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THE PYR-ALGORITHM FOR TIME SERIES MODELING OF TEMPERATURE VALUES AND ITS APPLICATIONS ON FULL-SCALE COMPARTMENT FIRE DATA

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Abstract: *We suggest a new method for the analysis of experimental time series. The order/disorder characteristics of the compartment fire are determined based on experimental data. From our analysis, we claim that the newly developed so-called PYR-algorithm is suitable to detect unusual data in full-scale fire experiments.*

Key words: *full-scale fire experiment; compartment fire; permutation entropy; time series analysis; PYR-algorithm; Data Driven Method of Imputation; redistributing algorithm*

1. INTRODUCTION

We aim to develop a new algorithm adequate to perform a local entropic analysis of the evolution of the temperature during a full-scale fire experiment. We compare our results with those obtained with known algorithms dedicated to the extraction of the underlying probabilities and we check their suitability to point out abnormal values and structure of the experimental time series. We apply our methods on a single experimental data set, as an illustration. Notwithstanding such a limitation, our investigation contributes with a perspective to the overall analysis and finding of an adaptive direction, by observing the evolution of the fire through a mathematical lens that enables venturing in novel and testable proposals, as a subsequent need for sharpness and accuracy in contemporary analysis of experimental data. For other recent research on the fire phenomena performed using entropic tools see [16] and [20].
The experimental data was provided from a full-scale fire experiment conducted at Fire Officers Faculty in Bucharest. The experiment has been carried out using a container (single-room compartment) which had the following dimensions: 12 m × 2.2 m × 2.6 m. A single ventilation opening, the front door of the container, remained open during the experiment. The walls and the ceiling were furnished with oriented strand boards (OSB). The ignition burner has been a wooden crib, made of 36 pieces of wood strips 2.5 cm × 2.5 cm× 30 cm, on which has been poured 500 ml ethanol. The crib was placed at 1.2 m below the ceiling. Six built-in K-type thermocouples, fixed at key locations, connected to a data acquisition logger, were recording the temperature values.
A complete description of the experimental framework (materials and methods) and data analysis can be found in [15]. Other recent results on the analysis of this data set and a detailed mathematical background can be found in [13] and [14].
In Section 2 we present the notation, notions, algorithms needed to perform the analysis, and develop the new PYR-algorithm.
Section 3 is a detailed presentation of the results and their interpretation.

2. NOTATION AND TERMINOLOGY

2.1 The classic entropy and statistical complexities

Shannon's entropy [19] is defined as $H(P) = -\sum_{i=1}^{n} p_i \log\, p_i$, where $P = (p_1, \dots, p_n)$ is a finite probability distribution. It is nonnegative and its maximum is $H(U) = \log\, n$, where $U =$

$\left(\frac{1}{n}, \dots, \frac{1}{n}\right)$. The natural logarithm is used, and the convention $0 \cdot \log 0 = 0$.

The Kullback-Leibler divergence [9] is defined by

$$D(P\|R) = \sum_{i=1}^{n} p_i(\log\, p_i - \log\, r_i),$$

where $P = (p_1, \dots, p_n)$ and $R = (r_1, \dots, r_n)$ are probability distributions. It is nonnegative and it becomes zero only for $P = R$. This is a consequence of the known inequality $\log a \leq a - 1$ for all $a > 0$ (the equality holds only for $a = 1$).

If the value 0 appears in probability distributions $P = (p_1, \dots, p_n)$ and $R = (r_1, \dots, r_n)$, it must be found in the same positions for significance. Otherwise one considers the conventions $0 \log \frac{0}{b} = 0$ for $b \geq 0$ and $a \log \frac{a}{0} = \infty$ for $a > 0$. These limitations are strong, such conditions are rarely encountered in practice.

The Jeffreys divergence [8] is defined by $D^J(P\|R) = D(P\|R) + D(R\|P)$. The same restrictions and conventions apply.

The Jensen-Shannon divergence (see [11] and [17]) is

$$JS(P\|R) = \frac{1}{2} D\left(P \left\| \frac{P+R}{2}\right.\right) + \frac{1}{2} D\left(R \left\| \frac{P+R}{2}\right.\right) = H\left(\frac{P+R}{2}\right) - \frac{H(P) + H(R)}{2}.$$

It holds $JS(P\|R) \leq \frac{1}{4} D^J(P\|R)$ (see [5] and [11]).

The disequilibrium-based LMC statistical complexity [12] is defined as $C(P) = D(P)\frac{H(P)}{\log n}$, where the disequilibrium $D(P)$ is $D(P) = \sum_{i=1}^{n}(p_i - \frac{1}{n})^2$.

The *Jensen-Shannon statistical complexity* [10], [23] is $C^{(JS)}(P) = Q_{(JS)}(P)\frac{H(P)}{\log n}$, where the disequilibrium $Q_{(JS)}(P)$ is

$$Q_{(JS)}(P) = k \cdot JS(P\|U).$$

Here the normalizing constant is $k = (\max_P JS(P\|U))^{-1}$ and $U = \left(\frac{1}{n}, \dots, \frac{1}{n}\right)$. For its computation, the maximum is attained for P such that there exists $i, p_i = 1$.

Remark 1 These two statistical complexities are defined in different ways, that is with or without normalizing the disequilibrium. Throughout this paper we have obeyed the known definitions, but we stress that one could normalize $D(P)$ in the same way as $JS(P\|U)$ has been normalized. The maximum of $D(P)$ is attained for P such that there exists $i, p_i = 1$. The proof of this fact follows using the same recipe as in [14]. It holds $\max_P D(P) = \left(1 - \frac{1}{n}\right)^2 + \frac{n-1}{n^2} = \frac{n-1}{n} < 1$, hence $D(P)$ is bounded. To normalize by a bounded maximum is not relevant to our current analysis.

The possibility to use the *Jeffreys statistical complexity* $C^J(P) \equiv D^J(P\|U)\frac{H(P)}{\log n}$ has been briefly mentioned in [18] as an alternative to the Jensen-Shannon complexity measure. One encounters the following drawback: since U has no zero components, the probability distribution P must have only strictly positive components, a fact which depends on the algorithms used to determine the underlying probability distribution. For fire experiments, the collected data yields sometimes, by various algorithms, some underlying probability distributions which have zero components. An alternative is to use the Jeffreys-Ferreri statistical complexity, as discussed in our recent paper [13].

2.2 Extraction of the probability distribution out of experimental data

The permutation entropy PE [2] is used for the analysis of time series based on comparisons of neighboring entries. For details on the PE-algorithm applied to the present experimental data see [15].

Let $T = (t_1, \dots, t_n)$ be a time series with distinct values.

Step 1. The increasing rearranging of the components of each j-tuple $(t_i, \dots, t_{i+j-1})$ as $\left(t_{i+r_1-1}, \dots, t_{i+r_j-1}\right)$ yields a unique permutation of order j denoted by $\pi = (r_1, \dots, r_j)$, an encoding pattern that describes the up-and-downs in the considered j-tuple.

Example 1 For the 5-tuple (2.4, 1.7, 3.5, 1.2, 4.6) the corresponding permutation (encoding) is (4, 2, 1, 3, 5).

Step 2. The number of j-tuples associated to this permutation is

$$k_\pi \equiv \#\{i : i \leq n - (j-1), (t_i, \dots, t_{i+j-1}) \text{ is of type } \pi\}.$$

The sum of these values is equal to the total amount of all consecutive j-tuples, that is $n-(j-1)$.

Step 3. The *permutation entropy of order j* is defined as $PE(j) \equiv -\sum_{\pi} p_{\pi} \log p_{\pi}$, where $p_{\pi} = \frac{k_{\pi}}{n-(j-1)}$.

This algorithm does not handle the j-tuples which contain equal values. For such cases, common approach in the literature is to rank the ties according to their *order of emergence* (to rank them according to their chronological order). See for instance [4] and [6]. We use this method to compute the *permutation entropy* $PE(j)$, for j =3, 4. We consider that the chronological ordering is an artificial convention, disregarding the evolution of the phenomenon that originated the data on which we perform the analysis.

In [2] the values of the time series are considered distinct. The authors propose to break the equalities by adding small random perturbations. We interpret this as mapping the j-tuples with ties to all suitable (compatible) permutation with equal probabilities. This method is used for what we further call the *Bandt-Pompe entropy* denoted by $BP(j)$ for j =3, 4. In our interpretation, this type of ranking equalities is coarser, so we are not surprised to obtain higher entropies.

In this paper we aim to provide an adequate/enhanced permutation type (fire-dynamics based) algorithm for fire and check its validity on our experimental raw data.

Data-Driven Method of Imputation DDMI presented in [21] and [22] requires to map the j-tuples with ties to all the suitable (compatible) permutations according to the so-called *a priori* probabilities (i.e. the relative frequencies of those permutations obtained when ignoring the patterns with ties). Foremost, one can interpret this as counting each j-tuple which contains equalities on behalf of its compatible permutations, proportionally to the amount of j-tuples with no equalities corresponding to those permutations.

To prepare the theoretical background of our approach we present, for reader's convenience, the *modified permutation entropy* (mPE) introduced in [3]. For distinct temperatures, one applies the PE-algorithm. The equal values are mapped onto the same symbol, that is the smallest time index: if $t_{i+r_1-1} = t_{i+r_2-1}$, $r_1 < r_2$ then both temperatures are represented by r_1 in the encoding symbol sequence (not a permutation anymore, as for PE). Therefore, for every permutation π used by the PE-algorithm, which corresponds to the j-tuples containing $t_i \leq t_k$, we will have two encodings in the mPE-algorithm, one for the j-tuples containing $t_i < t_k$, the other one for $t_i = t_k$, and their probabilities have the sum $p(\pi)$.

Example 2 $(2.3,\ 1,\ 3.3,\ 1,\ 5.7) \to (2,\ 2,\ 1,\ 3,\ 5)$

Based on the mPE-algorithm, we describe further steps aiming to redistribute the j-tuples from the encodings with ties to the encodings that are permutations of order j.

Hereafter, the notation used is $\mathcal{S}$ for the set of permutations of order j, $\mathcal{E}$ for the set with all encodings in the mPE(j) algorithm (therefore $\mathcal{E} \setminus \mathcal{S}$ contains the encodings with ties). Let $\kappa \in \mathcal{E} \setminus \mathcal{S}$, $p(\kappa)$ be the probability of κ computed by the mPE-algorithm,

$$p(\kappa) = \frac{\text{number of } j\text{−tuples of type } \kappa}{\text{total number of } j\text{−tuples}}.$$

We call the permutation $\sigma \in \mathcal{S}$ and the encoding $\kappa \in \mathcal{E} \setminus \mathcal{S}$ *compatible* if each j-tuple $(t_i, \dots, t_{i+j-1})$ of type κ satisfies the condition $t_{i+\sigma(1)-1} \leq \cdots \leq t_{i+\sigma(j)-1}$. We write $\sigma \sim \kappa$.

Remark 2 Note that if $\sigma \sim \kappa$, then κ and σ coincide as functions $\{1, \dots, j\} \to \{1, \dots, j\}$, up to the terms involved in ties.

Example 3 The constant encoding is compatible to all the permutations.

Example 4 The encoding $(1,1,3)$ is compatible to the permutations $(1,2,3)$ and $(2,1,3)$. The encoding $(1,2,2,2)$ is compatible to the permutations $(1,2,3,4)$, $(1,2,4,3)$, $(1,3,2,4)$, $(1,3,4,2)$, $(1,4,2,3)$ and $(1,4,3,2)$.

The number of permutations that are compatible to a given encoding $\kappa \in \mathcal{E} \setminus \mathcal{S}$ is the product of the factorials of the numbers of terms involved in each tie in κ.

We denote by $\kappa^{\uparrow}$ (respectively $\kappa^{\downarrow}$) the permutations which are compatible to κ and rank increasingly/chronologically (respectively decreasingly/reversed chronologically) the terms corresponding to each tie.

Redistributing algorithms (Partial Weights Algorithms, PW-algorithms):

Step 1.

It holds $\sum_{\sigma\sim\kappa,\sigma\in\mathcal{S}} p^*(\kappa,\sigma) = 1$, where

$$p^*(\kappa,\sigma) \equiv \frac{\text{number of } j\text{−tuples redistributed from } \kappa \text{ to } \sigma}{\text{total number of } j\text{−tuples of } \kappa}.$$

It holds $0 \le p^*(\kappa,\sigma) \le p(\kappa)$. Several ways to compute $p^*(\kappa,\sigma)$ correspond to known algorithms, as we show in our remark below. The numerators (which reduce the problem of finding adequate partial probabilities $p^*(\kappa,\sigma)$) should depend more on the phenomena which originated the data, but currently their determination involves general procedures as choices researchers make.

Step 2. Obtaining the new probability distribution:

$$\mathcal{p}(\sigma) \equiv p(\sigma) + \sum_{\kappa\in\mathcal{E}\setminus\mathcal{S},\sigma\sim\kappa} p^*(\kappa,\sigma)p(\kappa),$$

for all $\sigma \in \mathcal{S}$. It holds $\sum_{\sigma\in\mathcal{S}} \mathcal{p}(\sigma) = 1$.

Remark 3 Cases of interest:

- The PE-algorithm (with chronological ordering of ties) is the case $p^*(\kappa,\kappa^{\uparrow}) = 1$ (deterministic assignment). It is counting the j-tuples containing an equality $t_i = t_{i+1}$ on behalf of the compatible permutation which has $t_i < t_{i+1}$.
- The (Bandt – Pompe) BP -algorithm has the partial probabilities $p^*(\kappa,\sigma) = \frac{1}{\text{card}\{\tau:\,\tau\sim\kappa\}}$ for all $\sigma\sim\kappa$. It is counting the j-tuples containing equalities on behalf of all compatible permutations, randomly, that is, with equal probabilities (otherwise said adding random noise to break equalities).
- We note that the DDMI-algorithm uses, as the above described redistributing algorithms, the weights $p^*(\kappa,\sigma) = \frac{p(\sigma)}{\sum_{\tau\sim\kappa} p(\tau)}$ for all $\sigma\sim\kappa$. We stress that the DDMI-algorithm fails if there exist encodings κ such that $p(\kappa) \neq 0$ and $\sum_{\tau\sim\kappa} p(\tau) = 0$. The DDMI-algorithm appears as a quite natural way to redistribute the patterns with ties, however one has to interpret it carefully: if all $p^*(\kappa,\sigma)$ are established based only on a small number of j-tuples with distinct elements and if the number of j-tuples of type κ is significantly greater, this proportional redistributing method might exhibit some issues (as hiding the rare patterns which could characterize the phenomenon) and prove itself less relevant as expected.

When the number of the ties is small in comparison to the amount of measurements, all these algorithms yield similar results, as it will be visible in the analysis of our experimental data.

Obviously, these algorithms do not consider the nature of the phenomenon which originates the time series, a fact which would provide more insight for the researcher and would indicate in the most intuitive way how to deal with the j-tuples which contain equal values.

Remark 4 The coarsest redistributing algorithm we can think of is to *randomly redistribute* all the j-tuples with ties *to all* the existing $j!$ permutations (that is, to ignore the non-compatibilities). Another approach is to redistribute all the j-tuples with ties to all the existing $j!$ permutations, according to their a priori probabilities (as in the DDMI-algorithm). This is quite trivial, since it is equivalent to simply ignore all the j-tuples with ties (that is, one does not redistribute, but removes all the j-tuples with ties). Both methods provide only simplified and rough information on the data, with much information loss.

Remark One obtains the same results from the BP- and DDMI- algorithms in case when the permutations are $p-$ equiprobable (that would be an interesting time series, if obtained from some real-life measurements), since by both algorithms they become $\mathcal{p}$ −equiprobable. More mathematical conditions to get similar/same results from different algorithms can be inferred by the interested reader, but this goes well beyond our purpose here to discuss in detail, since such assumptions are unlikely to hold in the fire framework.

The class of PW-algorithms was inspired to us by these particular cases. However, according to the phenomenon under consideration (case when additional constraints and requirements may arise), researchers might need to review the definition of the partial probabilities $p^*(\kappa,\sigma)$ to fit a specific framework, to interpret the distinguishability of phases of the process and thus reveal the salient structural properties of the time series.

Since our further approach is based on the widely known evolution of the fire events, we will call it the *PYR-algorithm* (in Greek language, πῦρ means fire). We describe its encoding step below. Figure 1 shows the idealized time-temperature curve of the stages of a compartment fire. The lower curve shows a quasi-steady low-intensity fire. See [7].

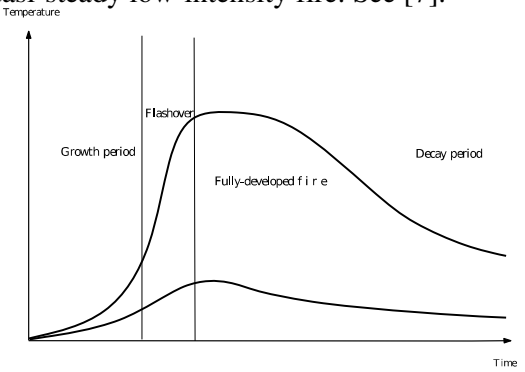

Fig. 1 Idealized time-temperature curve

Step 1. Let $k = \min\{s: t_s = \max(t_1, \dots, t_n)\}$. Each j-tuple $(t_i, \dots, t_{i+j-1})$ of type $\kappa \in \mathcal{E} \setminus \mathcal{S}$, is counted as a j-tuple of type $\kappa^{\uparrow}$ if $i < k$, otherwise it is considered on behalf of the permutation $\kappa^{\downarrow}$. So, we redistribute the j-tuples with ties to the two compatible permutations with either chronological or reversed chronological ranking of the terms involved in the ties. This step is a fine tuning of the previous encoding procedures, an adjustment made to fit specific types of data, inspired by the generally known evolution of the fire: the j-tuple is considered on the ascending trend before the maximum value of the temperature is reached, respectively on the descending trend afterwards. The basic idea behind our approach is to avoid generating unintended new fluctuations which would increase the erraticism of the time series.

Remark 5 Note that the tie $t_l = t_m,\ l < m$, in the j-tuple $(t_i, \dots, t_{i+j-1})$ is considered as $t_l < t_m$ if $i < k$, and $t_l > t_m$ if $i \geq k$, that is the same tie could have a different look in different j-tuples by applying the PYR-algorithm, a negligible issue of our method (it can happen at most $j - 2$ times). However, one could additionally check if $m < k$, if yes then order the tie chronologically, and if $l \geq k$ reverse the order. Still the case $l < k < m$ would remain not settled. We did not encounter this issue with our experimental data (therefore this remark is purely theoretical, for the interested reader), however we prefer the simplifying Step 1 (to check only if $i < k$, instead of scanning for the ties of each j-tuple containing t_k).

Step 2 and **Step 3** coincide with Step 1 and Step 2 in the PE-algorithm in the beginning of this section.

The resulting probability distribution may roughly but less arguably analyze the evolution of the temperature, and it turns our attention towards the features of the phenomenon originating the time series.

By the redistribution of the patterns with ties, in all four algorithms we have used, the resulting probability distribution is associated to $j!$ permutations. Note that the BP-, DDMI- and PYR-algorithms give the same result as the PE-algorithm for time series with distinct values or characterized by the sparsity of ties, that is by a small number of j-tuples with ties (a fact which we note at the thermocouple T5). The PYR- and PE- algorithms yield closer results when the ties are more frequent during the growing period of the fire.

Remark 6 An important difference between the DDMI- and the other discussed algorithms is that the permutations with 0 associated j-tuples in the mPE-algorithm remain with 0 associated j-tuples as a result of the DDMI-algorithm (with proportional redistribution), while this might (and usually does) change after the BP-, PE-, or PYR- algorithms (we encounter this case in Figure 9).

Remark 7 In case that the permutations $\sigma = (1,2, \dots, j)$ and $\tau = (j, j-1, \dots, 1)$ have 0 associated j-tuples after the mPE-algorithm, that is if there is no strictly increasing/decreasing j-tuple, we stress that *a bigger embedding dimension* j cannot solve this issue, while *a coarser time scale* might lose valuable information. For fire experiments, the DDMI-algorithm would *never be relevant* in this case, since one always anticipates the greatest numbers of associated j-tuples to the permutations σ and τ, therefore having 0 associated j-tuples would not agree to the real evolution of the temperature. However, in this case we intuitively claim that the PYR-algorithm would remain the most realistic approach, the

BP-algorithm would randomly redistribute the j-tuples with ties to σ and τ and probably also to other permutations, while the PE-algorithm would maintain 0 associated j-tuples to the permutation τ, another unrealistic approach from our perspective.

Remark 8 In case that all the permutations have at most 1 associated j-tuple after the mPE-algorithm, except eventually σ and τ, we consider that, if only a small number of encodings have ties, then a finer time scale could have been performing a better scanning of the evolution of the temperature.

Note that applying these algorithms to experimental fire data, the statistical complexities cannot be zero. The number of the encoding patterns which occur is > 1 and these patterns are not equiprobable: some patterns may be rare or locally forbidden (that is, one encounters such patterns at some thermocouples, but not in all time series), as discussed in [15].

In the next section we apply the above techniques and observe some meaningful aspects concerning the experimental data.

3. EXPERIMENTAL DATA ANALYSIS

The raw data set consists of six time-series having 3047 entries each. We are modeling the time series using information theory, and to assess the performance of the discussed statistical complexities and the PYR-algorithm.

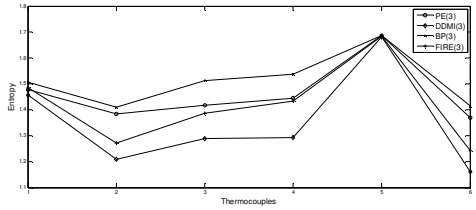

Fig. 2 The entropy for the embedding dimension $j = 3$

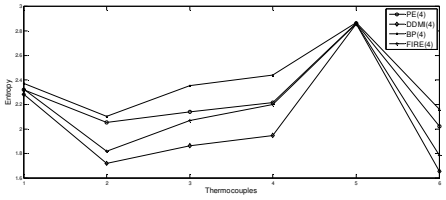

Fig. 3 The entropy for the embedding dimension $j = 4$

The most relevant aspect both for the entropies (Fig. 2 and Fig. 3) and for the statistical complexities (Fig. 4-9) is that we get similar plots regardless the embedding dimension and the encoding type algorithms, hence the analysis is reliable.

The results at T5 lie very close together, due to the small number of j-tuples with ties (j=3 and j=4), for all the discussed algorithms. (Table 1).

Table 1 - Number of j-tuples with ties

	T1	T2	T3	T4	T5	T6
j=4	513	1031	1371	1463	191	1172
j=3	319	756	1032	1135	109	903

As a measure of uncertainty, we see in Fig. 2 and Fig. 3 bigger entropy for the random redistributing (using equal weights) BP-algorithm, and smaller when the redistribution is proportional, with the DDMI-algorithm, which agrees with our intuition.

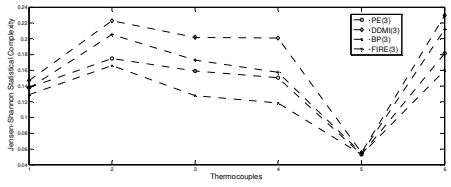

Fig. 4 Jensen-Shannon complexity for $j = 3$

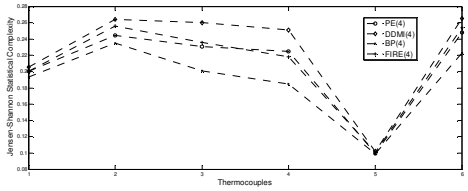

Fig. 5 Jensen-Shannon complexity for $j = 4$

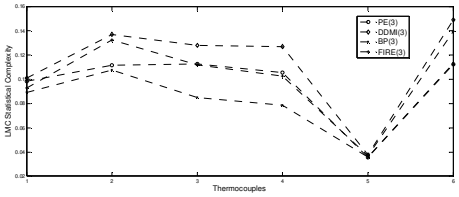

Fig. 6 LMC complexity for $j = 3$

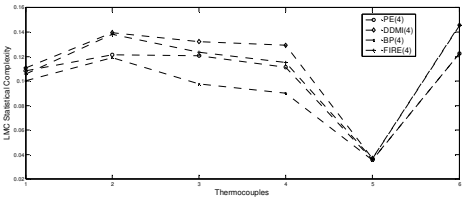

Fig. 7 LMC complexity for $j = 4$

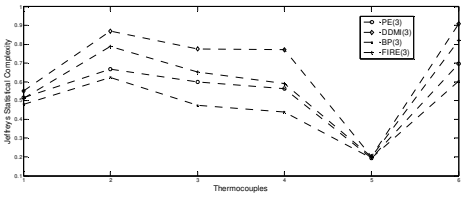

Fig. 8 Jeffreys complexity for $j = 3$

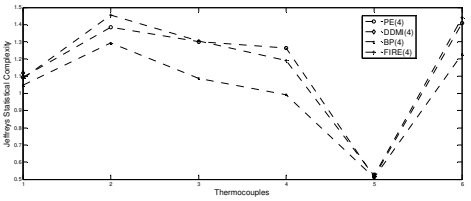

Fig. 9 Jeffreys complexity for $j = 4$

In Figure 9 we note the absence of the dotted line for DDMI(4), plotted only at T1 and T5, a fact due to the infinite values of the Jeffreys complexity at the thermocouples T2, T3, T4, T6. The DDMI-algorithm returned some 0 components in the new probability distribution. This is explained by the fact that the permutations with 0 associated j-tuples always remain, by the DDMI-algorithm, with 0 associated j-tuples. Note that the Jeffreys statistical complexity failed for an algorithm and worked for others, so the choice of the algorithm should not be underestimated.

The PYR-algorithm is not meant to change (and in our case it does not, but theoretically it could change it) the hierarchy established by other algorithms, we consider that it has the great advantage that one can trust more such procedures developed having as a starting point the phenomenon which originated the data. To ignore how the data has been produced is what all the future research should avoid.

Remark 9 Our analysis is based on several known statistical complexities: LMC, Jensen-Shannon and Jeffreys. It might be relevant sometimes to exploit only their corresponding disequilibrium formulae, which are interpreted in the recent literature as (roughly speaking) distances from the white noise. See for instance [1], where the author uses the LMC disequilibrium $D(P)$, with the probability distribution P obtained via the PE-algorithm.

To understand the cause of the tied values, we note that most of the j-tuples with ties appear in the decay period, however the corresponding percentages are quite different and we cannot conclude if this characterizes the fire dynamics or it should/could be explained as an artifact, an effect of confounding factors or by some inertia of the measuring devices. See Table 2. We let this as an open question for the interested reader.

Table 2 - Percentages of j-tuples with ties during the growth period, calculated from the total number of j-tuples with ties encountered at each thermocouple (see Table 1)

	T1	T2	T3	T4	T5	T6
j=4	20.47%	17.07%	36.76%	38.28%	56.02%	44.88%
j=3	24.14%	17.46%	39.83%	41.41%	55.96%	49.94%

4. CONCLUSIONS

The keys to evaluate experimental results is the appropriate selection of statistical data analysis techniques. Our main concern here are the experimental time series which contain ties, therefore we present particular cases of interest among the redistributing algorithms. In our framework, the PYR-algorithm provides the most suitable method and we use experimental data collected from a full-scale experiment to illustrate its enhanced fitness.

It is also noteworthy that our results indicate unusual data or an improperly calibrated at the thermocouple T5.

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PYR-algoritm pentru seriile de timp care modelează valorile temperaturii și aplicaţii ale lor pe date obţinute din incendiu de compartiment la scară reală

Dezvoltăm o nouă metodă pentru analiza seriilor de timp experimentale. Caracteristicile de ordine/dezordine ale unui incendiu de compartiment sunt investigate pe baza datelor experimentale. Din analiza noastră, concluzionăm că noul algoritm dezvoltat, numit PYR-algoritm, este adaptat să detecteze date neobişnuite în experimentele la foc.

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