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ANALYSIS OF HUMAN HIP MOVEMENT USING NONLINEAR TIMESERIES ANALYSIS METHODS

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Abstract: In this paper, measurements were acquired from a group of seven subjects with healthy right hips and left hips affected by an osteoarthritic (OA) process in first stage of evolution. The measurements consist in 126 time-series, where values represent the angles of the hip joint in sagittal plane, describing the flexion-extension movement of each hip joint, of each subject performing nine experimental tests of walking on treadmill, at three predefined speeds and three predefined incline angles. Using the experimental time-series, a measure of the local dynamic stability was estimated with Lyapunov exponents.

Key words: osteoarthritic hip, nonlinear time-series analysis, Lyapunov exponents, treadmill

1. INTRODUCTION

A very important feature of human movement especially of human gait, is its variability [1]. Classical methods of gait variability analysis no longer provide sufficient data. The use of nonlinear instruments enriches our understanding of variability by revealing additional information about its nature and its relation to performance and pathology [1-9], as well as to rehabilitation of human movements using orthotic systems and exoskeletons. [9-12] or rehabilitation robots [13-16].

In the present study, we apply methods and instruments of "nonlinear time-series analysis", in order to estimate the local dynamic stability of human hip joints by considering the computed values of short-term Lyapunov exponents obtained for each experimental time-series with measurements of the flexion-extension (fl-ex) angles of left and right hip joints of a sample of 7 patients performing 9 tests.

2. EXPERIMENTAL PROTOCOL

The equipment (Fig.1) used for data acquisition during experimental tests is the "Biometrics" system [17], often used in domains like gait analysis, biomechanical research, clinical medicine and rehabilitation, [3, 7-10, 13, 18-21].



Fig.1. Biometrics system mounted on a subject

All walking tests were executed inside biomechanics laboratory of INCESA research Centre, by a group of seven individuals having their left hip affected by osteoarthritis. A number of 6 electrogoniometers, one for each main joint of both lower limbs, were mounted on each subject. In this study, the time-series of fl-ex angles of both hips of each subject are analyzed in order to measure their local dynamic stability. Each subject performed 9 walking tests (T) (Table 1) on treadmill (TM), corresponding to 3 different speeds and 3 different inclines.

The duration of each experimental test was 1 minute, the biomechanical data collected simultaneously by 6 electrogoniometers at a sampling rate of 500 Hz being transferred to Biometrics software to be processed and analyzed (Fig.2).



Fig. 2. The experimental angular variation of rotation angles in frontal and sagittal plane for ankle, knee and hip

A number of 126 time-series (2 hips of the 7 subjects for 9 tests) representing consecutives fl-ex cycles of hip joints were collected.

 Table 1

 Speed and incline of the treadmill for each test.

	T 1	T 2	T 3	T 4	Т5	T 6	T 7	T 8	Т9
Speed [km/h]	2.5	5	7.5	2.5	5	7.5	2.5	5	7.5
Incline [o]	0	0	0	7	7	7	11	11	11

Before performing the tests, all subjects gave their permission in written, regarding data processing. The main anthropometric data (means and standard deviations) of the subjects sample are: Age - 36.5 (2.31); Height - 182.36 cm (1.94 cm); Mass - 78.63 kg (3.57 kg).

3. DATA PREPROCESSING

An important step in the "nonlinear timeseries analysis" is the reconstruction of the dynamics. Before doing this we need to consider that experimental time series often are corrupted by noise and non-stationarity. Therefore, we first must eliminate the factors that cover the behavior of interest and to identify the component that must be used to correctly reconstruct hip joints dynamics.

In order to separate unstructured variation (or noise) from structured variation (signal of interest) in the experimental data, we used a signal processing technique named SSA ("Singular Spectrum Analysis") [22]. SSA involves the construction of a trajectory matrix, decomposing it in a sum of component matrices (i.e. eigentriplets), grouping and reconstruction of time series components from the groups.

We applied the Toeplitz method of singular spectrum analysis (Toeplitz-SSA) for each acquired time-series of consecutives fl-ex angles of hip joint using the R package RSSA [23]. We illustrate the use of SSA for preprocessing of the fl-ex time-series acquired from the right hip joint of Subject 3 in Test 3. The following visual diagnostics generated by RSSA, are derived from each component of the eigentriplets defining each decomposed matrix. The plot in Fig.3 is the eigenspectrum and shows the square roots of the eigenvalues of the decomposed matrices in descending magnitude. Components of which associated values are along the steep portion of the plot form the basis of the deterministic signal.



Fig. 3. Eigenspectrum of the fl-ex angles left hip joint of Subject 3 in Test 3

Equal and successive values represent pairs, forming the core of potential harmonic oscillations, while values along the flat portion of the plot are associated with noise. The eigenspectrum of the flexion-extension angles left hip joint of Subject 3 in Test 3, has the expected hockey-stick shape, with steps at paired singular values (2; 3), (4; 5) and (6; 7) along the steep portion of the plot. The visual diagnostic presented in Fig.4. shows the eigenvectors corresponding to singular values along the eigenspectrum. The following pairs of eigenvectors (2; 3), (4; 5) and (6; 7), oscillate at a similar frequency in phase quadrature, providing further evidence for the harmonics. The percentages of each component represents the portion of total variance from the mean.



Fig. 4. Eigenvector plots of the fl-ex angles right hip joint of Subject 3 in Test 3.

In Fig.5. are presented the weighted correlations between the reconstructed time series from decomposed matrices (by diagonal averaging).



Fig. 5. Weighted correlation matrix of the flexionextension angles left hip joint of Subject 3 in Test 3.

Black shading indicates high correlation while white shading indicates statistical independence. Fig.4. indicates that paired groups (2; 3), (4; 5) and (6; 7), are highly correlated to each other and insignificantly correlated with the others and thus can be grouped independently. Although we can choose groups just by visual inspection of the w-correlation plot, we grouped component series using hierarchical clustering and weighted correlation as a proximity matrix [23].

In Fig.6. we present the original time series, isolated signal and the residual noise.

In order to be confident that the residual signal represents uncorrelated noise, we used surrogate data testing method. Based on this method we can test against uncorrelated noise by generating 99 surrogates with similar Fourier power spectrum (or linear correlations between data points) and distribution of probabilities, as the residual, using IAAFT algorithm [22-24] and maximum likelihood estimation of correlation dimension (Takens-Theiler estimator) as statistical discriminant indicator. In Fig.7 can be observed that there is no significant difference between the residual and the generated surrogates.



Fig. 6. The fl-ex angles right hip joint of Subject 3 in Test 3, reconstructed isolated signal and noise (unstructured variation) removed from the original time-series.

The value obtained for the statistical discriminant indicator for the original series, falls within the distribution of the values of the discriminant indicators estimated for the surrogates, which means that the residual is likely generated by linear stochastic dynamics and does not present interest for nonlinear dynamics analysis [25].



Fig. 7. Distribution of Correlation Dimension of residual eliminated in preprocessing phase and 99 surrogates generated using IAAFT.

4. NONSTATIONARITY

After preprocessing the data, and mitigating the impact of noise in obstructing the dynamic of interest, before calculating maximum Lyapunov exponents, it is necessary to make sure that the isolated signal is stationary.

The stationarity requirement for nonlinear analysis implies that the qualitative structure of system dynamics does not change (e.g. the transition from a chaotic behavior to a limit cycle behavior or vice versa), otherwise the application of any of the nonlinear analysis algorithms would be problematic [26].

To verify the stationarity of the isolated signal from hip flexion-extension angles we used an effective method proposed by [24]. The test is called "nonlinear cross-prediction" and it checks if there is a similar nonlinear behavior across time series segments by dividing it into several non-overlapping segments and applying nonlinear prediction methods to measure the ability of each segment to predict the others. If ability to cross-predict does not decrease significantly when segments are further apart in time, the time series is considered stationary for nonlinear analysis purpose. The efficiency coefficient Nash-Sutcliffe (nse) was used in order to measure the prediction skill:

$$nse = 1 - \frac{\sum_{i=1}^{N} (x_i - x_{pi})^2}{\sum_{i=1}^{N} (x_i - \bar{x})^2} = 1 - \left(\frac{RMSE}{\sigma}\right)^2$$
(1)

where *i*, represents the points in the segment, x_i is the corresponding value in the test segment, x_{pi} is the predicted value.

In Fig.8. the Nash-Sutcliffe efficiencies of 5 segments of the isolated signal from right hip joint of Subject 3 in Test 3 are shown. The values for each learning segment are close to 1, which means nonlinear prediction works very well on this data. In addition, the values do not deteriorate for more distant segments, so we can consider the signal stationary for the purpose of nonlinear analysis.



Fig. 8. Nash-Sutcliffe efficiencies (NSE) of 5 segments of the isolated signal from right hip joint of Subject 3 in Test 3.

5. NONLINEARIY TEST

It is very important to distinguish between the nonlinear structure of a time series (or the observed dynamics) and the dynamics underlying this structure. The nonlinear structure is characterized by the observed irregular behavior of the time series. But this irregular behavior can be as well generated by a regular linear dynamic, corrupted by noise or non-stationarity. Although at this point we already preprocessed the data, significantly mitigating the impact of noise and subjected the isolated signal to a nonlinear stationary test to exclude the possible change in dynamic structure, we do not expect that SSA has completely eliminated all the noise. Therefore, in order to be confident that our results characterize the underlying dynamics of the system, we first need to establish evidence of its existence. In this way we will exclude the possibility that the observed behavior is a product of a linear stochastic process affected by noise, resulting in a nonlinear structure.

To do this, we used surrogate data testing method (Fig.9). Because flexion-extension angles of the hip joint have a periodic behavior, they are incompatible with the null hypothesis of a linearly filtered noise consequently IAAFT [27] surrogates are not applicable in this case.



Fig. 9. The flexion-extension angle right hip of Subject 3, Test 3 (solid line) and one of the surrogate time-series generated with PPS (dashed line)

A more appropriate null hypothesis (which we want to reject), is that of a periodic orbit disturbed by uncorrelated noise. More precisely, this states that besides the periodic behavior there is no other deterministic behavior.

In order to generate surrogates in accordance with this null hypothesis, we used PPS algorithm introduced by Small et al. [28], which takes a random walk over the reconstructed attractor of the original time series. PPS algorithm generates surrogates preserving the most obvious deterministic traits (as, for example, cyclic trends), and destroying any detailed structure (for example, deterministic chaos). Fig.10., Fig.11. and Fig.12. show that the generated PPS surrogates have similar linear correlations between data points and similar distribution of probabilities as the original. We set the probability of false rejection $\alpha = 0.02$ and the rank-order test parameter k = 1. Consequently, we generated 99 surrogates for the two-tailed test (S = $(2k/\alpha) - 1$). We chose the maximum likelihood estimation of correlation dimension (Takens-Theiler estimator) as statistical discriminant and we reject the null hypothesis when the computed value for the original is either the lowest or the highest value.



Fig. 10. The power spectrum of the fl-ex angles right hip joint- Subject 3 in Test 3 and one of the surrogate time-series generated with PPS.



Fig. 11. The autocorrelation function of the fl-ex angles right hip joint of Subject 3 in Test 3 and one of the surrogate time-series generated with PPS.



Fig. 12. The distribution of the fl-ex angles right hip joint of Subject 3 in Test 3 and one of the surrogate time-series generated with PPS.

As Fig.13 and Fig.14 shows, we strongly reject the null hypothesis, since the MLE computed for the isolated signal is the lowest

value. Similar values were obtained for all time series of all subjects corresponding to the nine experimental tests.



Fig. 13. The Correlation sum of the fl-ex angles right hip joint of Subject 3 in Test 3 and the 99 surrogate time-series generated with PPS.



Fig. 14. Correlation Dimension of the flexion-extension angles right hip joint of Subject 3 in Test 3 and the 99 surrogate time-series generated with PPS.

Consequently, we have reasons to believe that the time series of angles of hip joint is not just a periodic orbit disturbed by uncorrelated noise.

6. STATE SPACE RECONSTRUCTION

After isolating a strong stationary signal for each of the time-series of fl-ex angles of hip joints. And with evidence of the presence of a nonlinear aperiodic determinism in the underlying dynamics. We can justifiably go further to reconstruct the dynamics from each isolated signal in order to estimate maximum Lyapunov exponents as a measure of local dynamic stability.

Applying the "Method of delays" [29] we reconstructed the state space for 126 time series of all subjects on all tests. In the original state space of the human hip joint, the coordinates represent all the state variables characterizing the hip joint and a trajectory in this space is defined by a sequence of values of these state variables. Each reconstructed state space is topologically identical to its original and was used to estimate Lyapunov exponents, which were preserved from the original state space.

In Fig.15 it can be observed that the m coordinates in each of the delayed vectors, are not strongly correlated (which would have caused the embedded dynamics to lie close to the main diagonal of the reconstructed space) and are not too independent (which would have caused the reconstruction to unfold off the subspace). This means that the selected time delay was neither too small nor to long, which is very important for obtaining a topologically correct reconstruction of the state space. Even though the embedding theorems require that the time-delay T to be any value bigger than zero and not a multiple of one of the orbit's periods, in practice we do not have real-valued arithmetic on infinite amount of noise free data and we are dealing with a limited number of noisy observations.



Fig. 15. Reconstructed State Space of Subject 3 in Test 3

The "Method of delays" and "TISEAN" software [30] were used in order to reconstruct

the state space from the available data for each collected time-series. Using vectors containing time-delayed values of a time-series, a trajectory in the new space can be drawn as follows:

$$x_t = (s_t, s_{t+d}, s_{t+2d}, \dots, s_{t+(m-1)d})$$
(2)

where m is an integer called "embedding dimension"; t - the sampling time; d - an appropriately chosen time delay.

The value of "d" is chosen as the first minima of "the average mutual information" (AMI) [31]. Another method used in practice is the value where autocorrelation function first reaches zero, but unlike AMI this does not take into account nonlinear correlations. The AMI plot for Test 3 performed by Subject 3 is shown in Fig.16.



Fig. 16. AMI function of the flexion-extension angles right hip joint of Subject 3 in Test 3.

The embedding dimension, "m", means the required number of values necessary to construct a valid point (or state) in the trajectory in the new space, which unfolds the structure of the dynamical system [32]. To estimate a valid value of m, for each time-series, we used the method called "False Nearest Neighbor" (FNN) method [33].

In Fig.17 the fraction of FNN is plotted against consecutive values of dimensions used for embedding the time-series of left hip fl-ext angles on Test 3 performed by Subject 3. The value of "m" should not be higher than the value where the fraction of FNN is zero, or is very small. For this paper a threshold of 0.1 was used. Embedding dimensions and time delays calculated for the 126 time-series were estimated in a similar manner.



Fig. 17. FNN of the flexion-extension angles right hip joint of Subject 3 in Test 3.

The "short-term Lyapunov exponent" (λ s), quantifies the average rate of divergence of the embedded trajectory constructed from delayed values of the hip joint fl-ex angles, over a period of 0.5 or 1 stride [29]. The smaller is λ s the more stable the movement is. The "lyap_r" routine of "TISEAN" package, which implements the "Rosenstein algorithm" [34], was used to compute the results. Plots of the average logarithmic divergence of reconstructed trajectories of the left hip of Subject 3 for each test are shown in Fig.18.



Fig. 18. Average logarithmic divergence for left hip of Subject 3 on all 9 tests.

Similar plots for left hip of all subjects on Test 6 are presented in Fig.19.



Fig. 19. Average logarithmic divergence for left hip of all subjects, Test 6.

7. RESULTS

Averaged λs over all 7 subjects grouped by test and hip are presented in (Table 2) and (Fig.120). The positive values of computed λs suggest that a chaotic structure is inherent to human hip motion.

Average values of λs by test and hip.								
Incline	Test	Speed	Left Hip	Right Hip				
0	Test 1	2.5km/h	0.17816	0.16931				
	Test 2	5km/h	0.16921	0.16525				
	Test 3	7.5km/h	0.19127	0.17918				
7	Test 4	2.5km/h	0.25124	0.22055				
	Test 5	5km/h	0.15624	0.16381				
	Test 6	7.5km/h	0.16914	0.15459				
11	Test 7	2.5km/h	0.31464	0.28784				
	Test 8	5km/h	0.17751	0.13764				
	Test 9	7.5km/h	0.23482	0.22733				

Table 2



Fig. 20. Plots of averaged values of the "short Lyapunov exponents" for both hips of the group on each test.

8. DISCUSSION

In this paper we calculate a deterministic nonlinear measure ("finite-time Lyapunov exponents") as an estimate of the local dynamic stability of the hip joint. In order to make a compelling case that the time-series of fl-ex angles of hip joint contain a deterministic nonlinear component, we first eliminated noise and nonstationary as drivers of the observed complexity. Next using surrogate data testing we eliminated linear stochastic dynamics as a possible cause, and prepared the ground for estimating finite-time Lyapunov exponents. Analyzing obtained λs values, we observe that for each test the values were positive, which indicates that human hips motions show chaotic characteristics. For the same test, under identical experimental conditions, the λ s corresponding to the affected joints are larger than the λs corresponding to healthy joints, which suggests that the motion of affected hips is more sensitive to local perturbations. So, the movement of the affected hips presents a more unstable dynamic comparing with the movement of healthy hips. This decrease in local stability can be caused by pain or instabilities determined by various injuries. These results confirm previous results which suggest an increased stability during treadmill locomotion [2, 4, 6, 31–33].

9. CONCLUSIONS

Traditional measures of variability are limited in describing the way the locomotor behavior responds to change. Recent studies has shown that some components of the measured signal produce by a variable of a dynamic system, which can attributed to a random behavior have in fact deterministic origins and are produced by some nonlinear interactions. In this study we investigate the temporal structure of variability by analyzing time-series of the hip joint movement to obtain new information about the sensitivity of the locomotor system to local perturbations. Larger values of λs obtained for affected hips suggest a higher instability and increased sensitivity while smaller values reflect a local stability.

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ANALIZA MIŞCĂRII ŞOLDULUI UMAN PRIN METODE DE ANALIZA A SERIILOR DE TIMP NELINIAR

Rezumat: În această lucrare, serii de timp experimentale ale unghiurilor de flexie-extensie ale articulațiilor șoldului uman au fost colectate de la un grup de șapte subiecți cu șoldul drept normal iar cel stâng afectat de un proces de osteoartritita în prima etapă a evoluției. S-au efectuat nouă teste experimentale, pe banda de alergat, la trei viteze diferite și trei unghiuri de înclinare diferite. Pe baza seriilor de timp experimentale, exponenții Lyapunov au fost estimați ca o măsură a stabilității dinamice locale.

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