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$CoFe_2O_4$ NANOFLUID FLOW OVER A ROTATING AND MOVING DISK

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Abstract: *The present model aims to investigate the unsteady flow of $CoFe_2O_4$ nanofluid over a rotating and moving disk under the influence of the magnetic field. The magnetic body force is directed normal to the disk. The similarity transformation is employed to transform the governing equations into a set of nonlinear coupled differential equations. In the absence of a magnetic field and vertical movement of the disk, the problem reduces into Von Karman swirling flow problem. The nonlinear coupled differential equations in the present model are solved by the finite element method using the element size as 0.0001. In this numerical simulation, results for radial, tangential and axial velocity distributions are obtained for different values of physical parameters used in the model. A comparative study of different types of nanofluid is presented in the study and compared with $CoFe_2O_4$ nanofluid.*
Keywords: *Nanofluid, nonlinear differential equations, unsteady flow, velocity, rotation.*

1. INTRODUCTION

The steady flow of magnetic fluid in a rotating system is important in the applications of rotating machinery systems. Nowadays flow in a rotating system might be helpful in medical and space applications[1–3]. The flow in a rotating system was originated from the Von Karman swirling problem[4]. Cochran and Benton presented the analytic solution for swirling flow[5,6]. Ram et al. developed the rotating disk problem for ferrohydrodynamic flow and provided the analytic solution through power series approximations [7].

Using the Shliomis model, the concept of negative viscosity of magnetic fluid was introduced[8]. Due to the importance of rotating flow, several aspects have been presented by the researchers available in the literature[9–11]. The available literature shows that the viscosity of magnetic fluid plays an important role in the flow characteristics[12–16].

Recently, different types of nanofluid flow in a rotating system have been studied by researchers[17–20]. The models for rotating fluid can be practically used if they are solved without sacrificing the relevant physical phenomena. Therefore, several numerical approaches and mathematical modeling have been developed for nanofluid flow[21–26]. The flow due to rotating disk is an ideal problem that has a limited number of applications.

Therefore, researchers extended the problem of rotating disk with time-dependent vertical movement[27]. The unsteady flow of nanofluid between two rotating disks has also been investigated by researchers[28–30].

The above literature review motivates the investigation of the unsteady flow in the presence of a disk. In the current study, the unsteady flow of water-based $CoFe_2O_4$ nanofluid over a rotating disk is presented. It is assumed that the disk rotates about z axis and moves vertically upward direction.

Similarity transformation is used to obtain a set of nonlinear coupled differential equations. The solution of the nonlinear coupled differential equations is obtained using the finite element method. The finite element simulation of the problem has been completed with the help of COMSOL Multiphysics.

The impact of Lorentz force, vertical movement of the disk, and rotation speed of the disk are studied on the velocity distributions.

The velocity distribution of $CoFe_2O_4$ nanofluid is compared with different nanofluids.

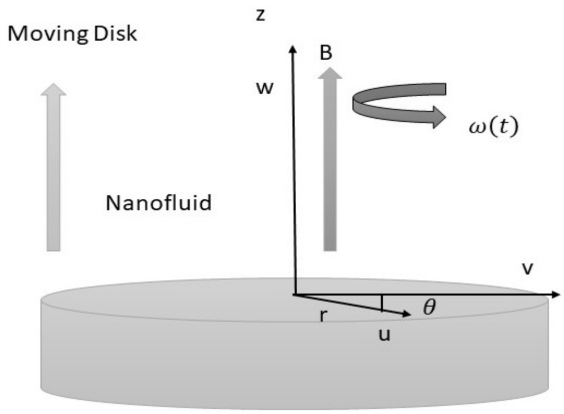

Figure 1: Flow configuration of nanofluid flow.

2. MATHEMATICAL FORMULATION

Figure 1 represents the flow configuration of the nanofluid flow over a rotating disk. The disk rotates about z axis and moves in the vertical direction. The components of velocity distributions are u, v, w in the directions of r, θ, z, respectively. The magnetic field is applied normal to the surface of the disk. The present flow is considered incompressible and axisymmetric. Therefore, the Navier-Stokes Equation for the present model in the cylindrical form is as follows:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r}\right] = -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial r} + \nu_{nf}\left\{\frac{\partial^2 u}{\partial z^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial r^2}\right\} - \frac{\sigma B^2}{\rho_{nf}}u \quad (2)$$

$$\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r}\right] = \nu_{nf}\left\{\frac{\partial^2 v}{\partial z^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial r^2} - \frac{v}{r^2}\right\} - \frac{\sigma B^2}{\rho_{nf}}v \quad (3)$$

The boundary conditions for the rotational flow with a vertical moving disk are as follows:

$$z = b(t); \quad u = 0,\ v = r\Omega(t), \quad w = \lambda\frac{db(t)}{dt};$$

$$z \to \infty; \quad u = 0, \quad v = 0 \quad (4)$$

In the above equations, u, v, w denotes the radial, tangential, and axial velocity distributions in the r, θ, z directions, respectively. p denotes the pressure, ν_{nf} denotes the kinematic viscosity of nanofluid, ρ_{nf} denotes the density of nanofluid, σ denotes the electric conductivity, B denotes the magnetic field, $b(t)$ denotes the position of the disk at time t, $\Omega(t)$ denotes the angular velocity of the disk, λ denotes the strength of wall permeability. Physical properties of the nanofluid can be used as:

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s, \quad \mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}} \quad (5)$$

φ denotes the volume concentration of the nanoparticle, ρ_f denotes the density of carrier liquid, ρ_s denotes the density of nanoparticles, μ_{nf} denotes the dynamic viscosity of nanofluid, μ_f denotes dynamic viscosity of carrier liquid.

The similarity transformation for the unsteady flow over a rotating and moving disk are as follows:

$$u = \frac{r\mu_f}{\rho_f b^2(t)}E(\eta),\ v = \frac{r\mu_f}{\rho_f b^2(t)}F(\eta), \quad w = \frac{\mu_f}{\rho_f b(t)}G(\eta),\ p = \frac{(\mu_f)^2}{\rho_f b^2(t)}P(\eta),\ \eta = \frac{z}{b(t)} - 1 \quad (6)$$

$E(\eta)$, $F(\eta)$, $G(\eta)$ and $P(\eta)$ represent the dimensionless radial velocity, tangential velocity, axial velocity, and pressure distribution.

Using Eq. (6), the governing equations of the model can be transformed into nonlinear coupled differential equations as:

$$\frac{dG}{d\eta} + 2E = 0 \quad (7)$$

$$\left[\frac{1}{(1-\varphi_1)^{2.5}\left(1-\varphi_1+\varphi_1\frac{\rho_s}{\rho_f}\right)}\right]\frac{d^2E}{d\eta^2} - G\frac{dE}{d\eta} + S\frac{(\eta+1)}{2}\frac{dE}{d\eta} + SE - E^2 + F^2 - M(1-\varphi_1)^{2.5}E = 0 \quad (8)$$

$$\left[\frac{1}{(1-\varphi_1)^{2.5}\left(1-\varphi_1+\varphi_1\frac{\rho_s}{\rho_f}\right)}\right]\frac{d^2F}{d\eta^2} - G\frac{dF}{d\eta} + S\frac{(\eta+1)}{2}\frac{dF}{d\eta} + SF - 2EF - M(1-\varphi_1)^{2.5}F = 0 \quad (9)$$

$$E(0) = 0, \quad F(0) = \omega, \quad G(0) = \lambda S,$$

$$E(\infty) = 0, \quad F(\infty) = 0 \quad (10)$$

The dimensionless quantiles and parameters used in the above equations are as follows:

$$b(t) = h\sqrt{\left(1 + \frac{\mu_{nf} S}{\rho_{nf} h^2} t\right)},\ S = 2\frac{\rho_{nf}\, b(t)\, \frac{db(t)}{dt}}{\mu_{nf}},$$

$$\omega = \frac{\rho_{nf}\omega(t) b^2(t)}{\mu_{nf}}, M = \frac{\sigma b^2(t)}{\mu_f} B^2 \qquad (11)$$

In the above equations, h denotes the initial position of the disk at $t = 0$, S denotes the expansion parameter for the disk movement and M denotes the magnetic interaction number.

3. NUMERICAL SOLUTION

The numerical solution of the nonlinear coupled differential equations is obtained through the finite element method. The equations are highly nonlinear therefore the normal mesh in COMSOL does not present a valid solution. For the grid-free solution, the element size is selected as 0.0001. Under the solution configuration, automatic highly Newton is selected and the maximum number of iterations is 25. The error in the solution is of the order 10^{-6}. If the solution is not still grid-free, the element size can be reduced until the accurate solution.

4.RESULTS AND DISCUSSION

Presents results show the distribution of radial, tangential, and axial velocity distributions for different values of physical parameters. The present problem represents the unsteady flow of nanofluid due to the vertical movement of the disk. At $S = 0$, the present problem reduces to an axisymmetric steady flow. The parameter S in the present flow measures the unsteadiness of the problem. However, at $\omega = 0$, the present problem represents only the vertical movement of the disk without any rotation of the disk. This case reduces the present flow into the two-dimensional flow. In the absence of the parameters S, φ, and M the present problem reduces to the original Von Karman swirling flow[5,6].

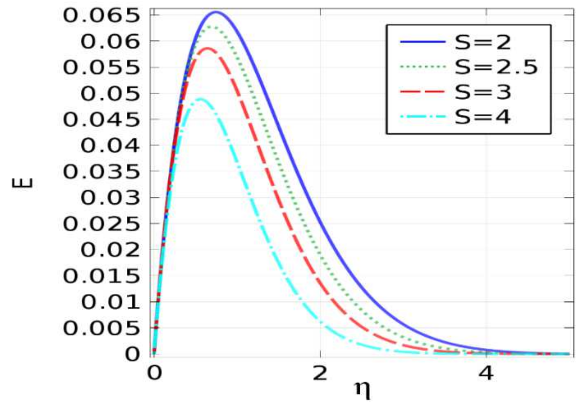

Figure 2: Radial velocity distribution for different values of S at $\varphi = 0.10, M = 2, \omega = 1.$

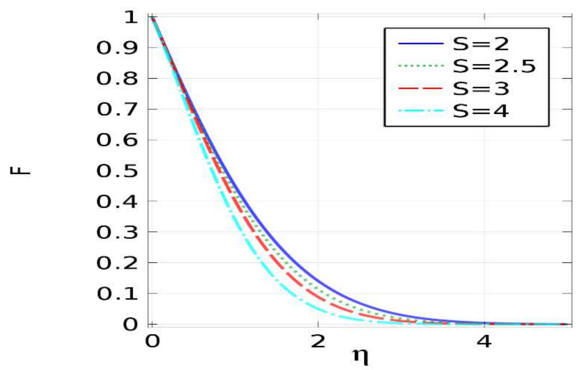

Figure 3: Tangential velocity distribution for different values of S at $\varphi = 0.10, M = 2, \omega = 1$

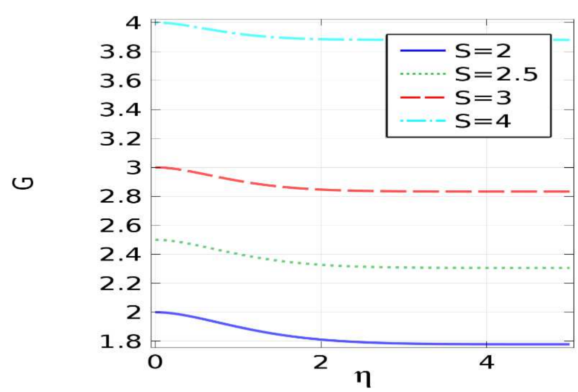

Figure 4: Axial velocity distribution for different values of S at $\varphi = 0.10, M = 2, \omega = 1$

Figures 2-4 show the radial, tangential, and axial velocity profiles for different values of S. Higher values of S represent the deceleration of rotation of the disk. Therefore, the radial and tangential velocities are reduced for higher values of S. The similarity solution is not shown for the negative values of S. The present results show that the similarity solution is valid in a local range not for all values of S. These results indicate that unsteady flow can be measured in a local region. Figures 5-7 represent the radial, tangential, and axial velocity distribution for different values of magnetic interaction numbers. At $M = 0$, the velocity distributions are shown without any influence of Lorentz force. However, escalating the values of M diminishes the radial, tangential, and axial velocity distributions. These results are evidence that the magnetic field in Navier-Stokes equations is directed in the opposite directions of the flow.

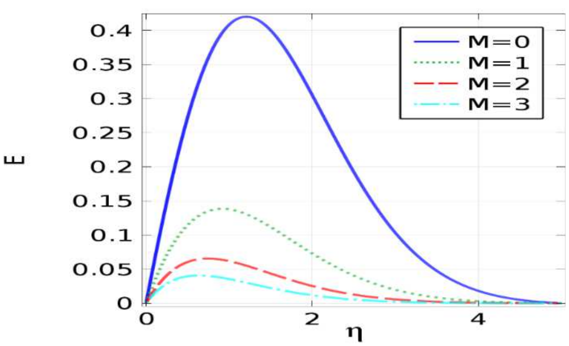

Figure 5: Radial velocity distribution for different values of M at $\varphi = 0.10, S = 2, \omega = 1$.

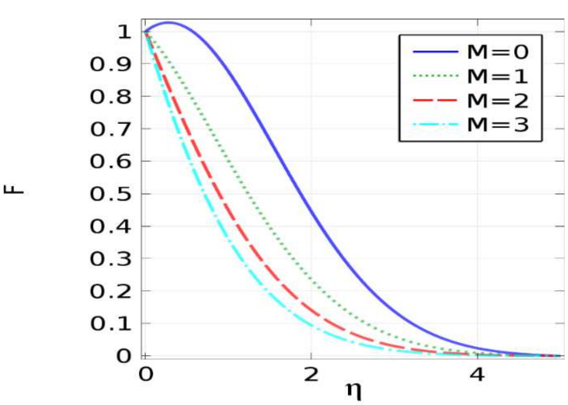

Figure 6: Tangential velocity distribution for different values of M at $\varphi = 0.10, S = 2, \omega = 1$.

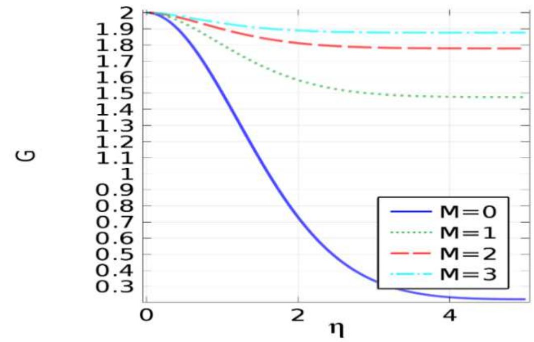

Figure 7: Axial velocity distribution for different values of M at $\varphi = 0.10, S = 2, \omega = 1$.

Figures 8-10 depicts the radial, tangential and axial velocity distributions for the different rotation speed of the disk. In the absence of the rotation, the three-dimensional boundary later flow reduces into two-dimensional boundary layer flow. Increasing the rotation speed enhances the impact of centrifugal force. This force creates a significant impact on the velocity distributions.

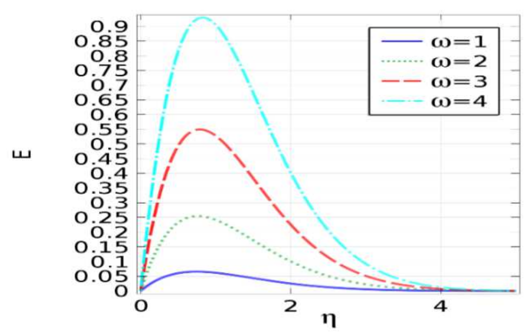

Figure 8: Radial velocity distribution for different values of ω at $\varphi = 0.10, S = 2, M = 2$.

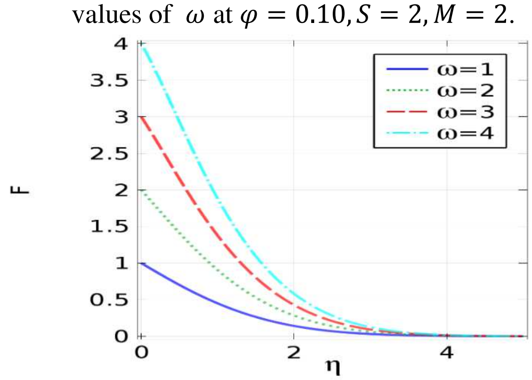

Figure 9: Tangential velocity distribution for different values of ω at $\varphi = 0.10, S = 2, M = 2$.

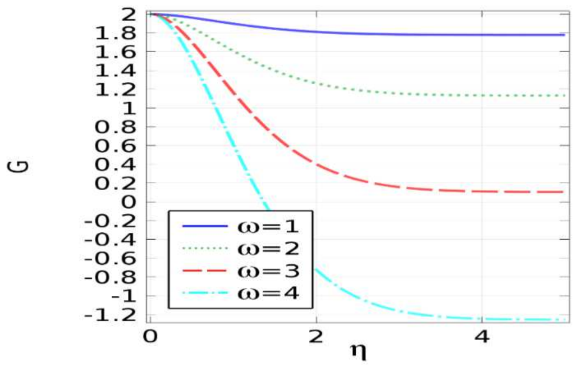

Figure 10: Axial velocity distribution for different values of ω at $\varphi = 0.10, S = 2, M = 2$.

Figures 11-13 shows the comparison of different nanofluids. In the present study, the base fluid is considered as water. The carrier liquids are mixed with $CoFe_2O_4$, Fe_2O_3, NiO and Co nano-particles, and the volume concentration of the nanoparticles are considered as 10%. The densities of these particles are 3594 kg/m^3, 5170 kg/m^3, 6670 kg/m^3 and 8860 kg/m^3, respectively. The densities of the carrier liquid are taken as 1024 kg/m^3. The present numerical results show that the nanoparticles having higher density have less radial, tangential, and axial velocities than the particles having less density. The density of the nanoparticles plays a significant role in the application nanofluid in rotating machinery.

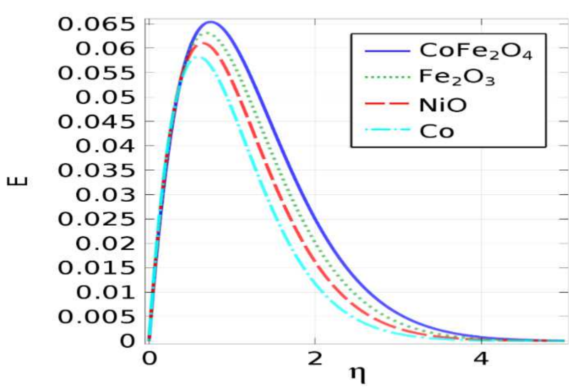

Figure 11: Radial velocity distribution for different nanofluids.

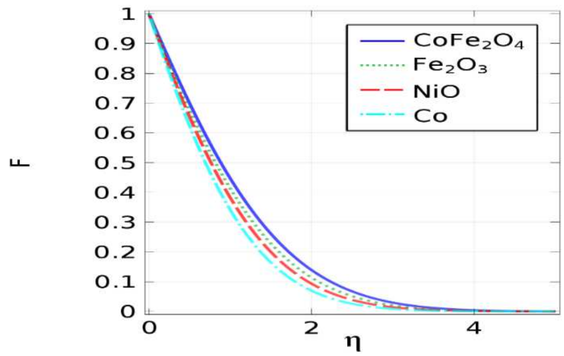

Figure 12: Tangential velocity distribution for different nanofluids.

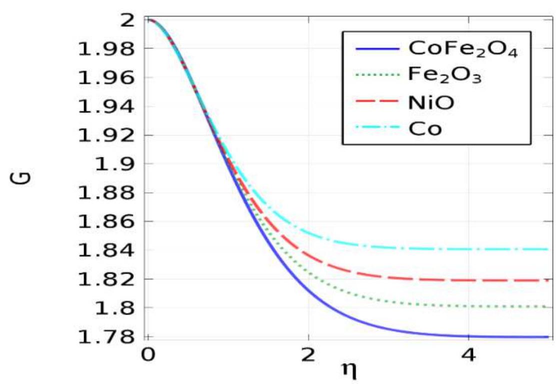

Figure 13: Axial velocity distribution for different nanofluids.

5. CONCLUSIONS

The flow of $CoFe_2O_4$ nanofluid over a rotating and moving disk is studied and the velocities of $CoFe_2O_4$ are compared withFe_2O_3, NiO, and Co nanofluid. The main findings of the present investigation are as follows:

- Increasing values of S decelerate the rotation of the disk.
  - Lorentz force plays a significant role in the reduction of velocity distribution since it is directed in the opposite direction of the flow.
  - Velocities can be increased by increasing the speed of the rotation of the disk.
  - Densities of nanoparticles play a crucial role in the velocity distributions.
  - The practical use of nanofluid in the rotating system can be optimized

through suitable types of nanoparticle selection.

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$CoFe_2O_4$ NANOFLUID DEBIT PESTE UN DISC ROTATIV ȘI ÎN MIȘCARE

Rezumat: *Modelul actual îşi propune să investigheze fluxul instabil de $CoFe_2O_4$ nanofluid peste un disc rotativ şi în mişcare sub influenţa câmpului magnetic. Forţa magnetică a corpului este direcţionată normal către disc. Transformarea similitudinii este utilizată pentru a transforma ecuaţiile de guvernare într-un set de ecuaţii diferenţiale cuplate neliniare. În absenţa unui câmp magnetic şi a mişcării verticale a discului, problema se reduce în von Karman problema fluxului de vârtej. Ecuaţiile diferenţiale cuplate neliniare din modelul actual sunt rezolvate prin metoda elementului finit folosind dimensiunea elementului ca 0,0001. În această simulare numerică, rezultatele pentru distribuţiile de viteză radiale, tangenţiale şi axiale sunt obţinute pentru diferite valori ale parametrilor fizici utilizaţi în model. Un studiu comparativ al diferitelor tipuri de nanofluid este prezentat în studiu şi comparativ cu $CoFe_2O_4$ nanofluid.*
Keywords: *Ecuaţii diferenţiale nanofluide, neliniare, debitinstabil, viteză, rotaţie.*

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