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CONTRIBUTIONS TO THE APPROXIMATION WITH POLYNOMIAL FUNCTIONS OF MATERIAL SYSTEMS VIBRATIONS. PART II: APPROXIMATION OF REAL VIBRATION ACCELERATIONS

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Abstract: *The paper presents the approximation of the graphic representations of the measured vibrations for a truck platform, through 10th degree polynomial functions. The approximation by interpolating Lagrange within four distinct measuring points is applied, using the Vandermonde matrix at each analyzed point. Approximation leads to truthful results, so it can be considered valid for the application of vibration analysis of material systems. The second part of the work realizes the approximation of the presentation of the vibrational rations of a real system with 10-degree polynomial functions, for four distinct cases of graphic alms.*

Key words: *mechanical vibrations, 10th degree polynomial functions, mathematical approximation.*

1. INTRODUCTION

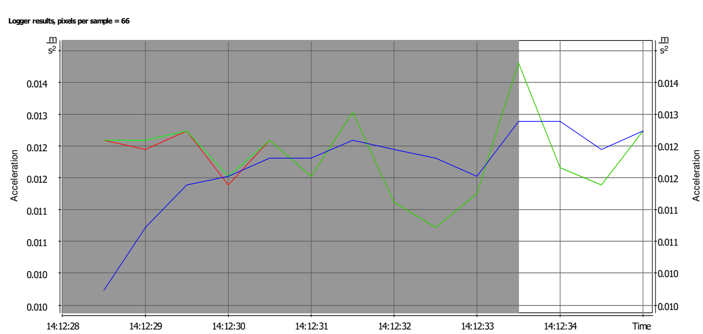
A mechanical system corresponding to a Mercedes-Benz Actros truck is considered. For which were made vibration measurements, that are presented in the first part of this paper, in the Chapter 3 [5].

For this system is formulated all the theory (Chapter 1 of the first part of the paper), that was necessary to approximate the measurement of the real mechanical system with polynomial functions [1], [2], [3].

The current work can only be considered together with the first part of the work, which gives the necessary clarifications on the approximation, as well as on the vibration measurements performed on the actual system of the truck.

From the first part of the work, only the graphic representations of the acceleration are taken into account in the four measuring points, which are presented in succession [4]:

1. Figure 1 for measurement point P1, which shall be called P1 vibrations measured;
2. Figure 2 for paragraph P2, which shall be called P2 vibrations measured;
3. Figure 3 for paragraph P4, which shall be called P4 vibrations measured;
4. figure 4 for paragraph P6, which shall be called P6 vibrations measured.



	Start	Duration	RMS	RMS	Max
Info	-	-	Ch1, P1 (HP1, Lin)	Ch2, P1 (HP1, Lin)	Ch3, P1 (HP1, 1 s)
Main cursor	8/22/2015 14:12:33.500	-	0.014 m/s^2	0.014 m/s^2	0.013 m/s^2
Inside blocks	8/22/2015 14:12:28.000	00:00:05.500	0.012 m/s^2	0.012 m/s^2	0.013 m/s^2
Outside blocks	8/22/2015 14:12:33.500	00:00:01.500	0.012 m/s^2	0.012 m/s^2	0.013 m/s^2

Fig. 1. P1 vibrations measurement

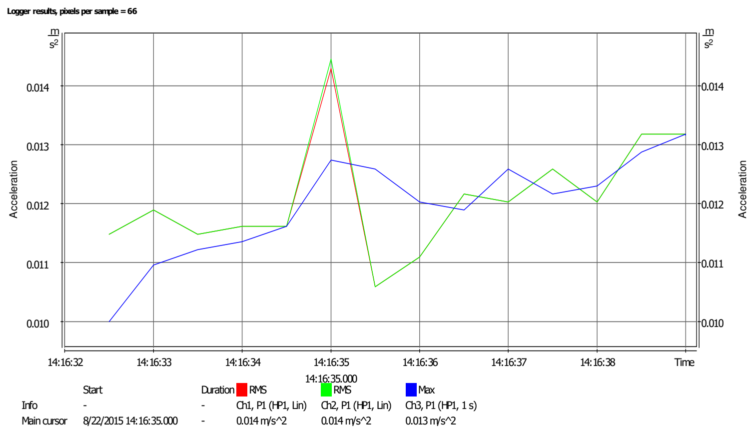


Fig. 2. P2 vibrations measurement

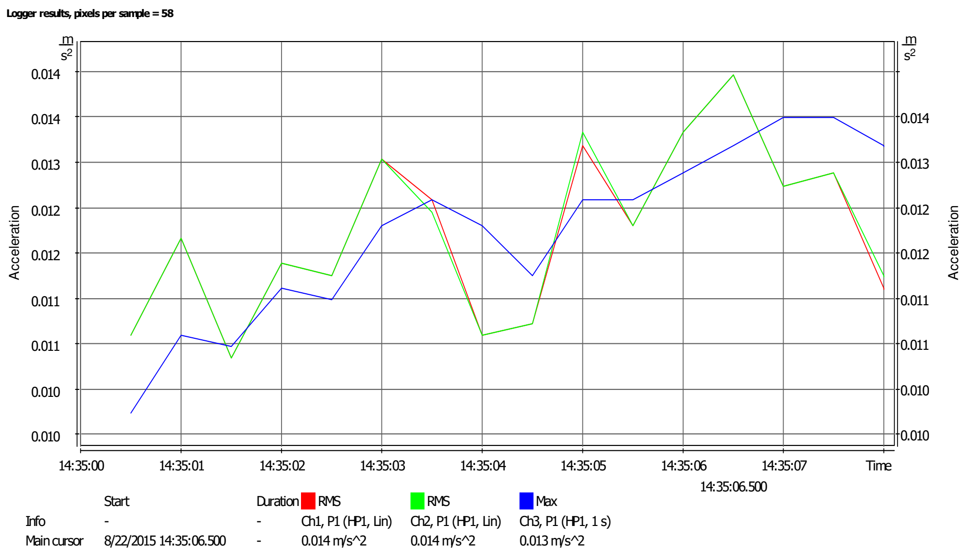


Fig. 3. P4 vibrations measurement

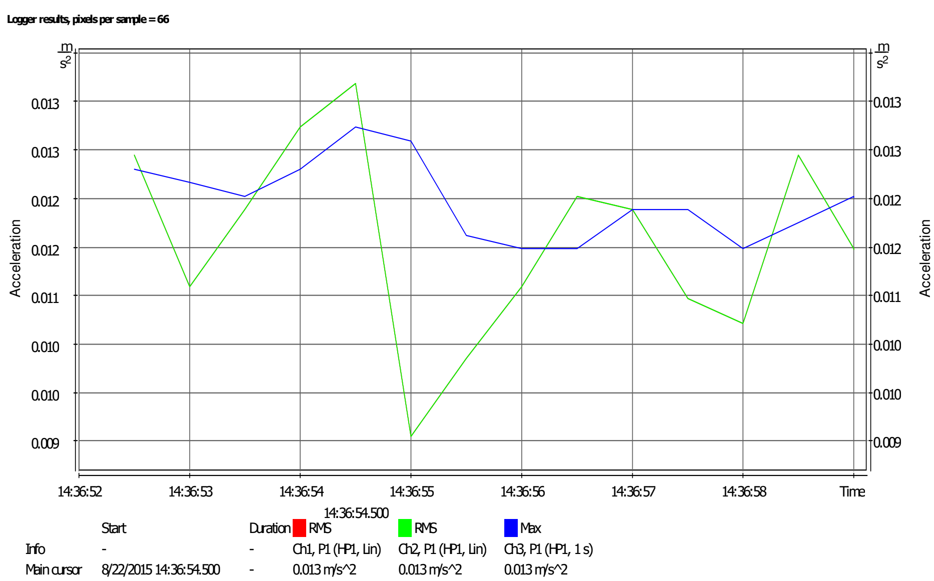


Fig. 4. P6 vibrations measurement

From each representation of the vibration acceleration measured, only the size presented as MAX in the measurements and which is represented by blue color will be considered.

To understand continuity in this work, the first part of the work resumes a little bit.

1.1. Numerical Approximation of Functions

A material system defined in a range of **[a, b]** is supposedly known in **n+1** distinct argument values $x_0, x_1, ..., x_n$, which are known of values $y_0, y_1, \dots, y_n$.

The Lagrange interpolation polynomial noted $L_n(x)$, lower grade or equal to **n,** for which the points considered the polynomial value corresponds to the value of the function. So:

$$L_n(x_i) = y_i, (i = 0, 1, 2, \dots, n) \quad (1)$$

Using the Kronecker's symbols, the function (1) can be expressed as:

$$L_n(x) = \sum_{i=0}^{n} y_i \frac{(x - x_0) \dots (x - x_{i-1})(x - x_{i+1}) \dots (x - x_n)}{(x_i - x_0) \dots (x_i - x_{i-1})(x_i - x_{i+1}) \dots (x_i - x_n)} \quad (2).$$

The relationship (2) contains the expression of polynomial interpolation with the n-degree Lagrange formula, in which polynomial coefficients were defined, as function values at each point in the definition range.

2. CALCULATION OF POLYNOMIAL COEFFICIENTS

In part I of this work was presented the general theory existing in the literature that calculates the polynomial coefficients [5].

Here, in the second part of the work will indicate a much easier scheme for the calculation of the coefficients of y_i (I = 1, 2, ...,n) of Lagrange's formula (2), called **Lagrange's coefficients**.

It is only made from the relationship (2) the coefficient of polynomial function, which is presented by the expression:

$$L_i^{(n)}(x) = \frac{(x - x_0) \dots (x - x_{i-1})(x - x_{i+1}) \dots (x - x_n)}{(x_i - x_0) \dots (x_i - x_{i-1})(x_i - x_{i+1}) \dots (x_i - x_n)} \quad (3)$$

Which you can write abbreviated in the form of:

$$L_i^{(n)}(x) = \frac{M_{n+1}(x)}{(x - x_i)\, N_{n+1}(x_i)} \quad (4)$$

Where

$$M_{n+1}(x) = (x - x_0) \dots (x - x_n) \quad (5)$$

$$N_{n+1}(x_i) = (x_i - x_0) \dots (x_i - x_{i-1}) (x_i - x_{i+1}) \dots (x_i - x_n) \quad (6)$$

With the explanations (5) and (6) Lagrange's formula can be written as follows:

$$L_n(x) = \sum_{i=0}^{n} L_i^{(n)}(x) y_i \quad (7)$$

The form of Lagrange coefficients is invariant in relation to a linear substitution

$$x = a\,t + b \quad (8)$$

In which **a** and **b** are constant, and **a≠0**.

For each point in the range, substitution can be placed, by:

$$x_j = a\,t_j + b \ (j = 0, 1, \dots, n) \quad (9)$$

Replace relationships (8) and (9) in relations (5) and (6), the result obtained is introduced in the relationship (3), in which divide the counter (numerator) and denominator with $\mathbf{a^n}$ and the relationship results:

$$L_i^{(n)}(t) = \frac{(t - t_0)(t - t_1) \dots (t - t_{i-1})(t - t_{i+1}) \dots (t - t_n)}{(t_i - t_0)(t_i - t_1) \dots (t_i - t_{i-1})(t_i - t_{i+1}) \dots (t_i - t_n)} \quad (10)$$

Or

$$L_i^{(n)}(t) = \frac{M_{n+1}(t)}{(t - t_i)\, N_{n+1}(t)} \quad (11)$$

For the calculation of **Lagrange's coefficients**, a scheme shall be drawn up as follows:

$$\begin{array}{ccccc} \underline{x - x_0} & x_0 - x_1 & x_0 - x_2 & \dots & x_0 - x_n \\ x_1 - x_0 & \underline{x - x_1} & x_1 - x_2 & \dots & x_1 - x_n \\ x_2 - x_0 & x_2 - x_1 & \underline{x - x_2} & \dots & x_2 - x_n \\ & & \cdot\ \cdot\ \cdot & & \\ x_n - x_0 & x_n - x_1 & x_n - x_2 & \dots & \underline{x - x_n} \end{array} \quad (12)$$

From the system (12) of the **n+1** relationships are made the notes:

1. The product of the items on the first line is noted with D_0;
2. From the second line with D_1;
3. And so on;
4. And the product of the elements on the main diagonal will be obvious M_{n+1} (x).

With the notes above, the relationship (10) established for each point, it transforms into the system:

$$L_i^{(n)}(x) = \frac{M_{n+1}(x)}{D_i} \text{ (I = 0, 1, …, n)} \qquad (13)$$

Accordingly

$$L_n(x) = M_{n+1}(x) \sum_{i=0}^{n} \frac{y_i}{D_i} \qquad (14)$$

In the case of equidistant points, Lagrange's coefficients can be simplified.

Putting:

$$x = x_0 + t\,h \qquad (15)$$

They will be

$$t_0 = 0, \; t_1 = 1, \; \ldots, \; t_n = n, and\; t = \frac{x - x_0}{h} \qquad (16)$$

The approach to calculating Lagrange's coefficients by equidistant point was the procedure by which coefficients of polynomial functions of grade 10 were calculated, for the approximation of graphic representations of the accelerations of real mechanical systems.

3. APPROXIMATION WITH POLYNOMIAL FUNCTIONS OF MATERIAL SYSTEMS VIBRATIONS

The material system is considered the engine of a truck, on which vibration measurements have been made with SVAN 958, which is a digital analyzer designed for dynamic monitoring, in accordance with ISO 10816, produced by SVANTEK.

Vibration measurements were performed in eight distinct points on the metal parts of the truck, in sequence:

P1 - gearbox grip screw;
P2 - the support screw of the engine block;
P3 - cardan– next to the gearbox;
P4 - trager (intercooler support)- on the heat sink holder;
P5- on chassis next to the engine;
P6 – on the screw of the intake gallery;
P7 – driver seat;
P8 – in the passenger seat.

The data obtained by measuring vibrations were processed and completed with the SVAN P++ program package and were graphically represented.

In Chapter 1, the points for which vibration measurements were made were specified, and the accelerating oscillograms measured with the vibrometer were represented in figures 1 – 4.

From each figure, the representation of the Max value was discussed, which was restored by blue color.

The theory of approximation of polynomial functions is applied in the stages:

1. Calculation of polynomial coefficients;
2. Representation of the vibration measurement graph for **Max** value;
3. Determination of polynomial function, through 10th degree of polynomials;
4. Their graphical Representation by overlapping over the graphic on all positions of the corresponding measurements.

3.1. Approximation of Polynomial Function for P1 Measurement Point

The values of the polynomial coefficients calculated for point P1 for vibration measurement are given in Table 1.

The graphical representation of the Max value of the acceleration measured for point P1, as well as the graphic representation of the polynomial interpolation function can be found in Figure 5.

Table 1.
Polynomial coefficients for approximating the graph point P1 measurement

Polynomial Coefficient	Value
p1	-3.62e-10
p2	2.74e-08
p3	-8.92e-07
p4	1.63e-05
p5	-0.00018
p6	0.001329
p7	-0.00616
p8	0.017868

p9	-0.03075
p10	0.028528
p11	9.68e-05

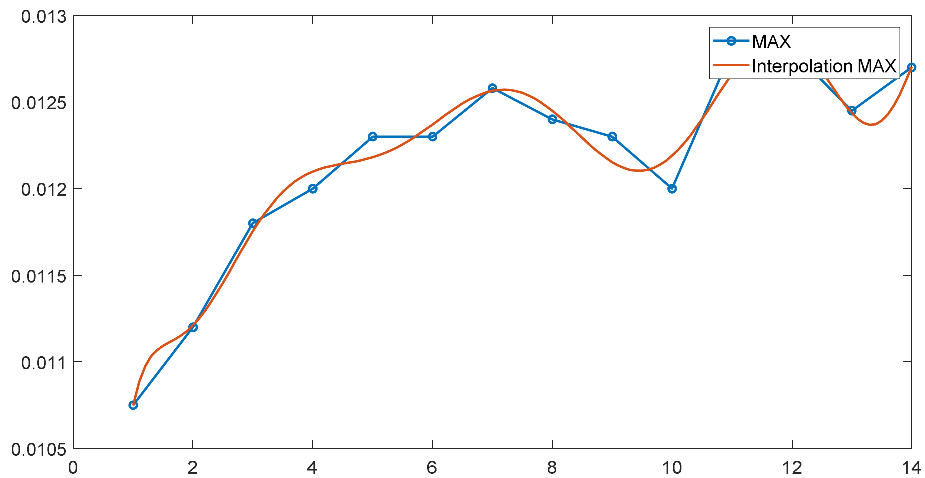


Fig. 5. The Interpolation with 10THDegree Polynomial Function for the Measurement Accelerations in the Point P1

3.2. Approximation of Polynomial Function for P2 Measurement Point

The values of the polynomial coefficients calculated for point P2 for vibration measurement are given in Table 2.

The graphical representation of the Max value of the acceleration measured for point P2, as well as the graphic representation of the polynomial interpolation function can be found in Figure 6.

Table 2.
Polynomial coefficients for approximating the graph point P2 measurement

Polynomial Coefficient	Value
p1	3.41e-10
p2	-3.02e-08
p3	1.14e-06
p4	-2.41e-05
p5	0.000314
p6	-0.00261
p7	0.013841
p8	-0.0458
p9	0.088938
p10	-0.08957
p11	4.54e-02

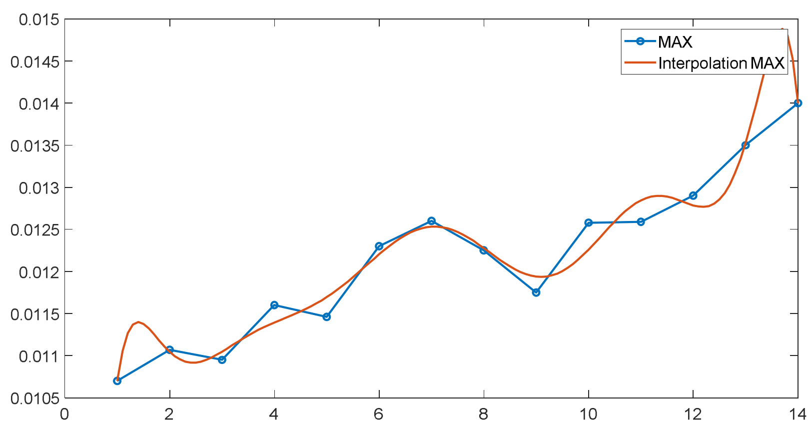


Fig. 6. The Interpolation with 10THDegree Polynomial Function for the Accelerations in the Point P2

3.3. Approximation of Polynomial Function for P4 Measurement Point

The values of the polynomial coefficients calculated for point P4 for vibration measurement are given in Table 3.

The graphical representation of the Max value of the acceleration measured for point P4, as well as the graphic representation of the polynomial interpolation function can be found in Figure 7.

Table 3.
Polynomial coefficients for approximating the graph point P4 measurement

Polynomial Coefficient	Value
p1	-9.84e-10
p2	7.17e-08
p3	-2.25e-06
p4	4.00e-05
p5	-0.00044
p6	0.003155
p7	-0.01465
p8	0.043393
p9	-0.0776
p10	0.074683
p11	-1.79e-02

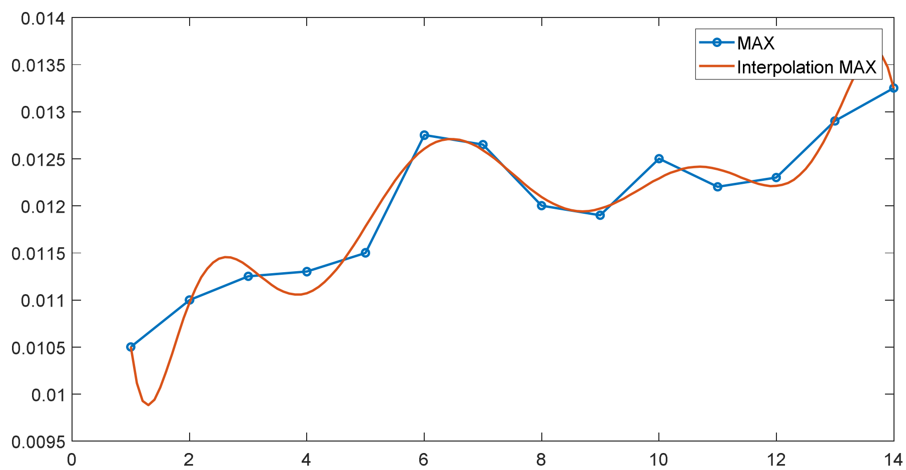


Fig. 7. The Interpolation with 10^{TH}Degree Polynomial Function for the Measurement Accelerations in the Point P4

3.4. Approximation of Polynomial Function for P6 Measurement Point

The values of the polynomial coefficients calculated for point P6 for vibration measurement are given in Table 4.

The graphical representation of the Max value of the acceleration measured for point P6, as well as the graphic representation of the polynomial interpolation function can be found in Figure 8.

Table 4.
Polynomial coefficients for approximating the graph point P6 measurement

Polynomial Coefficient	Value
p1	1.48719663019921e-10
p2	-1.28322325836032e-08
p3	4.63117652036476e-07
p4	-9.16674291481943e-06
p5	0.000109321303376113
p6	-0.000811464613656745
p7	0.00374367163716864
p8	-0.0104531712171015
p9	0.0167837399099930
p10	-0.0141076698095056
p11	0.0174936813212134

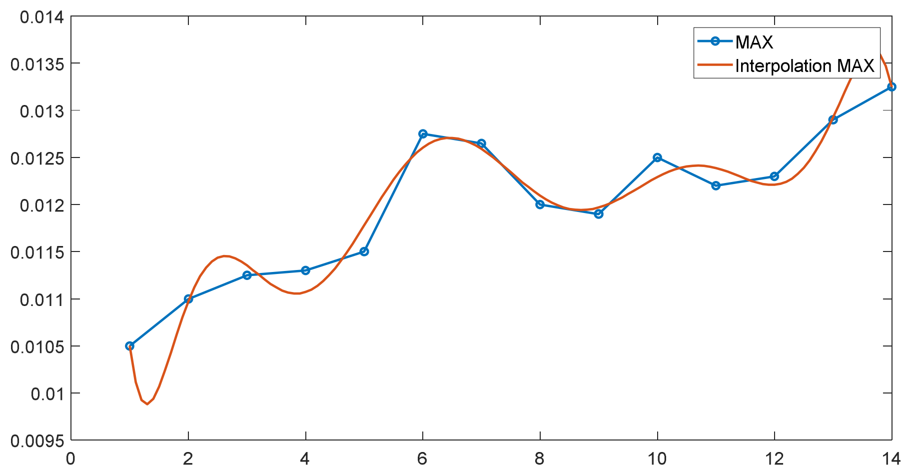


Fig. 8. The Interpolation with 10^{TH}Degree Polynomial Function for the Measurement Accelerations in the Point P6

4. CONCLUSIONS

1. In the experimental study are used only 4 measurement points, that are considered usable for the theory of approximation of graphic functions.
2. The approximation was made for the polynomial functions with 10^{th} degrees.
3. The general theory was realized using the demonstration as in first part of this paper [5].
4. For an easy in this part of the paper was apply a new owner method named calculation of coefficient by scheme of Lagrange's, as in chapter 1.

5.VIBRATION APPROXIMATION WITH 10^{TH} DEGREES OF POLYNOMIAL FUNCTIONS BY LAGRANGE'S COEFFICIENTS

From the experimental study presented the independent variable is time, and the variable dependence is the acceleration.

According to the measurements performed the measurement step is 10^{-3}s, and the representations were made according to the indications in each table.

It adopts **n=10** to solve the problem, and the determination of polynomial coefficients represents the second part of the work, through which a technical problem is solved, through analytical practices.

6. DISCUTIONS

The theory shown in chapter 1 is universally valid for any material system, whether measurements are made or not different if measurements are followed by graphic representations or not.

This work is tried and proves that the approximation with polynomial functions of the results of the measurements of a real mechanical system is possible and in this way, it is established that mathematics is not a sterile science, but it applies to activities in all possible circumstances.

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Contributii la aproximarea vibratiilor sistemelor materiale cu functii polinomiale. Partea II: Aproximarea acceleratiilor vibratiilor reale

Rezumat: Lucrarea prezinta aproximarea reprezentarilor grafice ale vibratiilor masurate ale unei platforme de camion, prin functii polinomiale de gardul 10. Se aplica aproximarea prin interpolarea Lagrange in patru puncte distincte de masurare, cu utilizarea matricei Vandermonde in fiecare punct analizat. Aproximarea duce la rezultate veridice, deci poate fi considerata valida pentru aplicarea analizei vibratiilor sistemelor materiale. Cea de a doua parte a lucrarii realizeaza punerea in evidenta a aproximarii prezentarii aceleratiilor vibrationale ale unui sistem real cu functii polinomiale de gradul 10, pentru patru cazuri distincte de reprentari grafice.

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