TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering
Vol. 63, Issue IV, November, 2020

MODELING SCENARIOS FOR RANDOM TIMES AVAILABILITY OF THE PRODUCTION SYSTEMS USING MARKOV CHAINS METHOD

Elena-Iuliana GINGU (BOTEANU), Miron ZAPCIU

Abstract: *Markov chains models are widely applied in industrial engineering. In this paper, Markov chains are used to achieve modeling scenarios for the maintenance parameters and performance evaluation of production systems. The Markov chains are built with the queuing theory and are well-known for their power of representation by given a small computing effort. This paper is focused on reveal some random behaviors with the help of modeling scenarios. Our contribution consists in the development of two modeling scenarios. In the first scenario, the availability of a system with the setup phase and scraps is calculated, then the machine behavior in a period of a month is evaluated using failure and repair rates generated by Linear Congruential Generator. In the second scenario, the availability of a system with the setup phase without scraps is designed and evaluated.*

Key words: *Modeling scenarios, Markov chains, estimation, availability*

1. INTRODUCTION

In the last years, production systems have been extensively studied. Most of the studies analyze random times with different methods of modeling and simulation. The basic idea is to analyze the behavior of production systems and, then, to design and implement scenarios of modeling in order to detect the maintenance parameters. A well-known method used for this purpose is the Markov Chains method. Several papers have been written in literature.

1.1 Literature Review

Gershwin S. et al. [1] considered a model with two-machine and one-buffer, and they calculated the transitions among up and down states with Markov chains. Borcsok J. et al. [2] proposed a method to calculate the MTTF (Mean Time To Failure) values with the help of Markov models. In [3], P. Fernandes et al. developed an algorithm based on a Markovian and Kronecker representation for production lines, that automatically generates the equivalent SAN (Stochastic Automata Networks) model for any K-station production line. S. M. Meerkov et al. [4] made an analytical and numerical examination of the transient matrix behavior of two-machine Bernoulli lines. They applied the Second Largest Eigenvalue (SLE) to evaluate the transients of the probabilities of buffer occupancy. For the transients of the production rate and WIP (Work-in-Process), they analyzed the problems with the Second Largest Eigenvalue (SLE) and Pre-Exponential Factors (PEF). In order to analyze the transient behavior of production systems, F. Ju et al. [5] studied a serial production line model with two Bernoulli reliability machines and a finite buffer. K. Takahashi et al. [6] used Markov chain to compare the performances between KCS (kanban control system) and the two kinds of DBR (drum-buffer-rope) systems. N.E. Abboud [7] applied a Markov production-inventory model to develop an algorithm for the cost function estimation. X. Wang et al. [8] investigated a manufacturing system with two series machines and a finite buffer. They proposed an algorithm using a semi-Markov decision process to obtain the control-limit maintenance. N. Nahas [9] proposed an EGD (Extended Great Deluge) algorithm to determine the optimal preventive maintenance and also, the optimal buffer allocation that will maximize the system throughput level. M. Regard [10] modeled a serial manufacturing line using Markov chain optimization methods with the purpose of the machine downtime diminution using the targeting problem. A. Matta et al. [11] presented a method to calculate the steady-state probabilities of the manufacturing system, which depends only on the failures and not on the buffer capacity. The behavior of the system was modeled with a discrete-time discrete-state Markov chain. Y. Guoa et al. [12] developed an experimental model for the evaluation of corrective and preventive maintenance. The scheduling maintenance objective was to minimize the schedule duration.

1.2 Outline and our contributions

Main goal of the paper. In this article, we focus on the Markov chain method in order to analyze the random times in the production lines. The main goal of this paper is to develop modeling scenarios with the purpose of analyzing the behavior of the system, by computing the availability of the line. We address these studies for the availability of a system and, as well as, for the failure rates and repair rates, because they are an everyday occurrence in industry and are particularly relevant to practitioners.

Structure of the paper. This paper is organized as follows: Section 1 presents the aim of the article and reviews the related literature. A general description of the random numbers and Linear Congruential Generator, the stochastic processes and the reliability theory are described in Section 2. In Section 3, we propose different scenarios of modeling necessary to evaluate the performance of particular production systems called lines. More precisely, we analyze some problems in these production lines and, we consider that it is useful to focus on random times. We describe two scenarios: modeling scenario for the availability of a machine with the setup phase and scraps and modeling scenario for the availability of a machine with the setup phase without scraps, and then, we propose the solutions for each scenario. For these scenarios, a Linear Congruential Generator is introduced in order to generate random failure and repair rates for a month and then, to evaluate the machines' behavior. Finally, we conclude and discuss further development ideas in Section 4.

1.3 Notations and assumptions

- λ is the failure rate
- μ is the repair rate
- $X = \{x_1, x_2, ..., x_r\}$ represent the system states
- p_{ij} represents the transition probability = transition rate $\cdot$ time interval
- dt represents the incremental time interval sufficiently small so that probability of two or more transitions during the interval is negligible
- λ_{ij} represents the transition rate from the state x_i to x_j
- $\pi_i(t) = P[X(t) = x_i]$, and the row vector: $\pi_i(t) = (\pi_1(t), \pi_2(t), ..., \pi_n t)$ is a probability distribution.

2. MODELING RANDOM TIMES METHODOLOGY USING MARKOV CHAINS

According to [2,13,14,15,16,17], we introduce a general mathematical description of random numbers, stochastic processes and reliability theory.

2.1 Pseudo-Random Number Generator

In many real production systems, accidental events appear. The time intervals between the arrival of parts in a flowline (IAT - Inter Arrival Time), the time to repair a machine (MTTR - Mean Time To Repair), or the time between failures (MTBF - Mean Time Between Failure) are not often predictable and cannot be fixed in a model. A solution to describe accidental events in a model could be random numbers.

Random Number Generator provides random numbers to generate independent and identically distributed (i.i.d.) numbers that are uniformly distributed in the interval (0; 1). Failure and repair times are independent and identically distributed with an exponential distribution.

In a Discrete Event Simulation, a common family of Random Number Generator algorithms is the class of congruential generators. Linear Congruential Generator is a well-known pseudo-random number generator algorithms. The algorithm of the generator starts with the seed value, and then the next values can be predicted, totally determining the sequence. The length of the sequence before repeating itself is very long, more than 2 billion in a 32-bit computer. For a good source of random numbers, it is necessary to involve a mathematical transformation of uniform random numbers.

Definition 1. The Linear Congruential Generator is defined by the relation:

$$X_{n+1} = (aX_n + c) \bmod m \quad (1)$$

where, X represents the sequence of pseudo-random values; m represents the modulo operation; a represents the multiplier; c represents the increment; X_0 represents the *seed*, i.e. the start value.

2.2 Basic Theory of stochastic Processes

A Markov chain is a stochastic process, which starts in one of the states $X = \{x_1, x_2, ..., x_r\}$ and moves successively from x_i to x_j with a probability denoted by p_{ij}.

To compute τ_{ij}, the average total amount of time, which is spent in the state x_j, when the initial state is x_i, let us consider the functions $p_{ij}(t) = P[X(t) = x_j / X(0) = x_i]$.

Definition 2. The average value is defined as:

$$\pi_{ij} = \int_0^{\infty} p_{ij}(t)dt \quad (2)$$

Definition 3. A Markov chain is defined by the formula:

$$\begin{aligned} &P[X(t_k) = x_k / X(t_{k-1}) = x_{k-1}, ...X(t_1) = x_1] = \\ &P[X(t_k) = x_k / X(t_{k-1}) = x_{k-1}] \end{aligned} \quad (3)$$

where, $P[./.]$ is the usual notation for a conditional probability, $X = \{x_1, x_2, ..., x_k\}$ is the domain of the variable and $t = \{t_1, t_2, ..., t_k\}$ is a parameter, $t_1 \le t_2 \le ... \le t_k$.

Definition 4. The graph of a Markov chain represents the diagram that has n vertices, with a corresponding state.

Definition 5. The balance equation at state x_i is:

$$\sum_{i,j=1}^{n} \pi_i \lambda_{ij} = \sum_{k,l=1}^{n} \pi_l \lambda_{lk} \quad (4)$$

Definition 6. The normalization equation is:

$$\sum_{i,=1}^{n} \pi_i = 1 \quad (5)$$

2.3 Mathematical description of Reliability Theory

Definition 7. The reliability function $R(t)$ is represented by the relation :

$$\begin{aligned} &R(t) = P[\textit{There is no failure on } [0,t] / \\ &\textit{The system works at } t = 0] \end{aligned} \quad (6)$$

Definition 9. The maintainability function is presented as:

$$\begin{aligned} &M(t) = 1 - P[\textit{There is no repair on } [0,t] / \\ &\textit{The system is down at } t = 0] \end{aligned} \quad (7)$$

Definition 10. The availability function is:

$$A(t) = P[\textit{The system works at } t = 0] \quad (8)$$

3. MODELING SCENARIOS ANALYSIS OF RANDOM TIMES - CASE STUDY

According to [15,18], we set the numerical data for a case study, a manufacturing line of a car headrest support work-piece with the following operations (Figure 1):

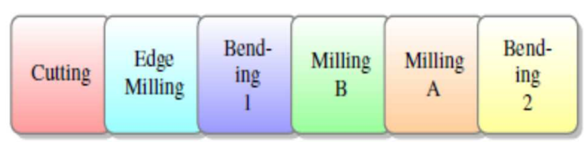

Fig. 1. Manufacturing line of the headrest support

The production rates for each machine from the case study are presented in Table 1.

Table 1

Production rate for each machine (parts/minute).

Operation	Production rate τ
Cutting	12.5
Edge Milling	7.01
Bending 1 & 2	4.55
Milling A & B	5.83

In this section, we introduce two modeling scenarios that have been considered for the analysis of random times and availability. These scenarios describe the calculation steps of the availability for a machine with the setup phase and with/without scraps. Then, the availability is calculated for each machine of the manufacturing system and the availability of the whole line is considered as an average of these results.

3.1 Modeling scenario for the availability of a system with the setup phase and scraps

To achieve a work-piece on a machine, it is necessary to execute a setup phase and then, the cycle time is achieved. These times are exponentially distributed with the rates τ and ν.

When the machine is working (in the state busy), the machine could fail with rate λ. When the machine is down, a repair is realized with the rate μ. If a failure happens, we assume that the work-piece under processing has lost (it is a scrap). We also assume, there are always raw work-pieces available before the machine.

For this scenario, we choose the bending 2 operation with the production rate $\tau = 4.55$, the setup rate $\nu = 1$, failure rate $\lambda = 0.00694$ and repair rate $\mu = 0.0924$ (the observation data). We need to represent by a Markov chain the behavior of the system and then to compute the availability of the system.

There are three states :

- x_1: The Machine is in set up phase,
- x_2: The Machine is working,
- x_3: The Machine is down and then is being repaired.

The graph of the Markov chain is designed in Figure 2.

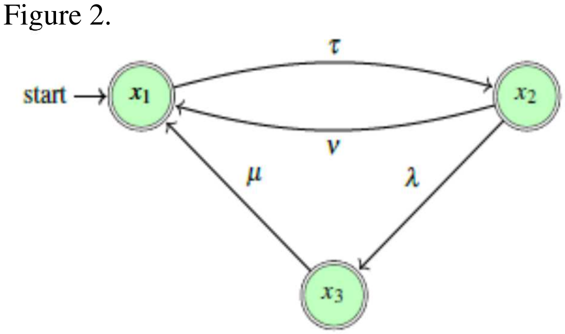

Fig. 2. Markov model with part loss

We write the balance equations (4) for states x_1 and x_3:

$$\pi_1 \tau = \pi_2 \nu + \pi_3 \mu;$$
$$\pi_3 \mu = \pi_2 \lambda;$$

with the normalizing equation (5):

$$\pi_1 + \pi_2 + \pi_3 = 1,$$

and using the numerical values of the problem, we will get:

$\pi_1 = 0.170706$, $\pi_2 = 0.771359$, $\pi_3 = 0.0579354$.

The availability of the system is:

$$A = (1 - \pi_3) = 0.942065 .$$

This modeling scenario was realized only for one failure with the corresponding repair rate to show the calculus methodology of the maintenance parameter.

In this article, we aim to evaluate the machine behavior in a period of one month using a random number generator, a Linear Congruential Generator, which produces random failure and repair rates.

We used the following setting for the LCG:

- the source: C++11's minstd_rand;
- the Minimal Standard minstd_rand generator;
- the modulus $m = 2^{31} - 1$;
- the multiplier $a = 48271$;
- the increment $c = 0$;
- the seed: for the failure rates and the repair rates, the seeds are considered as the observation data.

We applied the following seeds (the failure and repair rates observed): for cutting operation 0.00307 and 0.147, for edge milling operation 0.00892 and 0.0509, for bending 1 & 2 operations 0.00694 and 0.0924 and for milling A & B operations 0.00107 and 0.127.

The data for bending 1 & 2 operation are generated in Appendix 1.

The average availability produced in a month for the bending machine is:

$$A_{bending} = \frac{\sum_{i=1}^{30} A(i)}{30} = 0.921227.$$

Using C++ programming with modeling methodology and failure and repair rates generated by LCG, we show the availabilities of each machine produced in a month described in Figure 3.

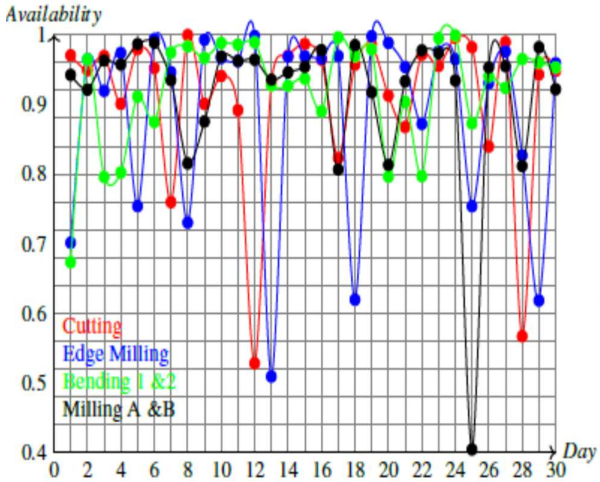

Fig. 3. The availabilities for modeling scenario with scraps

So, the availability for each machine and the results are showed in Table 2.

Table 2

Availability for each machine.

Operation	Availability
Cutting	0.910840
Edge Milling	0.890617
Bending 1 & 2	0.921227
Milling A & B	0.915206

The availability of the production line is calculated as an average of the availability for each machine, taking into account the set-up phase, the production rate, the failure rate and the repair rate:

$$A = \frac{\sum_{i=1}^{6} A(i)}{6} = 0.912387 .$$

3.2 Modeling scenario for the availability of a system with the setup phase without scraps

How is the Markov chain modified, if after a repair we presume that the operation on the work-piece was processed before the failure?

The new Markov chain graph is designed in Figure 4.

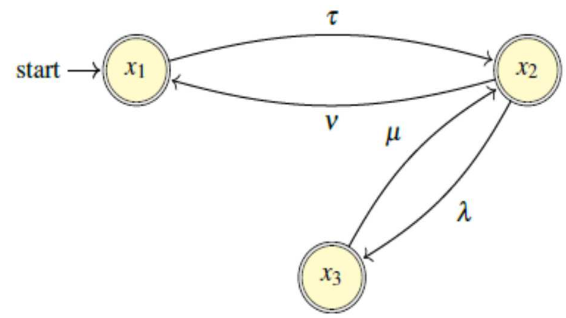

Fig. 4. Markov model with part machining

We write the balance equations (4) for states x_1 and x_3:

$$\pi_1 \tau = \pi_2 \nu;$$
$$\pi_3 \mu = \pi_2 \lambda;$$

with the normalizing equation (5):

$$\pi_1 + \pi_2 + \pi_3 = 1.$$

For this modeling scenario, we choose the milling A operation, using the numerical values of the problem, the production rate $\tau = 5.83$, the setup rate $\nu = 1$, failure rate $\lambda = 0.00107$ and repair rate $\mu = 0.127$.

We will get :

$\pi_1 = 0.145367$, $\pi_2 = 0.847492$, $\pi_3 = 0.00714029$.

The availability of the system is:

$$A = (1 - \pi_3) = 0.99286.$$

Using the methodology described above, we calculated the availability for each machine in a period of one month and the result are described in Table 3.

Table 3

Availability for each machine without scraps.

Operation	Availability
Cutting	0.913330
Edge Milling	0.910627
Bending 1 & 2	0.940382
Milling A & B	0.909652

The seeds chosen for the generator are: for cutting operation 0.00688 and 0.0627, for edge milling operation 0.00889 and 0.0295, for bending operation 0.00469 and 0.0773 and for milling A operation 0.00107 and 0.127.

We described in Appendix B the failure rates and the repair rates generated by Linear Congruential Generator for the Milling A & B operation.

The machine behavior in a period of one month is presented in Figure 5.

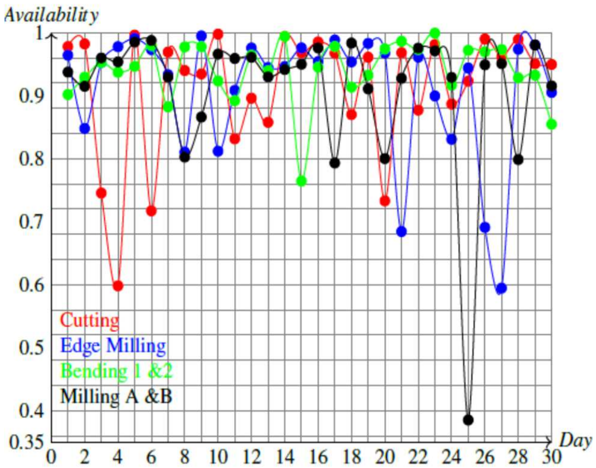

Fig. 5. The availabilities for modeling scenario without scraps

The availability of the production line will be:

$$A = \frac{\sum_{i=1}^{6} A(i)}{6} = 0.920671.$$

Therefore, the availabilities of the two experiments are predicted based on random failure and repair rates generated by Linear Congruential Generator. For that, a maintenance plan can be generated considering the production schedule for production-maintenance synchronizing purposes.

4. CONCLUSION

In this study, Markov chains have been analyzed in order to model the random times necessary to evaluate the availability of a production line. For that, we proposed two modeling scenarios and the calculation steps of the availability has been presented, detailed and examined using random values for failure and repair rates generated by the Linear Congruential Generator. The main aim of this article was to consider different scenarios in order to test a lot of ideas and alternatives using the concept: *What-if?*... We wanted to reveal the machines behavior and how much can be influenced the availability of the system depending on the random failure and repair rates. The two scenarios presented in this article evaluated the machines' behavior in a period of one month, and predicted the availability of the production line; 0.912387 for the modeling scenario with the setup phase and scraps and 0.920671 for the modeling scenario with the set up phase without scraps.

The methodology presented to create Markov chains models, which may be used to determine solutions in revealing the production system behavior and maintenance parameters for flow lines within a reasonable time achieved the goal of the paper. These results led us to believe that the integration of the Markov chains method and C++ programming in order to predict the maintenance parameters would provide interest to researchers and practitioners alike.

This paper and the modeling scenarios provided could be useful to practitioners where the machine failure and maintenance are factors in performance and afford a basis for further experimental study.

For new research directions, the extension of the modeling scenarios to longer lines with repairmen and assembly systems is proposed.

5. ACKNOWLEDGEMENT(S)

This work has been funded by the European Social Fund from the Sectoral Operational Programme Human Capital 2014-2020, through the Financial Agreement with the title "Scholarships for entrepreneurial education among doctoral students and postdoctoral researchers (Be Antreprenor!)", Contract no. 51680/09.07.2019 - SMIS code: 124539.

The authors would like to thank Diana Maimut and Daniel Popescu for their helpful comments.

6. REFERENCES

[1] Gershwin S., Fallah-Fini S., *A General Model and Analysis of a Discrete Two-Machine Production Line*, Analysis of Manufacturing Systems, AMS, 1-6, 2007.

[2] Borcsok J., Ugljesa E., and Machmur D., *Calculation of MTTF values with Markov models for safety instrumented systems*, Proceedings of the 7th conference on WSEAS international conference on

applied computer science, 14 , 30-35, 2007.

[3] Fernandes P., O'Kelly M., Papadopoulos C., *Sales A., Analysis of exponential reliable production lines using Kronecker descriptors*, International Journal of Production Research, 51 , No 14,4240--4257, 2013.

[4] Meerkov M., Zhang L., *Transient behavior of serial production lines with Bernoulli machines*, IIE Trans., 40 , No. 3, 297–312, (2008).

[5] Ju F., Li J., Horst J., *Transient analysis of serial production lines with perishable products: Bernoulli reliability model*, IEEE Transactions on automatic control, 62 , No 2, 694-707, 2016.

[6] Takahashi K., Morikawa K., Chen Y.C., *Comparing kanban control with the theory of constraints using Markov chains*, International Journal of Production Research, 45, No 16, 3599-3617, 2007.

[7] Abboud N.E., *A discrete-time Markov production-inventory model with machine breakdowns*, Computers & Industrial Engineering, 39, No 1-2, 95-107, 2001.

[8] Wang X., Wang H., Qi C., *Multi-agent reinforcement learning based maintenance policy for a resource constrained flow line system*, Journal of Intelligent Manufacturing, 27, No 2, 325-333, 2016.

[9] Nahas N., *Buffer allocation and preventive maintenance optimization in unreliable production lines*, Journal of Intelligent Manufacturing, 28,No. 1, 85-93, 2017.

[10] Regard M., *Process Chain Optimization in a Smart Factory*, Degree project electrical engineering, 2018.

[11] Matta A., Simone F., *Analysis of two-machine lines with finite buffer, operation-dependent and time-dependent failure modes*, International Journal of Production Research, 54, No. 6, 1850-1862, 2016.

[12] Guo, Yunsong and Lim, Andrew and Rodrigues, Brian and Yu, Shanhu, *Machine scheduling performance with maintenance and failure*, Mathematical and computer modelling, 45, No 9-10, 1067-1080, 2007.

[13] Altiok T., Melamed B., *Simulation Modeling and Analysis with Arena*, Elsevier, 2010.

[14] Buzacott J.A. and Shanthikumar J.G., *Stochastic Models of Manufacturing Systems*, Prentice-Hall, Englewood Cliffs, 1993.

[15] Gingu E.I., Zapciu M., *Analytical method to evaluate the maintenance parameters of a production line using Markov chains*, Conference Proceedings of the Academy of Romanian Scientists Productica Scientific Session, 27th - 28th may 2016, 8 , No 1, 10-19.

[16] Nelson B. L., *Foundations and Methods of Stochastic Simulation*, 187, DOI 10.1007/978-1-4614-6160-9, International Series in Operations Research & Management Science, Springer New York Heidelberg Dordrecht London, 2013.

[17] Smith D.J., *Reliability, maintainability and risk: practical methods for engineers*, Butterworth-Heinemann, 2017.

[18] Gingu E.I., Zapciu M., *Markov chains and decomposition method used for synchronizing the manufacturing production rate with real market demand*, U.P.B. Sci. Bull., Series D, 79, Iss. 1, ISSN 1454-2358, 163-174, 2017.

Scenarii de modelare pentru disponibilitatea timpilor aleatori ai sistemelor de producție care utilizează metoda lanțurilor Markov

Rezumat.. *În această lucrare, se folosesc lanțurile Markov pentru a realiza scenarii de modelare a parametrilor de întreținere și evaluarea performanței sistemelor de producție. Contribuția autorilor constă în dezvoltarea a două scenarii de modelare. În primul scenariu, se calculează disponibilitatea unui sistem cu faza de setare configurată și cu rebuturi, apoi se evaluează comportamentul mașinii într-o perioadă de o lună folosind ratele de defectare și reparații generate de*

Generatorul Liniar Congruențial. În cel de-al doilea scenariu, este proiectată și evaluată disponibilitatea unui sistem cu faza de setare configurată, dar fără rebuturi.

Elena-Iuliana GINGU (BOTEANU), PhD, University "Politehnica" of Bucharest, Romania and Advanced Technologies Institute, e-mail: iuliana_boteanu@yahoo.com.
Miron ZAPCIU, Professor, University "Politehnica" of Bucharest, Faculty of Industrial Engineering and Robotics, Romania and Academy of Romanian Scientists, e-mail: miron.zapciu@upb.ro.

Appendix A. Failure and repair rate generated by Linear Congruential Generator - Modeling scenario 1 (Bending operation)

Day (8 hours)	Failure rates (failures/min)	Repair rate (repairs/min)
1.	0.00806	0.0136
2.	0.00858	0.1920
3.	0.00458	0.0146
4.	0.00889	0.0295
5.	0.00637	0.0535
6.	0.00892	0.0509
7.	0.00227	0.0704
8.	0.00307	0.1470
9.	0.00335	0.0791
10.	0.00194	0.1300
11.	0.00164	0.0929
12.	0.00172	0.1260
13.	0.00816	0.0849
14.	0.00967	0.1000
15.	0.00840	0.1020
16.	0.00293	0.0194
17.	0.000908	0.1850
18.	0.00701	0.1790
19.	0.00426	0.1620
20.	0.00804	0.0257
21.	0.00309	0.0237
22.	0.00828	0.0266
23.	0.00121	0.1990
24.	0.000204	0.1500
25.	0.00494	0.0277
26.	0.00543	0.0696
27.	0.00845	0.0836
28.	0.00779	0.1750
29.	0.00606	0.1200
30.	0.00453	0.0751

Appendix B. Failure and repair rate generated by Linear Congruential Generator - Modeling scenario 2 (Milling A operation)

Day (8 hours)	Failure rates (failures/min)	Repair rate (repairs/min)
1.	0.00676	0.0938
2.	0.00680	0.0677
3.	0.00851	0.1880
4.	0.00156	0.0297
5.	0.00289	0.1780
6.	0.00835	0.0611
7.	0.00632	0.0771
8.	0.000797	0.0030
9.	0.00339	0.0203
10.	0.00464	0.1220
11.	0.00886	0.1920
12.	0.00645	0.1470
13.	0.00603	0.0741
14.	0.00440	0.0657
15.	0.00339	0.0596
16.	0.00376	0.1400
17.	0.00972	0.0345
18.	0.00101	0.0557
19.	0.00813	0.0769
20.	0.00537	0.0199
21.	0.00381	0.0452
22.	0.00308	0.1130
23.	0.00615	0.1920
24.	0.00646	0.0781
25.	0.00640	0.00371
26.	0.00415	0.0715
27.	0.00473	0.0850
28.	0.00643	0.0236
29.	0.00232	0.1070
30.	0.00776	0.0777