TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering
Vol. 63, Issue III, September, 2020

COMPOUND AMORTIZATION FOR COUPLED DYNAMIC SYSTEMS

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Abstract: *For a linear dynamic system (m, c, k) with several degrees of freedom q_i, $i = \overline{1,n}$, amortization may be assessed only based on the modal amortization ratio $\zeta^{(r)}$ corresponding to the vibration modulus r, in the direction of the q_r degree of freedom for i = r . If a dynamic structure consists of several systems j = 1, 2, 3,..,s with linear viscoelastic connections between them, that is the dynamic systems are physically coupled by the three parametric quantities m, c, k, there must be assessed the equivalent amortization or the compound amortization $\zeta_{eq}^{(r)}$ depending on the physical structure of the coupled systems. The research conducted for this purpose highlighted the possibility of establishing an analytical relationship meant to ensure the calculation of the modal amortization ratio for a dynamic structure composed of several systems coupled among them. This article presents the equivalent modal amortization ratio for the linear parallelly coupled viscoelastic systems.*
Key words *viscous amortization, compound dynamic structure, linear dynamic system, equivalent modal amortization, structural compound modal amortization.*

1. INTRODUCTION

The subject consists in the modal approach of a dynamic system with several degrees of freedom for which the modal amortization ratio is established with the hypothesis of the linear behavior of the viscoelastic parameters (*c, k*) and of the proportionality of the amortization with the mass and rigidity of the system according to Rayley model.

Thus, the modal amortization ratio is determined for a dynamic system *j,* where $j = \overline{1,s}$,, that is $\zeta_j^{(r)}$corresponding to the vibration mode *r.*

For the dynamic structure composed of $j = \overline{1,s}$ dynamic systems, the equivalent amortization ratio $\zeta_{eq}^{(r)}$ is established as global parameter for the vibration mode *r,* where $r = \overline{1,n}$. [1 – 4]

2. THE MODAL AMORTIZATION RATIO FOR A LINEAR DYNAMIC SYSTEM

We consider a linear dynamic system with n degrees of freedom q_i, $i = \overline{1,n}$, having the inertia matrix *M,* the linear viscous amortization matrix *C,* the linear rigidity matrix *K* and the excitation given by the vector of the harmonic perturbing forces as $f = [F_1(t), F_2(t), \dots, F_n(t)]^T$. Marking with $q, \dot{q}$ și $\ddot{q}$ the vectors of the instantaneous displacements, the instantaneous velocities, and the generalized instantaneous accelerations, as physical units which characterize the overall motion of the dynamical system, we have the equation of the dynamic equilibrium as a matrix as [5 – 7]

$$M\ddot{q} + C\dot{q} + Kq = f \tag{1}$$

We apply the linear transformation of passing into the modal coordinates $x = [x_1, x_2, \dots, x_n]^T$from generalized coordinates (physical) $q = [q_1, q_2, \dots, q_n]^T$ as follows

$$q = \phi x \tag{2}$$

where $\phi = [\phi_1, \phi_2, \dots, \phi_r, \dots, \phi_n]$ is the modal matrix consisting of own vectors $\phi_1, \phi_2, \dots, \phi_r, \dots, \phi_n$ corresponding to the motions according to the degrees of freedom $i = \overline{1,n}$, where the own vector according to the vibration mode *r* is

$$\phi_r = [\phi_{1r}, \phi_{2r}, \dots, \phi_{ir}, \dots, \phi_{nr}]^T \tag{3}$$

Thus, the modal matrix is

$$\phi = \begin{array}{c} \begin{matrix} \phi_1 & \phi_2 & & \phi_r & & \phi_n \end{matrix} \\ \begin{bmatrix} \phi_{11} & \phi_{12} & \dots & \phi_{1r} & \dots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \dots & \phi_{2r} & \dots & \phi_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ \phi_{i1} & \phi_{i2} & \dots & \phi_{ir} & \dots & \phi_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ \phi_{n1} & \phi_{n2} & \dots & \phi_{nr} & \dots & \phi_{nn} \end{bmatrix} \begin{matrix} \\ 1 \\ 2 \\ i \\ n \\ \\ \end{matrix} \end{array} \tag{4}$$

Inserting the linear transformation (2) in (1) we have

$$M\phi\ddot{x} + C\phi\dot{x} + K\phi q = f \quad (5)$$

which multiplied to the left by ϕ^T leads to the expression of motion in modal coordinates thus leads to the expression of motion in modal coordinates as follows

$$\phi^T M\phi\ddot{x} + \phi^T C\phi\dot{x} + \phi^T K\phi q = \phi^T f \text{ or}$$
$$M_0\ddot{x} + C_0\dot{x} + K_0 x = F \quad (6)$$

Matrices M_0, C_0 și K_0 are diagonal as

$$M_0 = \phi^T M\phi = Diag[m_i], i = 1, 2, \dots, r, \dots, n$$
$$C_0 = \phi^T C\phi = Diag[c_i], i = 1, 2, \dots, r, \dots, n \quad (7)$$
$$K_0 = \phi^T K\phi = Diag[k_i], i = 1, 2, \dots, r, \dots, n$$

r the vibration mode $I = r$, we have

$$M_0^r = \phi_r^T M\phi_r = m_r$$
$$C_0^r = \phi_r^T C\phi_r = c_r \quad (8)$$
$$K_0^r = \phi_r^T K\phi_r = k_r$$

The modal differential equation, depending on the modal vectors x_r with $r = \overline{1,n}$, can be written as follows

$$m_r\ddot{x}_r + c_r\dot{x}_r + k_r x_r = f_r \quad (9)$$

where $c_r = 2\zeta_r\omega_r m_r$ is the model amortization of order r. In this case, the modal amortization ratio ζ_r can be written as

$$\zeta_r = \zeta^r = \frac{c_r}{2\omega_r m_r} \quad (10)$$

or taking into account relations (8) we have [8 – 11]

$$\zeta^{(r)} = \frac{1}{2\omega_r}\frac{\phi_r^T C\phi_r}{\phi_r^T M\phi_r} \quad (11)$$

3. THE EQUIVALENT AMORTIZATION RATIO FOR SEVERAL PARALLELLY COUPLED DYNAMIC SYSTEMS

In case of a structural assembly consisting of $j = \overline{1,s}$ coupled linear dynamic systems, for vibration mode, the equivalent modal amortization c_{eq}^r of the parallel grouping, can be written as [8, 9, 12, 13]

$$c_{eq}^r = c_1^r + c_2^r + \cdots + c_j^r + \cdots c_s^r = \sum_{j=1}^{s} c_j^r \quad (12)$$

For the dynamic system j, based on the relation (10) we have

$$c_j^r = 2\zeta_j^r\omega_j^r m_j^r \quad (13)$$

where we insert $\omega_j^r = \sqrt{\frac{k_j^r}{m_j^r}}$ so that relation (13) can be written as

$$c_j^r = 2\zeta_j^r\sqrt{k_j^r m_j^r} \quad (14)$$

and relation (12) emerges as

$$c_{eq}^r = 2\sum_{j=1}^{s}\zeta_j^r\sqrt{k_j^r m_j^r} \quad (15)$$

For the assembly consisting of s dynamic systems, *the modal equivalent amortization ratio* ζ_{eq}^r corresponding to the vibration mode r, the equivalent modal amortization c_{eq}^r may be written down as

$$c_{eq}^r = 2\zeta_{eq}^r\sum_{j=1}^{s}\sqrt{k_j^r m_j^r} \quad (16)$$

It emerges from the identity condition of (15) and (16)

$$\zeta_{eq}^r\sum_{j=1}^{s}\sqrt{k_j^r m_j^r} = \sum_{j=1}^{s}\zeta_j^r\sqrt{k_j^r m_j^r}$$

from where we have

$$\zeta_{eq}^r = \frac{\sum_{j=1}^{s}\zeta_j^r\sqrt{k_j^r m_j^r}}{\sum_{j=1}^{s}\sqrt{k_j^r m_j^r}}, j = 1, 2, \dots, s \quad (17)$$

where ζ_{eq}^r is the equivalent modal amortization ratio of the whole assembly consisting of $j = \overline{1,s}$ parallelly coupled linear dynamic systems.

3.1 The equivalent modal amortization ratio according to the modal mass

In relation (17) we replace $k_j^r = m_j^r\omega_r^2$ and we obtain [14 – 17]

$$\zeta_{eq}^r = \frac{\sum_{j=1}^{s}\zeta_j^r\sqrt{\omega_r^2\left(m_j^r\right)^2}}{\sum_{j=1}^{s}\sqrt{\omega_r^2\left(m_j^r\right)^2}} \quad \text{or}$$
$$\zeta_{eq}^r = \frac{\sum_{j=1}^{s}\zeta_j^r m_j^r}{\sum_{j=1}^{s} m_j^r} \quad (18)$$

and in the developed form for system $j = \overline{1,s}$ we have

$$\zeta_{eq}^r = \frac{\zeta_1^r m_1^r + \zeta_2^r m_2^r + \cdots + \zeta_j^r m_j^r + \cdots + \zeta_s^r m_s^r}{m_1^r + m_2^r + \cdots + m_j^r + \cdots + m_s^r} \quad (19)$$

Considering that m_j^r is the modal mass of r order for the dynamic system j, we have

$$m_j^r = \phi_r^T M_j\phi_r$$

in which case relation (18) can be written as

$$\zeta_{eq}^r = \frac{\sum_{j=1}^{s}\zeta_j^r(\phi_r^T M_j\phi_r)}{\sum_{j=1}^{s}\phi_r^T M_j\phi_r} \quad (20)$$

or

$$\zeta_{eq}^r = \frac{\sum_{j=1}^{s}\zeta_j^r(\phi_r^T M_j\phi_r)}{\sum_{j=1}^{s}(\phi_r^T M_j\phi_r)_j} \quad (21)$$

3.2 The equivalent modal amortization ratio according to the modal mass

In relation (17) we replace $k_j^r = m_j^r\omega_r^2$ and we obtain [18 - 21]

$$\zeta_{eq}^{r} = \frac{\sum_{j=1}^{s} \zeta_j^r \sqrt{\omega_r^2 \left(m_j^r\right)^2}}{\sum_{j=1}^{s} \sqrt{\omega_r^2 \left(m_j^r\right)^2}} \quad \text{or}$$

$$\zeta_{eq}^{r} = \frac{\sum_{j=1}^{s} \zeta_j^r m_j^r}{\sum_{j=1}^{s} m_j^r} \qquad (22)$$

and in the developed form for system $j = \overline{1, s}$ we have

$$\zeta_{eq}^{r} = \frac{\zeta_1^r m_1^r + \zeta_2^r m_2^r + \cdots + \zeta_j^r m_j^r + \cdots + \zeta_s^r m_s^r}{m_1^r + m_2^r + \cdots + m_j^r + \cdots + m_s^r} \qquad (23)$$

Considering that m_j^r is the modal mass of *r* order for the dynamic system *j*, we have

$$m_j^r = \phi_r^T M_j \phi_r$$

in which case relation (18) can be written as

$$\zeta_{eq}^{r} = \frac{\sum_{j=1}^{s} \zeta_j^r (\phi_r^T M_j \phi_r)}{\sum_{j=1}^{s} \phi_r^T M_j \phi_r} \qquad (24) \quad \text{or}$$

$$\zeta_{eq}^{r} = \frac{\sum_{j=1}^{s} \zeta_j^r (\phi_r^T M_j \phi_r)}{\sum_{j=1}^{s} (\phi_r^T M_j \phi_r)_j} \qquad (25)$$

Relation (25), without demonstration, is used in the GP-101-04 Technical Regulation “Guide for the design of passive seismic insulation systems (supports, dissipators) of buildings”, approved by Order no. 736/19.04.2004 of MTCT, published in M.Of. Part I, no. 874/24.09.2004 bis.

4. CONCLUSION

In the case of the coupled mechanical systems with several degrees of freedom, when the inertial, amortization and rigidity characteristics for each component are known, the determination of the equivalent modal amortization is an essential objective in assessing the dissipation capacity on specific own modes.

This article has approached the analysis methods for establishing the equivalent or compound modal amortization, being finalized the calculation relations for the following situations:

a) the equivalent modal amortization based on the modal masses of the system and the individual amortization rate for each component;

b) the equivalent modal amortization based on the modal rigidities of the system and the individual amortization rate for each component;

c) presented calculation method was applied to the dynamic analysis of the Bechtel viaduct from Gilău, Cluj - Napoca, on the Transylvania highway.

Given the above, I mention that this analysis can be done numerically with specialized programs using the algorithm and the approach presented in this article.

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Amortizarea compusă pentru sisteme dinamice cuplate

Rezumat Pentru un sistem dinamic liniar (*m, c, k*) cu mai multe grade de libertate q_i, $i=\overline{1,n}$, amortizarea poate fi evaluată numai pe baza raportului de amortizare modală $\zeta^{(r)}$ corespunzătoare modulului *r* de vibrație, pe direcția gradului de libertate q_r pentru $i = r$. În situația în care o structură dinamică este alcătuită din mai multe sisteme $j = 1, 2, 3,..,s$ cu legături vâscoelastice liniare între ele, adică sistemele dinamice sunt cuplate fizic prin cele trei mărimi parametrice *m, c, k,* trebuie evaluată amortizarea echivalentă sau amortizarea compusă $\zeta_{eq}^{(r)}$dependentă de structura fizică a sistemelor cuplate. Cercetările efectuate în acest scop au evidențiat posibilitatea stabilirii unei relații analitice menite să asigure calculul raportului de amortizare modală pentru o structură dinamică alcătuită din mai multe sisteme cuplate între ele. În articolul prezent va fi determinat raportul de amortizare modală echivalentă pentru sisteme liniare vâscoelastice cuplate în paralel.

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