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GEOMETRICAL AND KINEMATICAL CONTROL FUNCTIONS FOR A MOBILE ROBOT USED IN EXTINGUISHING FIRES

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Abstract: *In the last decades, the mobile robots are probably the most spectacular and representative category of robotic systems, especially due to the attempt to copy and approach models from the living world.*
One adjacent area to the use of robots is preventing and fire-fighting, recently, in developed countries; mobile systems are used to detect, isolate and extinguish fires in excessively toxic environments, as well as to rescue the victims of such disasters. In the paper are presented in analytical form, the control functions for geometry and kinematics for a proposed mobile prototype, having mounted water cannon.
Key words: *mobile robot, mathematical modeling, direct kinematic modeling, control functions.*

1. INTRODUCTION

The field of mobile robots has developed a lot in recent years and currently has applications in most fields of activity. Unlike fixed robots that can work in a limited space, mobile robots have the ability to operate in a space that goes far beyond their own dimensions. In the general case, a mobile robot is a vehicle equipped with a certain degree of autonomy capable of performing a class of useful tasks.[1],[2]

In this context, the paper presents lapidary, a prototype of a mobile robotic system, designed for difficult interventions, being endowed with intelligent systems with functions similar to human functions, but also intelligent sensorial systems for gathering information from the environment and intervention in dangerous situations.

As shown in Figure 1, the hybrid prototype of the system used in fire-fighting consists of a mobile platform with four active wheels, and a mechanism for orienting the water jet, with two degrees of freedom. For optimal intervention, the prototype proposed for development is equipped with a high-resolution camera for locating outbreaks, as well as light sources.

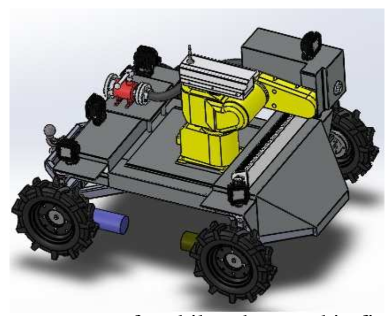

Fig.1 The prototype of mobile robot used in fire-fighting

In the first part, the direct kinematic equations for the mobile robot are presented, and then the geometric control functions for the orientation of the water-cannon mechanism.

2. GEOMETRICAL MODELING OF THE MOBILE ROBOT PROTOTYPE

The Kinematics of the mobile robots, consisting in study of position and motion estimation, which can be set or read from the encoders of robots. In keeping with the fact that the robot`s wheels are imposing constraints in robot motion, the establishing of kinematic functions, in general, is based on studying the motion of each robot wheel [3],[4],[5].

2.1 Direct kinematic modeling of mobile robot in Cartesian coordinates

According to [5],[6] and [7], the motion will be studied with respect to two frames (see Figure 2): {0}- fixed coordinate system (global reference frame), and {R}- robot's coordinating system, fixed in the point R.

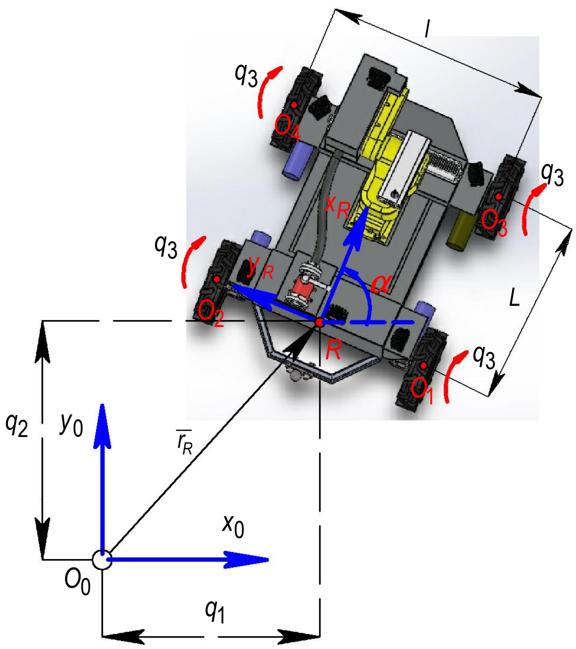

Fig.2 The independent parameters of the mobile robot

In keeping with the fact that the robot is executing a translational displacement, according to Figure 2, the robot motion law with respect to fixed reference frame is:

$$ \underset{(3\times1)}{{}^0\bar{X}} = \begin{bmatrix} \bar{r}_R \\ \alpha = ct. \end{bmatrix} = \begin{bmatrix} x_R(t) = q_1(t) \\ y_R(t) = q_2(t) \\ \alpha = ct. \end{bmatrix} \tag{1} $$

As an important remark, considering that there is no slipping constraint along either of the two robot axes, the robot's movement is olonomous.

Taking into account that the robot performs a translational movement, the geometrical parameters that are expressing the movement of the system attached to the robot are: x_R, y_R and φ, representing the angular displacement of the wheels.

The column vector of the operating velocities of the robot relative to the fixed system is:

$$ {}^0\dot{X} = \begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ 0 \end{bmatrix} \tag{2} $$

In this case, all points belonging to the mobile robot have the same velocities, equal to $\bar{v}$, and are oriented to axis x_R .

According to Figure 3, if the wheels of the mobile robot rotate without slipping then ***I*** is the instantaneous center of rotation, so that the point R, the origin of the mobile system $\{R\}$ will have the velocity:

$$ v_R = v = r \cdot \dot{q}_3 \tag{3} $$

where r is the radius of the wheel.

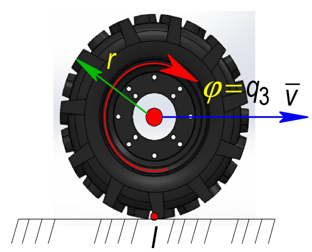

Fig.3 The velocities of the wheels

The expression which describing the motion of $\{R\}$, with respect to fixed reference frame is:

$$ {}^0\dot{x}_R = v_R \cdot cos\,\alpha = r \cdot \dot{q}_3 \cdot cos\,\alpha $$
$$ {}^0\dot{y}_R = v_R \cdot sin\alpha = r \cdot \dot{q}_3 \cdot sin\alpha, \tag{4} $$

representing the motion law expressions for the mobile robot.

According to figure 3, the position vector for each wheel, is expressed as:

$$ \bar{r}_{O_1} = \begin{pmatrix} q_1 + \frac{l}{2} \cdot \sin\alpha \\ q_2 - \frac{l}{2} \cdot \cos\alpha \\ r \end{pmatrix} \qquad \bar{r}_{O_2} = \begin{pmatrix} q_1 - \frac{l}{2} \cdot \sin\alpha \\ q_2 + \frac{l}{2} \cdot \cos\alpha \\ r \end{pmatrix} \tag{5} $$

$$\bar{r}_{O_3} = \begin{pmatrix} q_1 + \frac{l}{2} \cdot \sin\alpha + L \cdot \cos\alpha \\ q_2 - \frac{l}{2} \cdot \cos\alpha + L \cdot \sin\alpha \\ r \end{pmatrix} \quad \bar{r}_{O_4} = \begin{pmatrix} q_1 - \frac{l}{2} \cdot \sin\alpha + L \cdot \cos\alpha \\ q_2 + \frac{l}{2} \cdot \cos\alpha + L \cdot \sin\alpha \\ r \end{pmatrix} \quad (6)$$

On the basis of previous expressions, there are obtained:

$$v_{0_i} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ 0 \end{bmatrix}, \dot{v}_{0_i} = \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ 0 \end{bmatrix}; i = 1 \rightarrow 4 \quad (7)$$

which are characterizing the direct kinematic model (DKM) in Cartesian coordinates for the mobile structure having an essential role in determining the equations of direct dynamics for the mobile robot.

2.2 Direct kinematic modeling of mobile robot in polar coordinates

Further, in the paper, will be established the direct kinematic model for the mobile robot, using polar coordinate system, if the robot would have orientation [3]. According to Figure 4, there is considered a goal point, where is fixed a reference frame $\{G\}$; ρ represents the polar radius; -ψ-the angle between the polar radius and the system attached to the goal position.

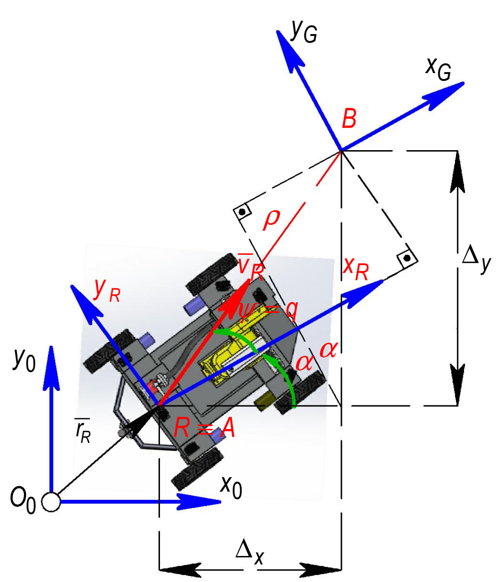

Fig.4 The parameters of the mobile robot in polar coordinate system

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$$\begin{cases} \rho = \sqrt{\Delta x^2 + \Delta y^2}, \\ tg(\psi + \alpha) = \frac{\Delta y}{\Delta x}; \quad \psi = q \end{cases} \quad (8)$$

In keeping with (8), there can be written the expression:

$$\psi = -\alpha + \text{Atan2}(\Delta y; \Delta x). \quad (9)$$

After performing some calculus in (8), results:

$$2 \cdot \rho \cdot \dot{\rho} = 2 \cdot \Delta x \cdot \Delta \dot{x} + 2 \cdot \Delta y \cdot \Delta \dot{y} \quad (10)$$

Where: $\dot{\rho} = \frac{1}{\rho} \cdot (\Delta x \cdot \Delta \dot{x} + \Delta y \cdot \Delta \dot{y});$ (11)

In keeping with Figure 4, the previous expression is rewritten as:

$$\dot{\rho} = \frac{1}{\rho} \cdot v_R \cdot (\Delta x \cdot cos(\alpha + \psi) + \Delta y \cdot sin(\alpha + \psi)); \quad (12)$$

And will conduct to:

$$\dot{\rho} = v_R; \quad (13)$$

Hence, if the robot would have orientation, the movement in polar coordinates would have been expressed as:

$$[\dot{\rho} \quad \dot{\psi} \quad \dot{\beta}]^T = [v_P \quad \dot{q} \quad 0]^T. \quad (14)$$

In order to establish the kinematic control functions, there are known the coordinates of two point A and B:

$${}^A\bar{X} = \begin{pmatrix} x_p^A & y_p^A & \alpha \end{pmatrix}^T; \ {}^B\bar{X} = \begin{pmatrix} x_p^B & y_p^B & 0 \end{pmatrix}^T \quad (15)$$

Also is assume to be known $\Delta t = t_B - t_A$, the displacement time between the points A and B, in this case there can be written:

$$\Delta x = x_B - x_A; \ \Delta y = y_B - y_A; \quad (16)$$

Substituting (3) in (8) there is obtained:

$$\left\{ \dot{\rho} = r \cdot \dot{\varphi}; \alpha + q = arctg\left(\frac{\Delta y}{\Delta x}\right) \right\}; \quad (17)$$

On the basis of previous expression, there is determined:

$$q = -\alpha + arctg\left(\frac{\Delta y}{\Delta x}\right); \dot{\varphi} = \frac{\dot{\rho}}{r}; \quad (18)$$

If the displacement between A and B is elementary and finite then the relation (18) is transformed into the form represented below:

$$q = -\alpha + \text{arctg}\left(\frac{\Delta y}{\Delta x}\right); \; \frac{\Delta \varphi}{\Delta t} = \frac{\Delta \rho}{\Delta t \cdot r}; \quad (19)$$

$$\Delta \rho = \rho = \sqrt{\Delta x^2 + \Delta y^2}\,. \quad (20)$$

thus establishing the kinematic control functions of the robot, if it would have orientation, thus having a parallel-plane motion.

3. Geometrical modeling for the water cannon mechanism

On the mobile platform is fixed a mechanism which is orienting the water jet in order to put out the fire. The mechanism is executing two rotations, one in horizontal plane, and the other in vertical plane, hence, with respect to the robot, the water cannon having two degrees of freedom, as resulting from the Figure 5.

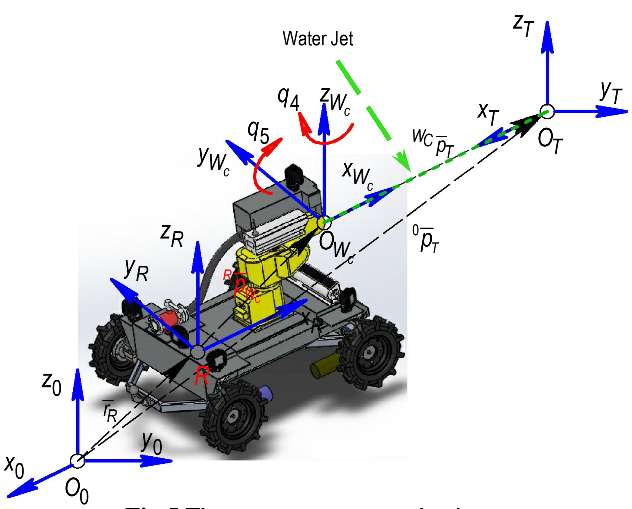

Fig.5 The water cannon mechanism

For the geometrical study, there is considered the reference frame, $\{W_C\}$, attached to the robot`s water cannon, the position vector of the with respect to the frame $\{R\}$ being:

$$^R\bar{p}_C = \begin{bmatrix} ^Rx_C & 0 & ^Rz_C \end{bmatrix}^T; \quad (21)$$

According to the Figure 5, there is considered another reference frame $\{T\}$, attached to the target point, which has to be reached by the water jet. Hence, $\{T\}$ with respect to $\{W_C\}$, is :

$$^{W_C}\bar{p}_T = \begin{bmatrix} ^{W_C}x_T & ^{W_C}y_T & ^{W_C}z_T \end{bmatrix}^T; \quad (22)$$

The water cannon orientation is realized by two independent rotations, around z_{W_C} and y_{W_C}, according to:

$$R\left(\bar{z}_{W_C}; q_4\right) = \begin{bmatrix} \cos q_4 & -\sin q_4 & 0 \\ \sin q_4 & \cos q_4 & 0 \\ 0 & 0 & 1 \end{bmatrix};$$
$$R\left(\bar{y}_{W_C}; q_5\right) = \begin{bmatrix} \cos q_5 & 0 & \sin q_5 \\ 0 & 1 & 0 \\ -\sin q_5 & 0 & \cos q_5 \end{bmatrix}. \quad (23)$$

Thus, the orientation of $\{W_C\}$ with respect to $\{R\}$ is:

$$^R_{W_C}[R] = R\left(\bar{z}_{W_C}; q_4\right) \cdot R\left(\bar{y}_{W_C}; q_5\right) =$$
$$= \begin{bmatrix} \cos q_4 \cdot \cos q_5 & -\sin q_4 & \cos q_4 \cdot \sin q_5 \\ \sin q_4 \cdot \cos q_5 & \cos q_4 & \sin q_4 \cdot \sin q_5 \\ -\sin q_5 & 0 & \cos q_5 \end{bmatrix}. \quad (24)$$

In keeping with the geometry, the position of $\{T\}$ according to $\{R\}$ is expressed as:

$$^R\bar{p}_T = {}^R\bar{p}_{W_C} + {}^R_{W_C}[R] \cdot {}^{W_C}\bar{p}_T = \begin{bmatrix} ^Rx_{W_C} & 0 & ^Rz_{W_C} \end{bmatrix}^T +$$
$$+ {}^R_{W_C}[R] \cdot \begin{bmatrix} ^{W_C}x_T & ^{W_C}y_T & ^{W_C}z_T \end{bmatrix}^T. \quad (25)$$

$$ {}^{R}p_T = \begin{bmatrix} {}^{R}x_T \\ {}^{R}y_T \\ {}^{R}z_T \end{bmatrix} = \begin{bmatrix} {}^{R}x_{W_C} + {}^{W_C}x_T \cdot \cos q_4 \cdot \cos q_5 - \\ - {}^{W_C}y_T \cdot \sin q_4 + {}^{W_C}z_T \cdot \cos q_4 \cdot \sin q_5 \\ \hline {}^{W_C}x_T \cdot \sin q_4 \cdot \cos q_5 + \\ + {}^{W_C}y_T \cdot \cos q_4 + {}^{W_C}z_T \cdot \sin q_4 \cdot \sin q_5 \\ \hline {}^{R}z_C - {}^{W_C}x_T \cdot \sin q_5 + {}^{W_C}z_T \cdot \cos q_5 \end{bmatrix} \quad (26) $$

Knowing the position of frame $\{T\}$ with respect to $\{R\}$, it can be written:

$$ {}^{0}\bar{p}_T = \bar{r}_R + {}^{0}_{R}[R] \cdot {}^{R}p_T = \begin{bmatrix} x_R + {}^{R}x_T \cdot \cos\alpha - {}^{R}y_T \cdot \sin\alpha \\ y_R + {}^{R}x_T \cdot \sin\alpha + {}^{R}y_T \cdot \cos\alpha \\ z_R + {}^{R}z_T \end{bmatrix} \quad (27) $$

And represents the position vector of the frame $\{T\}$ with respect to fixed frame $\{0\}$.

3.1 The geometrical control functions for orienting the water cannon mechanism

According to previous paragraph and [8], there is assumed be known the position the position of reference frame $\{T\}$ of the water jet target point, the vector ${}^{0}\bar{p}_T = \left({}^{0}x_T \quad {}^{0}y_T \quad {}^{0}z_T \right)^T$ respectively the situation parameters (position and orientation) with respect to fixed reference frame of the robot. Further will be established the orienting angles of the water cannon. Hence, the position of the water canon with respect to $\{0\}$ is:

$$ {}^{0}\bar{p}_{W_C} = \bar{r}_R + {}^{0}_{R}[R] \cdot {}^{R}\bar{p}_{W_C} = \begin{bmatrix} x_R \\ y_R \\ z_R \end{bmatrix} + {}^{0}_{R}[R] \cdot \begin{bmatrix} {}^{R}x_{W_C} \\ 0 \\ {}^{R}z_{W_C} \end{bmatrix} = \begin{bmatrix} x_R + {}^{R}x_T \cdot \cos\alpha \\ y_R + {}^{R}x_T \cdot \sin\alpha \\ z_R + {}^{R}z_T \end{bmatrix} \quad (28) $$

On the basis of input data, and previous expression, the position of target point with respect to $\{W_C\}$ is:

$$ {}^{W_C}\bar{p}_T = \begin{bmatrix} {}^{W_C}x_T \\ {}^{W_C}y_T \\ {}^{W_C}z_T \end{bmatrix} = {}^{0}_{W_C}[R]^T \cdot \left({}^{0}\bar{p}_T - {}^{0}\bar{p}_{W_C} \right); \quad (29) $$

After performing the calculus, there is obtained:

$$ \begin{bmatrix} {}^{W_C}x_T \\ {}^{W_C}y_T \\ {}^{W_C}z_T \end{bmatrix} = \begin{bmatrix} \cos q_5 \cdot \left[\cos(\alpha + q_4) \cdot \left({}^{0}x_T - x_R - {}^{R}x_T \cdot \cos\alpha \right) \right. \\ \left. + \sin(\alpha + q_4) \cdot \left({}^{0}y_T - y_R - {}^{R}x_T \cdot \sin\alpha \right) \right] - \\ - \sin q_5 \cdot \left({}^{0}z_T - z_R - {}^{R}z \right) \\ \hline \cos(\alpha + q_4) \cdot \left({}^{0}y_T - y_R - {}^{R}x_T \cdot \sin\alpha \right) - \\ - \sin(\alpha + q_4) \cdot \left({}^{0}x_T - x_R - {}^{R}x_T \cdot \cos\alpha \right) \\ \hline \sin q_5 \cdot \left[\cos(\alpha + q_4) \cdot \left({}^{0}x_T - x_R - {}^{R}x_T \cdot \cos\alpha \right) + \right. \\ \left. + \sin(\alpha + q_4) \cdot \left({}^{0}y_T - y_R - {}^{R}x_T \cdot \sin\alpha \right) \right] + \\ + \cos q_5 \cdot \left({}^{0}z_T - z_R - {}^{R}z \right) \end{bmatrix} \quad (30) $$

According to Figure 5, the axis x_{W_C} has the same orientation as x_T, in this case, the water jet has to reach a target point in $y_T O_T z_T$ plane. Hence, in order to obtain the generalized coordinates q_4 and q_5, .are equalized with zero the projections on y_{W_C} and z_{W_C} from (30). Is obtained the following:

$$ q_4 = \frac{\pi}{2} - \alpha; \quad q_5 = 0 \quad (31) $$

representing the orientation angles for the water cannon, in order to reach a target point.

4. CONCLUSIONS

Starting from the idea of the potential of terrestrial mobile robots to fire-fight in conditions that would be dangerous to humans, the paper presented such a prototype that can be controlled remotely with the help of a remote control for both movement and operation of discharge and targeting equipment.

Unlike the serial robots, in the kinematics of the mobile robots the mathematical models are different, in the first time due to holonomic links. The mathematical model presented in the paper is based on the hypothesis of the equations of plane motion, the robot being subjected to holonomic links. The equations of the kinematic model for the mobile robot were determined in Cartesian coordinates. The kinematics equations were also determined in polar coordinates, assuming that the robot would also have orientation, since the polar coordinates allow the easy determination of the kinematic control for t mobile structures.

Also, in the paper, for the water cannon, mounted on mobile structure it has been presented a mathematical model for orientation

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Functii de control geometric si cinematic pentru o structura mobila utilizata la stingerea incendiilor

In ultimele decenii, roboții mobili sunt probabil cea mai spectaculoasă și reprezentativă categorie de sisteme robotice, în special datorită încercării de a copia și aborda modele din lumea vie. O zonă adiacentă folosirii roboților este prevenirea și combaterea incendiilor, astfel ca în țările dezvoltate; sistemele mobile sunt utilizate pentru detectarea, izolarea și stingerea incendiilor în medii excesiv de toxice, precum și pentru salvarea victimelor unor astfel de dezastre. În lucrare sunt prezentate sub formă analitică, ecuațiile pentru control geometric și cinematic pentru un prototip mobil, cat si pentru tunul de apă, montat pe structura mobilă.

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