION

The frames are meandering beams that can be studied by different methods, including the Transfer-Matrix Method (TMM).

Similarity with orthodontic problems, makes the study of frames a very important problem, with practical applications in life domains, in the field of health, especially in dentistry. Classical frames calculus is presented in [5].

In [1], [2] and [3] are presented studies about some biomechanics applications on femoral bone by different methods.

The importance for the health field of dental bridges in England and Wales is presented in [4]. [9] gives us the basis of frames calculus by TMM.

Another calculus about exponential matrix in robot geometrical modeling is presented in [10]. [6] and [7] present studies about Co-Cr dental bridges using different methods. In [8] is presented studies about integrated construction and simulation for crowns and bridges. Studies about design of dental bridges are presented in [11]. Study of bending beams on elastic environment by TMM is presented in [12].

2. THE DENTAL BRIDGE AS A FRAME

As a fixed prosthesis, the dental bridge is very often used in orthodontics, for the morphological restoration of the missing teeth-partial edentation, or for the protection and remodeling teeth that have suffered major damages. Dental bridges have the role of compensating for mastcatory, aesthetic and phonetic functions of absent teeth and to prevent complications that may occur as a result of tooth loss. A dental bridge consists of two components, meaning aggregation elements-at least two, the bridge body-which replaces missing teeth and the pole teeth. Aggregation elements represent dental crowns which the bridge rests on the pole teeth (Fig. 1.).

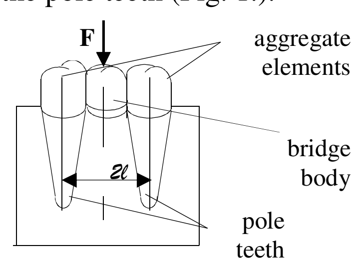

Fig. 1. A dental bridge

Dental bridges can by in medial extension-an extension to the front of the mouth (anterior part) or in distal extension-an extension to the back of the mouth cavity (posterior part).

The pole teeth can be assimilated with the two vertical parts of frame and the aggregation elements, together with the bridge body, can be assimilated with the horizontal part of frame.

The dental bridge is required by an uniformly distributed theoretical force. The most unfavorable case is given by a concentrated vertical force acting in the middle of horizontal opening of frame, equivalent to the uniformly distributed load (Fig. 2.).

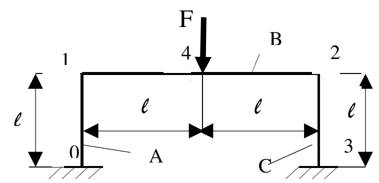

Fig. 2. A dental bridge as a frame

For this work, we consider the two vertical poles of frame embedded at inferior ends.

3. FRAME CALCULUS WITH TMM

Analytical calculus for frame with two vertical poles embedded at inferior ends and with a vertical concentrated load in the middle of horizontal part of frame is based of theory Dirac's and Heaviside's functions and operators [9].

We can consider vertical poles of length l and horizontal part of length $2l$, trying a similarity with a dental bridge with only one missing tooth (Fig. 1. and Fig. 2.). The charge density for a concentrated vertical load at the middle is:

$$q(x) = -F \cdot \delta(x-l) \qquad (1)$$

We note the vertical poles with A and C and the horizontal part with B. The embedded supports are noted with 0 and 3, the left section of the horizontal part of the frame is marked with 1 and the right section is marked with 2, (Fig. 2.). We note with: E - modulus of longitudinal elasticity (Young modulus), A - the area of frame transversal section, I - the moment of inertia. We must calculate the displacements of two nodes, 1 and 2, for the frame. After the general approach presented in [9], we have, for each of the three parts of the frame, the following matrix:

$$[T_{00}^{A}]=\begin{bmatrix} \frac{12EI}{l^3} & 0 & \frac{6EI}{l^2} \\ 0 & -\frac{EA}{l} & 0 \\ \frac{6EI}{l^2} & 0 & -\frac{4EA}{l} \end{bmatrix} \qquad (2)$$

$$[T_{01}^{A}]=\begin{bmatrix} \frac{12EI}{l^3} & 0 & \frac{6EI}{l^2} \\ 0 & \frac{EA}{l} & 0 \\ \frac{6EI}{l^2} & 0 & -\frac{2EA}{l} \end{bmatrix} \qquad (3)$$

$$[T_{10}^{A}]=\begin{bmatrix} \frac{12EI}{l^3} & 0 & -\frac{6EI}{l^2} \\ 0 & \frac{EA}{l} & 0 \\ \frac{6EI}{l^2} & 0 & -\frac{2EI}{l} \end{bmatrix} \qquad (4)$$

$$[T_{11}^{A}]=\begin{bmatrix} -\frac{12EI}{l^3} & 0 & -\frac{6EI}{l^2} \\ 0 & -\frac{EA}{l} & 0 \\ -\frac{6EI}{l^2} & 0 & -\frac{4EI}{l} \end{bmatrix} \qquad (5)$$

$$[T_{00}^{B}]=\begin{bmatrix} -\frac{EA}{2l} & 0 & 0 \\ 0 & -\frac{3EI}{2l^3} & -\frac{3EI}{2l^2} \\ 0 & -\frac{3EI}{2l^2} & -\frac{2EI}{l} \end{bmatrix} \qquad (6)$$

$$[T_{01}^{B}]=\begin{bmatrix} -\frac{EA}{2l} & 0 & 0 \\ 0 & \frac{3EI}{2l^3} & -\frac{3EI}{2l^2} \\ 0 & \frac{3EI}{2l^2} & -\frac{EI}{l} \end{bmatrix} \qquad (7)$$

$$[T_{10}^{B}]=\begin{bmatrix} \frac{EA}{2l} & 0 & 0 \\ 0 & \frac{3EI}{2l^3} & \frac{3EI}{2l^2} \\ 0 & -\frac{3EI}{2l^2} & -\frac{EI}{l} \end{bmatrix} \qquad (8)$$

$$[T_{11}^{B}]=\begin{bmatrix} -\frac{EA}{2l} & 0 & 0 \\ 0 & -\frac{3EI}{2l^3} & \frac{3EI}{2l^2} \\ 0 & \frac{3EI}{2l^2} & -\frac{2EI}{l} \end{bmatrix} \qquad (9)$$

and vectors:

$$\{U_{02}^{A}\}=\begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad (10); \quad \{U_{12}^{A}\}=\begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad (11); \quad \{U_{02}^{B}\}=\begin{Bmatrix} 0 \\ -\frac{F}{4} \\ -\frac{Fl}{24} \end{Bmatrix} \quad (12); \quad \{U_{12}^{B}\}=\begin{Bmatrix} 0 \\ -\frac{F}{4} \\ \frac{Fl}{24} \end{Bmatrix} \quad (13)$$

with following observations:

$$\begin{cases} [T_{00}^{A}]=[T_{00}^{C}] \\ [T_{01}^{A}]=[T_{01}^{C}] \\ [T_{10}^{A}]=[T_{10}^{C}] \\ [T_{11}^{A}]=[T_{11}^{C}] \end{cases} \quad (14) \quad \text{and} \quad \begin{cases} \{U_{02}^{A}\}=\{U_{02}^{C}\} \\ \{U_{12}^{A}\}=\{U_{12}^{C}\} \end{cases} \quad (15)$$

Because supports 0 and 3 are embedded, the displacements within them are known, with following displacement vectors:

$$\{D_0\}=\begin{Bmatrix} x_0 \\ y_0 \\ \omega_0 \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}=\{D_3\}=\begin{Bmatrix} x_3 \\ y_3 \\ \omega_3 \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (16)$$

We must write the balance of nodes 2 and 3 with help of two condensed matrix equations:

$$\begin{cases} [T_{11}^{A}]\cdot\{D_1\}+\{U_{12}^{B}\}+[T_{00}^{B}]\cdot\{D_1\}+[T_{01}^{B}]\cdot\{D_2\}+\{U_{02}^{B}\}=0 \\ [T_{10}^{A}]\cdot\{D_1\}+\{U_{12}^{B}\}+[T_{11}^{B}]\cdot\{D_2\}+[T_{11}^{C}]\cdot\{D_2\}=0 \end{cases} \qquad (17)$$

with:

$$\{D_1\}=\begin{Bmatrix} x_1 \\ y_1 \\ \omega_1 \end{Bmatrix} \quad (18) \qquad \text{and} \qquad \{D_2\}=\begin{Bmatrix} x_2 \\ y_2 \\ \omega_2 \end{Bmatrix} \quad (19)$$

The developed matrix equations are:

$$\begin{cases} \begin{bmatrix} -\frac{12EI}{l^3} & 0 & -\frac{6EI}{l^2} \\ 0 & -\frac{EA}{l} & 0 \\ -\frac{6EI}{l^2} & 0 & -\frac{4EI}{l} \end{bmatrix} \begin{Bmatrix} x_1 \\ y_1 \\ \omega_1 \end{Bmatrix} + \begin{Bmatrix} 0 \\ -\frac{F}{4} \\ -\frac{Fl}{24} \end{Bmatrix} + \begin{bmatrix} -\frac{EA}{2l} & 0 & 0 \\ 0 & -\frac{3EI}{2l^3} & -\frac{3EI}{2l^2} \\ 0 & -\frac{3EI}{2l^2} & -\frac{2EI}{l} \end{bmatrix} \begin{Bmatrix} x_1 \\ y_1 \\ \omega_1 \end{Bmatrix} + \\ + \begin{bmatrix} \frac{EA}{2l} & 0 & 0 \\ 0 & \frac{3EI}{2l^3} & -\frac{3EI}{2l^2} \\ 0 & \frac{3EI}{2l^2} & -\frac{EI}{l} \end{bmatrix} \begin{Bmatrix} x_2 \\ y_2 \\ \omega_2 \end{Bmatrix} + \begin{Bmatrix} 0 \\ -\frac{F}{4} \\ -\frac{Fl}{24} \end{Bmatrix} = 0 \\ \begin{bmatrix} -\frac{EA}{2l} & 0 & 0 \\ 0 & \frac{3EI}{2l^3} & -\frac{3EI}{2l^2} \\ 0 & \frac{3EI}{2l^2} & -\frac{EI}{l} \end{bmatrix} \begin{Bmatrix} x_1 \\ y_1 \\ \omega_1 \end{Bmatrix} + \begin{Bmatrix} 0 \\ -\frac{F}{4} \\ -\frac{Fl}{24} \end{Bmatrix} + \begin{bmatrix} \frac{EA}{2l} & 0 & 0 \\ 0 & -\frac{3EI}{2l^3} & \frac{3EI}{2l^2} \\ 0 & \frac{3EI}{2l^2} & -\frac{2EI}{l} \end{bmatrix} \begin{Bmatrix} x_2 \\ y_2 \\ \omega_2 \end{Bmatrix} + \\ + \begin{bmatrix} -\frac{12EI}{l^3} & 0 & -\frac{6EI}{l^2} \\ 0 & -\frac{EA}{l} & 0 \\ -\frac{6EI}{l^2} & 0 & -\frac{4EI}{l} \end{bmatrix} \begin{Bmatrix} x_2 \\ y_2 \\ \omega_2 \end{Bmatrix} = 0 \end{cases} \quad (20)$$

or:

$$\begin{cases} \begin{bmatrix} -\frac{12EI}{l^3}-\frac{EA}{2l} & 0 & -\frac{6EI}{l^2} \\ 0 & -\frac{EA}{l}-\frac{3EI}{2l^3} & -\frac{3EI}{2l^2} \\ -\frac{6EI}{l^2} & -\frac{3EI}{2l^2} & -\frac{6EI}{l} \end{bmatrix} \begin{Bmatrix} x_1 \\ y_1 \\ \omega_1 \end{Bmatrix} + \begin{bmatrix} -\frac{EA}{2l} & 0 & 0 \\ 0 & -\frac{3EI}{2l^3} & -\frac{3EI}{2l^2} \\ 0 & -\frac{3EI}{2l^2} & -\frac{2EI}{l} \end{bmatrix} \begin{Bmatrix} x_2 \\ y_2 \\ \omega_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ \frac{F}{2} \\ \frac{Fl}{12} \end{Bmatrix} \\ \begin{bmatrix} -\frac{EA}{2l} & 0 & 0 \\ 0 & \frac{3EI}{2l^3} & -\frac{3EI}{2l^2} \\ 0 & \frac{3EI}{2l^2} & -\frac{EI}{l} \end{bmatrix} \begin{Bmatrix} x_1 \\ y_1 \\ \omega_1 \end{Bmatrix} + \begin{bmatrix} -\frac{12EI}{l^3}-\frac{EA}{2l} & 0 & -\frac{6EI}{l^2} \\ 0 & -\frac{3EI}{2l^3}-\frac{EA}{l} & \frac{3EI}{2l^2} \\ -\frac{6EI}{l^2} & \frac{3EI}{2l^2} & -\frac{6EI}{l} \end{bmatrix} \begin{Bmatrix} x_2 \\ y_2 \\ \omega_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ \frac{F}{4} \\ \frac{Fl}{24} \end{Bmatrix} \end{cases} \quad (21)$$

which gives us a system of six equations with six unknowns, the unknowns being the six displacements, three in each nodes, 1 and 2, (22):

$$\begin{cases} -\left(\frac{12EI}{l^3}+\frac{EA}{2l}\right)x_1 - \frac{6EI}{l^2}\omega_1 - \frac{EA}{2l}x_2 = 0 \\ -\left(\frac{3EI}{2l^3}+\frac{EA}{l}\right)y_1 - \frac{3EI}{2l^2}\omega_1 + \frac{3EI}{2l^3}y_2 - \frac{3EI}{2l^2}\omega_2 = \frac{F}{2} \\ -\frac{6EI}{l^2}x_1 - \frac{3EI}{2l^2}y_1 - \frac{6EI}{l}\omega_1 + \frac{3EI}{2l^2}y_2 - \frac{EI}{l}\omega_2 = \frac{Fl}{12} \\ -\frac{EA}{2l}x_1 - \left(\frac{12EI}{l^3}+\frac{EA}{2l}\right)x_2 - \frac{6EI}{l^2}\omega_2 = 0 \\ \frac{3EI}{2l^3}y_1 - \frac{3EI}{2l^2}\omega_1 - \left(\frac{3EI}{2l^3}+\frac{EA}{l}\right)y_2 + \frac{3EI}{2l^2}\omega_2 = \frac{F}{4} \\ \frac{3EI}{2l^2}y_1 - \frac{EI}{l}\omega_1 - \frac{6EI}{l^2}x_2 + \frac{3EI}{2l^2}y_2 - \frac{6EI}{l}\omega_2 = \frac{Fl}{24} \end{cases} \quad (22)$$

This is a linear system, that has following form, (23):

$$\begin{cases} a_{11}x_1 + a_{12}y_1 + a_{13}\omega_1 + a_{14}x_2 + a_{15}y_2 + a_{16}\omega_2 = b_1 \\ a_{21}x_1 + a_{22}y_1 + a_{23}\omega_1 + a_{24}x_2 + a_{25}y_2 + a_{26}\omega_2 = b_2 \\ a_{31}x_1 + a_{32}y_1 + a_{33}\omega_1 + a_{34}x_2 + a_{35}y_2 + a_{36}\omega_2 = b_3 \\ a_{41}x_1 + a_{42}y_1 + a_{43}\omega_1 + a_{44}x_2 + a_{45}y_2 + a_{46}\omega_2 = b_4 \\ a_{51}x_1 + a_{52}y_1 + a_{53}\omega_1 + a_{54}x_2 + a_{55}y_2 + a_{56}\omega_2 = b_5 \\ a_{61}x_1 + a_{62}y_1 + a_{63}\omega_1 + a_{64}x_2 + a_{65}y_2 + a_{66}\omega_2 = b_6 \end{cases} \quad (23)$$

with unknown's coefficients determinant (24):

$$\begin{bmatrix} a_{11} & 0 & a_{13} & a_{14} & 0 & 0 \\ 0 & a_{22} & a_{23} & 0 & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & 0 & a_{35} & a_{36} \\ a_{41} & 0 & 0 & a_{44} & 0 & a_{46} \\ 0 & a_{52} & a_{53} & 0 & a_{55} & a_{56} \\ 0 & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix} \quad (24)$$

and with the free terms vector (25):

$$\begin{Bmatrix} 0 \\ b_2 \\ b_3 \\ 0 \\ b_5 \\ b_6 \end{Bmatrix} \quad (25)$$

The system solutions determine the displacement vectors given by the relations (18) and (19). The results will allow, in practice, the construction of dental bridges to ensure the best resistance.

4. CONCLUSION

This work exposed an analytical calculus for frame with two vertical poles embedded at inferior ends and with a vertical concentrated load in the middle of horizontal part of frame, using the Transfer-Matrix Method.

This approach can be applied for different frames, in occurrence for dental bridges with medial extension or in distal extension, with a larger body, that is, with more than one missing tooth. Original contribution is the similarity between the dental bridge and the frame, the frame being studied with TMM. Algorithm can by program, we hope that will be presented in future works and validate theoretical results with other numerical methods and by experimental tests.

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Asupra studiului cadrelor prin Metoda Matricelor de Transfer (MMT) – similitudinea între punţile dentare şi cadre

Rezumat: Similitudinea cu problemele ortodontice face ca studiul cadrelor să fie o problemă foarte importantă, cu aplicații practice în domeniile vieții, în domeniul sănătății, în special în stomatologie. Acest articol prezintă un calcul analitic pentru un cadru, utilizând Metoda Matricelor de Transfer (MMT). Cadrul are doi stâlpi verticali, încastraţi la extremitățile inferioare și solicitaţi cu o forţă verticală concentrată, în mijlocul părții orizontale a cadrului. Cei doi dinți stâlpi pot fi asimilați cu cele două porţiuni verticale ale cadrului, iar elementele de agregare, împreună cu corpul podului dentar, pot fi asimilate cu partea orizontală a cadrului. Se pot determina deplasările elementelor de agregare. Rezultatele vor permite, în practică, construcția unor punți dentare ce vor asigura cea mai bună rezistență. Putem rezolva această problemă foarte ușor pe un soft adecvat.

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