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# QUATERNION-BASED APPROACH FOR SOLVING THE DIRECT KINEMATICS OF A MODULAR HYPER REDUNDANT ROBOT 

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#### Abstract

The paper presents a new approach for solving the direct kinematics of a modular high dexterity robot by integrating the quaternions of the mobile platforms in the calculation algorithm. The information related to the mobile platforms quaternions are obtained from dedicated IMU sensors. These data are then used to determine the position and orientation of all mobile platforms and provide the necessary data for path planning and robot control algorithms. The developed method was tested using Matlab for a 10 DOF modular hyper redundant robot. The obtained numerical results validated the proposed approach.


Key words: high dexterity robot, quaternion, IMU sensor.

## 1. INTRODUCTION

Hyper redundant robots are a class of robots that have a large number of degrees of freedom (DOF) ( $M \gg 6$ ) [1]. This type of robots is characterized by the fact that for an imposed position and orientation for the end effector an infinite number of joint values $\theta_{i}$ exists that allows reaching that pose [2]. This offers new benefits like the ability to operate in workspaces with obstacles, increased robustness and the ability to perform complex movements.

In the last decades, due to its advantages, this type of robots has been used in a large number of applications from different fields: medical noninvasive procedures [3], agriculture [4], inspection in hard accessible or contaminated areas [5], search and rescue activities [6], teleoperation [7] etc.

Due to the increase number of kinematic elements and joints that form this type of robots, the kinematic models are more complex comparing to the traditional robots.


Fig. 1. Kinematic mapping between robot spaces

Different approaches are proposed in the literature, but in general the kinematics is divided in two mappings among three spaces: actuator space, joint space and task space [8] (fig. 1). In the case of forward kinematics first individual modules are analyzed in the actuator space. The obtained result are used to create individual transformation matrix that are then multiplied in order to obtain the kinematic for all modules.

In the paper a new approach is proposed for the direct kinematics of a modular hyper redundant robot. The method uses a set of quaternions that define the orientation of the mobile platforms for each module. In this case the analyze starts from joint space reducing in this way the complexity of the obtained number of calculations and the time needed for computing the kinematics of the robot.

## 2. PROPOSED APPROACH

### 2.1 Robot description

In this chapter an example of a hyper redundant modular robot is presented. The schematic of the robot is depicted in figure 2. The robot is comprised from a set of $n$ modules that are interconnected.


Fig. 2. The hyper redundant robot schematic representation

Each module is made up of two platforms that are connected through a universal joint. The module has 2 DOF which are independently controlled by 4 bellows which works antagonist two by two. The parameters $q_{i j}$ represents the pressure in each bellows, modifying the values of these parameters allows to directly control the orientation of each mobile platform ( $\theta_{x i}$ and $\theta_{y i}$ ).

In order to determine the orientation of the robot platforms, an Inertial Measurement Unit (IMU) is connected to each platform. The sensor provided the quaternions that define the absolute orientation of the platforms relative to magnetic north. These values are further used in the kinematic analysis.

### 2.2 Kinematic analysis

The proposed algorithm calculates the position and relative orientation for each platform and end effector, using the platforms quaternions $Q_{i}$ and geometrical parameters of the robot. These data are then used by the control algorithm and for path planning.

For each platform a local coordinate system $O_{i} x_{i} y_{i} z_{i}\{i=0 . . n\}$ is attached. The coordinate system $O_{0} x_{0} y_{0} z_{0}$, attached to the fixed platform is considered the global coordinate system (fig. 3). Using the IMU sensor, a set of $Q_{i}$ $\{i=0 . . n\} q u a t e r n i o n s$ are measured at each sample time. The quaternion is defined by 4 elements, as seen in eq. 1 .

$$
\begin{equation*}
Q_{i}=(w, x, y, z) \tag{1}
\end{equation*}
$$

The quaternions are used to obtain the rotation matrices $R_{P i}$ for the all $i$ platforms $\{i=0 . . n\}$. The general form of the rotation matrix $R_{P i}$ is presented in (2),

$$
R_{P i}=\left[\begin{array}{lll}
r_{11} & r_{12} & r_{13}  \tag{2}\\
r_{21} & r_{22} & r_{23} \\
r_{31} & r_{32} & r_{33}
\end{array}\right],(i=0 . . n)
$$

where the $r_{k l}\{k=1 \ldots 3 ; l=1 \ldots 3\}$ terms are obtained from the quaternion elements:

$$
\begin{aligned}
& r_{11}=1-2\left(y^{\wedge} 2+z^{\wedge} 2\right) \\
& r_{12}=2(x y-w z) \\
& r_{13}=2(w y+x z) \\
& r_{21}=2(x y+w z) \\
& r_{22}=1-2\left(x^{\wedge} 2+z^{\wedge} 2\right) \\
& r_{23}=2(y z-w x) \\
& r_{31}=2(x z-w y) \\
& r_{32}=2(w x+y z) \\
& r_{33}=1-2\left(x^{\wedge} 2+y^{\wedge} 2\right)
\end{aligned}
$$

Using the $R_{P i}$ matrices, the relative rotations between the platforms of one module are computed using the eq. 3 .
$R_{R i}=R_{P i} * \operatorname{transpose}\left(R_{P i-1}\right) ;\{i=1 . . n\}$
From the rotation matrix $R_{R i}$ the angles $\theta_{x l}, \theta_{y l}$ and $\theta_{z I}$ are calculated using eq. 4 .

$$
\begin{align*}
& \theta_{y i}=\operatorname{atan2} 2\left(-r_{31}, \sqrt{r_{11}^{2}+r_{21}^{2}}\right. \\
& \theta_{z i}=\operatorname{atan} 2\left(\frac{r_{21}}{\cos \left(\theta_{y i}\right)}, \frac{r_{11}}{\cos \left(\theta_{y i}\right)}\right)  \tag{4}\\
& \theta_{x i}=\operatorname{atan2}\left(\frac{r_{32}}{\cos \left(\theta_{y i}\right.}, \frac{r_{33}}{\cos \left(\theta_{y i}\right)}\right)
\end{align*}
$$

If $\theta_{x i}, \theta_{y i}$ are known, all the other kinematic parameters of the mobile platforms could be determined.


Fig. 3. The schematic for a functional module

The cartesian position of the center of each platform $O_{i}\{i=1 \ldots n\}$ in the global coordinate system can be determined using the homogeneous transform ${ }_{0}^{i} T$ of the robot.

For a module $i$ the transformation matrix ${ }_{0}^{i} T$ is calculated based on the previous transformation matrix ${ }_{0}^{i-1} T$ that is multiplied by the transformation matrix of the module ${ }_{i-1}^{i} T$.

$$
\begin{equation*}
{ }_{0}^{i} T={ }_{0}^{i-1} T_{i-1}{ }_{1}^{i} T \tag{5}
\end{equation*}
$$

The ${ }_{i-1}^{i} T$ transformation matrix is obtained from the rotation matrix ${ }_{i-1}^{i} R$ and the position vector ${ }_{i-1}^{i} P$ (eq. 6).

$$
{ }_{i-1}^{i} T=\left[\begin{array}{cccc}
{ }_{i-1}^{i} R & & { }_{i-1}^{i} P  \tag{6}\\
0 & 0 & 0 & 1
\end{array}\right]
$$

Where the rotation matrix ${ }_{i-1}^{i} R$ is calculated by multiplying the individual rotations on each axis, using the angles $\theta_{x i} \theta_{y i}$ and $\theta_{z i}$ :

$$
\begin{equation*}
{ }_{i-1}^{i} R=R_{x} R_{y} R_{z} \tag{7}
\end{equation*}
$$

The position vector ${ }_{i-1}^{i} P$ is calculated using the $d_{0}$ and $d_{l}$ distances.

## 3. NUMERICAL RESULTS

The proposed method was tested using a model of a hyper redundant robot with 10 DOF which was implemented in Matlab/Simulink. The modules that compose the robot are identical and were developed using Multibody library from Simscape.


Fig. 4. The robot virtual model


Fig. 5. The Simulink model of a module
The obtained model for one module is presented in figure 5. The model inputs are the kinematic parameters for the universal joint and the output is the quaternion of the mobile platform. The model allows also to measure the orientation angles and cartesian position of the mobile platform. These values are used to validate the obtained results from the proposed approach. The kinematics of the robot can be observed also in a Matlab window that contain a virtual space where the virtual model of the robot is displayed in 3D (fig. 4).

The proposed algorithm is implemented in Matlab/Simulink using m-code. The algorithm uses the quaternions from the virtual robot model and computes the position and orientation of all robot platforms. A graphic user interface was developed to represent the robot's platforms in a 3D graph and to draw the trajectory of the robot (fig.6).

Using the presented setup several experiments where developed. The calculated results (the cartesian position and relative orientations for each mobile platform) where compared with the values that were measured from the virtual robot.

In figure 7 the simulation results for the end effector position are presented. It can be observed that the outputs of proposed algorithm


Fig. 6. Computed positions and orientations


Fig. 7. Computed positions and orientations
are identical with the values measured from the virtual model of the robot.

## 4. CONCLUSION

The paper presented a new approach to solve the direct kinematics of a hyper redundant robot using quaternions. The obtained mathematical equations were implemented in Matlab Simulink where were teste using a virtual robot. The results validated the proposed method. The algorithm is intended to be used next for the development of the robot control algorithms.

## 5. ACKNOWLEDGEMENT

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## Abordare bazată pe quaternioni pentru rezolvarea cinematicii directe a unui robot modular hyper-redundant

Rezumat: Lucrarea prezintă o nouă abordare pentru rezolvarea problemei cinematice directe pentru un robot modular hyper-redundant prin integrarea quaternionilor platformelor mobile în algoritmul de calcul. Informațiile referitoare la quaternionii platformelor mobile sunt obținute de la senzori IMU dedicați. Aceste date sunt apoi utilizate pentru calcularea poziției și orientării tuturor platformelor mobile și pentru a furniza date necesare generării de traiectorii și controlul robotului. Metoda dezvoltată a fost testată utilizând Matlab pentru un robot hyper redundant modular cu 10 GDL. Rezultatele numerice obținute validează abordarea propusă.

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