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CONTRIBUTIONS ON THE ANALYTICAL CALCULUS OF SIMPLE DENTAL BRIDGE ASSIMILATED WITH A BEAM EMBEDDED AT BOTH ENDS BY TRANSFER-MATRIX METHOD (TMM)

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Abstract: *Through this article, we propose a special approach to orthodontic problems, by assimilating the dental bridges with beams studied by Transfer-Matrix Method (TMM), an original approach. For practical engineering problems and for bio-engineering problems, in occurrence, in dentistry, in orthodontics, beams calculus is very interesting and easy to study by (TMM). We hope to present that in future research, together with experimental validation of results.*
Key words: *simple dental bridge, aggregation elements, bridge body, pole teeth, embedded beam, Transfer-Matrix Method (TMM).*

1. INTRODUCTION

Using the Transfer-Matrix Method (TMM) for bio-engineering problems, in occurrence, in dentistry, in orthodontics, is an original approach. TMM is used for practical engineering problems too, that is very easy to program. Analytical beams calculus is very interesting to study by TMM too. This work presents a calculus for a simple dental bridge assimilated with a beam, embedded at both ends (Fig. 2.), by TMM. A simple dental bridge consists of two pole teeth on the extremities and a tooth in the middle, which replaces the missing tooth.

2. ABOUT THE DENTAL BRIDGE

The dental bridge is a fixed prosthesis, used in dentistry, to compensate partial edentations (missing teeth). Lack of teeth lead to serious chewing problems, morphological, aesthetic and phonetic problems too. Edentations produce important changes in oral cavity, which can seriously disrupt the functions of the dental apparatus. The components of a dental bridge are as follows: at least two aggregation elements, pole teeth–at least two too, on which the aggregation elements are placed and, the bridge body, which replaces missing teeth (Fig. 1. and Fig. 2.).

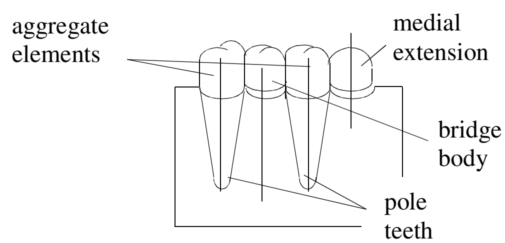

Fig. 1. A dental bridge with medial extension

An extension to anterior part of mouth (an extension to front of mouth) leads to construction of a dental bridge in medial extension (Fig. 1.). If there is an extension to posterior part of oral cavity, a distal extension dental bridge will be required.

3. SOME PREVIOUS STUDIES ABOUT DENTAL BRIDGES

Practical applications in orthodontics make the similarity between dental bridges and beams very important, proof are works in this field so far. In [4] is presented the importance of dental bridges for health field in England and Wales. Studies about Co-Cr dental bridges using different methods are presented in [6] and [7]. [8] presents studies about integrated construction and simulation for crowns and bridges. [11] presents studies about design of dental bridges. Fatigue of zirconia for dental bridges is presented in [10] and [12]. [15] gives us effects of small-grit grinding and glazing on mechanical behaviors and ageing resistance of dental zirconia. In another field of health, in orthopedics, are presented studies about some biomechanics applications on femoral bone by different methods, for example in [1], [2] and [3]. [5] gives us the classical beams calculus. The bases for calculus by Transfer-Matrix Method is presented in [9]. [11] gives us studies about exponential matrix calculus. Bending beams on elastic environment studies by TMM are presented in [13]. [14] presents a buckling calculus of straight bars on elastic environment by TMM for dental implants.

4. SIMILARITY BETWEEN A SIMPLE DENTAL BRIDGE AND A BEAM EMBEDDED ON BOTH ENDS

Aggregation elements and body of dental bridge are made together and can thus be considered to be similar to a beam (Fig. 2. and Fig. 3.).

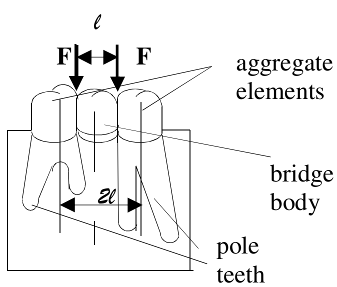

Fig. 2. Dental bridge with a single body

Also, the aggregation elements are stiffened on the pole teeth, allowing assimilation of the stiffeners with embedded supports.The dental bridge must be constructed so that at maximum loads, the bridge body will not touch the gum. We consider a dental bridge consisting of two aggregation elements and a single bridge body between them (meaning a missing tooth), (Fig. 2.). Dental bridge as a beam is requested by uniformly distributed theoretical load. We consider the most unfavorable requests, that is given by two equal concentrated vertical forces, equivalent to the uniformly distributed load, which act as follows: one in section between left aggregation element and dental bridge body, and the other, in section between dental bridge body and right aggregation element (Fig. 2. and Fig. 3.). We can study the calculus of beam, embedded at both ends, and loaded with two symmetrically concentrated forces arranged against the middle of the beam, which acts as in Figure 3.

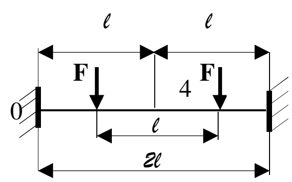

Fig. 3. Dental bridge as a beam

This study can be done with functions and operators of Dirac and Heaviside, using the Transfer-Matrix Method (TMM), after [9].

5. CALCULUS OF EMBEDDED BEAM AT BOTH ENDS BY TMM

Analytical calculus for beams presented in Fig. 3., by TMM is based on theory of Dirac's and Heaviside's functions and operators, [9]. It is considered that only the bending moment and the cutting force act, the action of an external axial force is not taken into account. Density of charge for the two concentrated vertical loads, acting in the sections *2* and *3*, is:

$$q(x) = -F\delta\left(x - \frac{l}{2}\right) - F\delta\left(x - \frac{3l}{2}\right) \qquad (1)$$

Embedded supports are noted with *0* and *1* and the middle of the beam with *4*, (Fig. 3.). For a section *x*, it is associated a state vector $\{U\}_x$, with four elements: $\{U\}_x=\{M(x),T(x),\omega(x),v(x)\}^{-1}$. At the origin, for the section *0*, we have the state vector: $\{U\}_0=\{M_0,T_0,\omega_0,v_0\}^{-1}$. We can write a matrix relation between the state vector of section *x* and the state vector for origin section *0*, $\{U\}_x$:

$$\{U\}_x= [T]_x\{U\}_0 + \{U_e\}_x \qquad (2)$$

with following notations: $[T]_x$ is the Transfer-Matrix between the section *0* and the section *x* and $\{U_e\}_x$ is the vector for the free term at the section *x,* which depends on external loads, *E* is modulus of longitudinal elasticity (Young modulus), A is area for transversal section of beam, I is moment of inertia. After the general approach presented in [9], we can write the developed relationship (2) as (3):

$$\begin{Bmatrix} M(x) \\ T(x) \\ \omega(x) \\ v(x) \end{Bmatrix} = \begin{bmatrix} 1 & x & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{x}{EI} & \frac{x^2}{2EI} & 1 & 0 \\ \frac{x^2}{2EI} & -\frac{x^3}{6EI} & x & 1 \end{bmatrix} \begin{Bmatrix} M_0 \\ T_0 \\ \omega_0 \\ v_0 \end{Bmatrix} + \begin{Bmatrix} -F\left(x-\frac{l}{2}\right)-F\left(x-\frac{3l}{2}\right) \\ -F-F \\ -\frac{F}{2EI}\left(x-\frac{l}{2}\right)^2-\frac{F}{2EI}\left(x-\frac{3l}{2}\right)^2 \\ -\frac{F}{6EI}\left(x-\frac{l}{2}\right)^3-\frac{F}{6EI}\left(x-\frac{3l}{2}\right)^3 \end{Bmatrix} \tag{3}$$

For the section at the right end of the beam we can write relation (3), referring to $x=2l$ and we obtain (4):

$$\begin{Bmatrix} M(2l) \\ T(2l) \\ \omega(2l) \\ v(2l) \end{Bmatrix} = \begin{bmatrix} 1 & 2l & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{2l}{EI} & -\frac{2l^2}{EI} & 1 & 0 \\ \frac{2l^2}{EI} & -\frac{4l^3}{3EI} & 2l & 1 \end{bmatrix} \begin{Bmatrix} M_0 \\ T_0 \\ \omega_0 \\ v_0 \end{Bmatrix} + \begin{Bmatrix} -F\left(2l-\frac{l}{2}\right)-F\left(2l-\frac{3l}{2}\right) \\ -2F \\ -\frac{F}{2EI}\left(2l-\frac{l}{2}\right)^2-\frac{F}{2EI}\left(2l-\frac{3l}{2}\right)^2 \\ -\frac{F}{6EI}\left(2l-\frac{l}{2}\right)^3-\frac{F}{6EI}\left(2l-\frac{3l}{2}\right)^3 \end{Bmatrix} \tag{4}$$

or:

$$\begin{Bmatrix} M(2l) \\ T(2l) \\ \omega(2l) \\ v(2l) \end{Bmatrix} = \begin{bmatrix} 1 & 2l & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{2l}{EI} & -\frac{2l^2}{EI} & 1 & 0 \\ \frac{2l^2}{EI} & -\frac{4l^3}{3EI} & 2l & 1 \end{bmatrix} \begin{Bmatrix} M_0 \\ T_0 \\ \omega_0 \\ v_0 \end{Bmatrix} + \begin{Bmatrix} -2Fl \\ -2F \\ -\frac{5Fl^2}{4EI} \\ -\frac{7Fl^3}{12EI} \end{Bmatrix} \tag{5}$$

Now, we can put the conditions on the two ends of the beam, meaning on the two embedded supports:

- for $x=0$: $\begin{cases} \omega_0 = 0 \\ v_0 = 0 \end{cases}$ (6)
- for $x=2l$: $\begin{cases} \omega(2l) = 0 \\ v(2l) = 0 \end{cases}$ (7)

(6) and (7) are replaced in relation (5) and we obtain:

$$\begin{Bmatrix} M(2l) \\ T(2l) \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} 1 & 2l & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{2l}{EI} & -\frac{2l^2}{EI} & 1 & 0 \\ \frac{2l^2}{EI} & -\frac{4l^3}{3EI} & 2l & 1 \end{bmatrix} \begin{Bmatrix} M_0 \\ T_0 \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} -2Fl \\ -2F \\ -\frac{5Fl^2}{4EI} \\ -\frac{7Fl^3}{12EI} \end{Bmatrix} \tag{8}$$

So, we have now a linear system of four equations with four unknowns of form:

$$\begin{cases} M(2l) = M_0 + 2lT_0 - 2Fl \\ T(2l) = T_0 - 2F \\ 0 = \frac{2l}{EI}M_0 - \frac{2l^2}{EI}T_0 - \frac{5Fl^2}{4EI} \\ 0 = \frac{2l^2}{EI}M_0 - \frac{4l^3}{3EI}T_0 - \frac{7Fl^3}{12EI} \end{cases} \tag{9}$$

with solutions (10):

$$\begin{cases} M_0 = -\frac{3Fl}{8} \\ T_0 = F \\ M(2l) = -\frac{3Fl}{8} \\ T(2l) = -F \end{cases} \tag{10}$$

solutions identical to those obtained from [5], applying twice equation of the three moments of Clapeyron. Now, we can calculate in each section of beam, with (3), the four elements of the state vector corresponding to the section x, so also for $x=l/2$ and $x=3l/2$, at the boundary between pole teeth and the body of dental bridge (i.e. the missing tooth). Therefore, the maximum cutting force is at boundary between aggregation elements and body of dental bridge, and the maximum moment is taken over by the two poles teeth.

6. CONCLUSIONS

Through this article, we propose a special approach to orthodontic problems, by assimilating the dental bridges with beams, studied by TMM, an original approach. TMM is very easy to apply for beams calculus, algorithm can be programmed. We hope to present that in future research, together with experimental validation of results.

7. REFERENCES

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Contribuţii asupra calculului analitic al punţii dentare simple asimilată cu o grindă încastrată la ambele capete prin Metoda Matricelor de Transfer (MMT)

Rezumat: Prin acest articol, se propune o abordare specială a problemelor ortodontice, prin asimilarea punților dentare cu grinzi, studiate prin Metoda Matricelor de Transfer (MMT), o abordare originală. Pentru problemele practice din inginerie și pentru problemele din bio-inginerie, în ocurenţă, în stomatologie, în ortodonţie, calculul grinzilor este foarte interesant și ușor de studiat prin MMT. Sperăm să prezentăm această abordare în cercetările viitoare, împreună cu validarea experimentală a rezultatelor.

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