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# A METHOD FOR THE ANALYSIS OF VIBRATIONS OCCURRING WITHIN SOME MECHANICAL SYSTEMS WHICH CONTAIN SPUR GEARS

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***Abstract:*** *Starting from the mathematical model created for the relative displacement of two cog wheels inside of a gearing (model which is expressed under the form of a second-order system of differential equations with parameter'shaped or possibly non-linear excitations) we will present in this work an original method of integrating the matrix movement equation which does make use of the procedure of rendering discrete the time interval during which the concerned movement is observed. The applying of the Laplace integrated transformations which are unilateral in respect to time should lead us towards rendering algebraic the concerned model. This fact could hugely simplify the problem. Furthermore, we should be as well able to provide the graphical representations of the relative displacements for the considered gearing mechanism.*

***Key words:*** *modeling, vibration, Laplace transformations, differential equations, gearing.*

## 1. INTRODUCTION

An important part in the conception of the new products is represented by the estimations made of the vibrations'intensity level respectively of the noise level for the concerned mechanical systems as well as by their control. Within a system a lot of vibration'sources might exist such as for example frictions and shocks. The reducing devices endowed with gearing mechanisms may equally become sources of current vibrations because they are frequently made use of in industry. They do constitute the objects of a lot of studies aiming to define their respective cinematic and dynamic behaviours. In order to model the vibrations and noises generated by the gearing mechanisms various approaches have been tested. Therefore we might meet: rigid body simplified models, multibody flexible models or models exclusively made of 2D or 3D finite elements.

The specialized literature does point out the necessity of integrating for the gearing mechanism both the elasticity of its components and the descriptions of its vibrations into an one and only tridimensional model able to reflect its dynamic behaviour. Should the gearing mechanisms suffer deformations they would involve the existence of some special deformations usually occurring nearby the contact points the functioning of which does require for a great finesse. We are consequently encouraged to turn our attention towards a model which should be simplified in respect to the models made of finite tridimensional elements but which could yet remain precise enough in what does concern the study of the elasticity of the elements involved into the gearing mechanism and would as well simultaneously allow the integration of the relatively complex geometries held by the deformable cog wheels. As we have already demonstrated the main source of excitation insofar the components which do enable the transmission of power are concerned is constituted by the gearing mechanism. The instantaneous movements of each wheel are represented through six degrees of freedom (three translations and three rotations). The fluctuations undergone by both the transmission error and the intrinsic rigidity of the gearing mechanism are the main causes of the excitations which are associated to them. For the transmission error it is particularly indispensable to distinguish the effects due to elastic deformations from the effects due to the gearing of the non-conjugated profiles. There are several scientific schools (see [5]) which do make use of these size units in order to describe the interface created through the gearing mechanism. Nowadays the research activities do focus upon the development of some multidisciplinary experimental, theoretical and numerical competencies which would have to be relied upon when the structures and elements of the machines – or more extensively the mechanical systems – should be conceived. The purposes of actual researches are the ones of improving our knowledge of the behaviour shown by the materials and structures, of developing some models and instruments that could be useful in the designing process concerning the structures and the machines as well as to take advantage from an already existing technical culture insofar the methodologies of analysis, conception and fabricating could be concerned. These researches are relying upon the domains of the science of materials, of the branches of non-linear mechanics which do respectively involve solid bodies, fluids and coupled systems, of acoustics, of the techniques made use of in forming and processing, of the experimental measurement methods and of the numerical modelling.

## 2. MODELING OF VIBRATIONS FOR A GEARING MECHANISM

### 2.1 Mechanical model

In figure 1, let us consider the mass of the vibrating system as being concentrated in M. The system's elasticity should be substituted by a spring owning the elastic constant k while the cushioning should be substituted by a damper which owns the cushioning constant c. The origin of the Ox axis is considered to be the intersection point between the gearing line and the symmetry axis of the tooth.

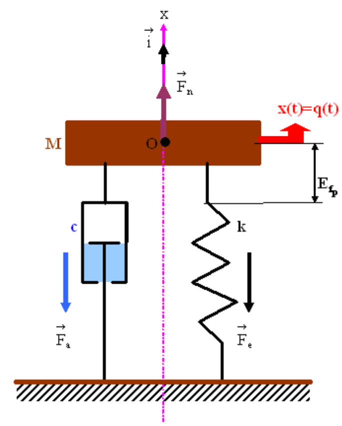


**Fig. 1.** The mechanical model of the mechanism

### 2.2. Mathematical model of the movement

In order to determine the relative displacements in the sense of the gearing line for the two respective cog wheels while the teeth do make contact either upon the active or inactive side of the tooth as well as the relative displacements of the cog wheels during the lost touch intervals (gearing discontinuities) let us consider the mechanical model bearing one freedom degree which is illustrated in figure 2.1. In the formalisms of Newton or respectively Lagrange we do have the respective mathematical models of the vibrations sustained by a cog wheels gearing mechanism for the cases of: a contact held upon the active side of the tooth (see (2.1)); a contact held upon the unactive side of the tooth (see (2.2)) and the one corresponding to some gearing discontinuities (see (2.3)):

$$\ddot{q}+2n\dot{q}+\omega_n^2 q=a_0+a_1\cos(\omega_0 t)+a_2\sin(\omega_0 t) \tag{2.1}$$

$$\ddot{q}+2n\dot{q}+\omega_n^2 q=a_0-a_1\cos(\omega_0 t)-a_2\sin(\omega_0 t) \tag{2.2}$$

$$\ddot{q}+2n\dot{q}=a_0 \tag{2.3}$$

where:

$$2n=\frac{c}{M};\ \omega_n^2=\frac{k}{M};\ a_0=\frac{F_n}{M};\ a_1=\frac{KE_{0,f_p}}{M}\sin\alpha;$$

$$a_2=\frac{KE_{0,f_p}}{M}\cos\alpha;$$

$E_{0,f_p}$ - the amplitude of the shaping error $E_{f_p}$ held by the tooth profile (the reduced shaping error) the variation of which is supposed to be a sinusoidal one;

$\omega_0$ - the pulse of the shaping error held by the tooth profile;

$\alpha$ - the initial phase;

$F_n$ - the static force which does act normally upon the tooth;

$M$ - the reduced mass $M = \frac{M_I M_{II}}{M_I + M_{II}}$;

$M_I$ - the mass of the pinion reduced to its basis circle $M_I = \frac{4I_1}{D_{b_1}^2}$;

$M_{II}$ - the mass of the cog wheel reduced to its basis circle $M_{II} = \frac{4I_2}{D_{b_2}^2}$;

$I_1$ - the inertial moment of the pinion in respect to its rotation axis;

$I_2$ - the inertial moment of the cog wheel in respect to its rotation axis;

$D_{b_1}$ - diameter of the basis circle for the pinion;

$D_{b_2}$ - diameter of the basis circle for the cog wheel.

## 2.3. Dinamic response

Should we apply the Laplace transformation to the equations (2.1), (2.2), (2.3) under the initial conditions:

$$q(0) = q_0,\ \dot{q}(0) = v_0,$$

the result would consist in the algebraic equations:

$$s^2\tilde{q}(s) - sq_0 - v_0 + 2ns\tilde{q}(s) - 2nq_0 + \omega_n^2\tilde{q}(s) = $$
$$= \frac{1}{s}a_0 + a_1\frac{s}{s^2+\omega_0^2} + a_2\frac{\omega_0}{s^2+\omega_0^2}$$

$$s^2\tilde{q}(s) - sq_0 - v_0 + 2ns\tilde{q}(s) - 2nq_0 + \omega_n^2\tilde{q}(s) = $$
$$= \frac{1}{s}a_0 - a_1\frac{s}{s^2+\omega_0^2} - a_2\frac{\omega_0}{s^2+\omega_0^2} \tag{2.1(1)}$$

$$s^2\tilde{q}(s) - sq_0 - v_0 + 2ns\tilde{q}(s) - 2nq_0 + \omega_n^2\tilde{q}(s) = $$
$$= \frac{1}{s}a_0 - a_1\frac{s}{s^2+\omega_0^2} - a_2\frac{\omega_0}{s^2+\omega_0^2} \tag{2.2(1)}$$

$$s^2\tilde{q}(s) - sq_0 - v_0 + 2ns\tilde{q}(s) - 2nq_0 = \frac{1}{s}a_0, \tag{2.3(1)}$$

bearing the solutions:

$$\tilde{q}(s) = \frac{a_0(s^2+\omega_0^2) + a_1 s^2 + a_2\omega_0 s + s(sq_0+v_0+2nq_0)(s^2+\omega_0^2)}{s(s^2+\omega_0^2)(s^2+2ns+\omega_n^2)} \tag{2.1(2)}$$

$$\tilde{q}(s) = \frac{s^2 q_0 + (v_0+2nq_0)s + a_0}{s^2(s+2n)}. \tag{2.3(2)}$$

$$\tilde{q}(s) = \frac{a_0(s^2+\omega_0^2) - a_1 s^2 - a_2\omega_0 s + s(sq_0+v_0+2nq_0)(s^2+\omega_0^2)}{s(s^2+\omega_0^2)(s^2+2ns+\omega_n^2)}. \tag{2.2(2)}$$

Should we apply the reversed Laplace transformation in respect to time to the relationships (2.1(2)), (2.2(2)) respectively (2.3(2)) the result would consist in the dynamic response provided by the vibrating system under the forms of the time functions:

$$q(t) = \frac{a_0}{\omega_n^2} + e^{-nt}\left\{\left[q_0 - \frac{a_0}{\omega_n^2} + \frac{2na_2\omega_0 + a_1(\omega_0^2-\omega_n^2)}{(\omega_0^2-\omega_n^2)^2 + 4n^2\omega_0^2}\right]\cosh\left(\sqrt{n^2-\omega_n^2}\,t\right) + \right.$$
$$+\frac{1}{\sqrt{n^2-\omega_n^2}}\left[nq_0 + v_0 - \frac{na_0}{\omega_n^2} + \frac{a_2\omega_0(2n^2+\omega_0^2-\omega_n^2) - na_1(\omega_0^2+\omega_n^2)}{(\omega_0^2-\omega_n^2)^2+4n^2\omega_0^2}\right]\cdot$$
$$\left.\cdot\sinh\left(\sqrt{n^2-\omega_n^2}\,t\right)\right\} + \frac{1}{(\omega_0^2-\omega_n^2)^2+4n^2\omega_0^2}\cdot\left\{\left[-2na_2\omega_0 + a_1(\omega_n^2-\omega_0^2)\right]\cos(\omega_0 t) + \right.$$
$$\left. + \left[2na_1\omega_0 + a_2(\omega_n^2-\omega_0^2)\right]\sin(\omega_0 t)\right\} \tag{2.1(3)}$$

$$q(t) = \frac{a_0}{\omega_n^2} + e^{-nt}\left\{\left[q_0 - \frac{a_0}{\omega_n^2} + \frac{-2na_2\omega_0 + a_1(\omega_n^2-\omega_0^2)}{(\omega_0^2-\omega_n^2)^2 + 4n^2\omega_0^2}\right]\cosh\left(\sqrt{n^2-\omega_n^2}\,t\right) + \right.$$
$$+\frac{1}{\sqrt{n^2-\omega_n^2}}\left[nq_0 + v_0 - \frac{na_0}{\omega_n^2} + \frac{-a_2\omega_0(2n^2+\omega_0^2-\omega_n^2) + na_1(\omega_0^2+\omega_n^2)}{(\omega_0^2-\omega_n^2)^2+4n^2\omega_0^2}\right]\cdot$$
$$\left.\cdot\sinh\left(\sqrt{n^2-\omega_n^2}\,t\right)\right\} + \frac{1}{(\omega_0^2-\omega_n^2)^2+4n^2\omega_0^2}\cdot\left\{\left[2na_2\omega_0 - a_1(\omega_n^2-\omega_0^2)\right]\cos(\omega_0 t) - \right.$$
$$\left. - \left[2na_1\omega_0 + a_2(\omega_n^2-\omega_0^2)\right]\sin(\omega_0 t)\right\} \tag{2.2(3)}$$

$$q(t) = \frac{a_0}{2n}t - e^{-2nt}\frac{2nv_0 - a_0}{4n^2} + \frac{4n^2 q_0 + 2nv_0 - a_0}{4n^2}. \tag{2.3(3)}$$

Should it be:

$$\sqrt{n^2-\omega_n^2} = i\sqrt{\omega_n^2 - n^2}$$

then with:

$$\cosh\left(\sqrt{n^2-\omega_n^2}\,t\right)=\cosh\left(i\sqrt{\omega_n^2-n^2}\,t\right)=\cos\left(\sqrt{\omega_n^2-n^2}\,t\right)$$

$$\sinh\left(\sqrt{n^2-\omega_n^2}\,t\right)=\sinh\left(i\sqrt{\omega_n^2-n^2}\,t\right)=i\sin\left(\sqrt{\omega_n^2-n^2}\,t\right),$$

the time functions ($2.1^{(3)}$) and ($2.2^{(3)}$) would become:

$$\begin{aligned}q(t)=&\frac{a_0}{\omega_n^2}+e^{-nt}\left\{\left[q_0-\frac{a_0}{\omega_n^2}+\frac{2na_2\omega_0+a_1\left(\omega_0^2-\omega_n^2\right)}{\left(\omega_0^2-\omega_n^2\right)^2+4n^2\omega_0^2}\right]\cos\left(\sqrt{\omega_n^2-n^2}\,t\right)+\right.\\&+\frac{1}{\sqrt{\omega_n^2-n^2}}\left[nq_0+v_0-\frac{na_0}{\omega_n^2}+\frac{a_2\omega_0\left(2n^2+\omega_0^2-\omega_n^2\right)-na_1\left(\omega_0^2+\omega_n^2\right)}{\left(\omega_0^2-\omega_n^2\right)^2+4n^2\omega_0^2}\right]\cdot\\&\left.\cdot\sin\left(\sqrt{\omega_n^2-n^2}\,t\right)\right\}+\frac{1}{\left(\omega_0^2-\omega_n^2\right)^2+4n^2\omega_0^2}\cdot\left\{\left[-2na_2\omega_0+a_1\left(\omega_n^2-\omega_0^2\right)\right]\cos(\omega_0 t)+\right.\\&\left.+\left[2na_1\omega_0+a_2\left(\omega_n^2-\omega_0^2\right)\right]\sin(\omega_0 t)\right\}\end{aligned}$$

$$\begin{aligned}q(t)=&\frac{a_0}{\omega_n^2}+e^{-nt}\left\{\left[q_0-\frac{a_0}{\omega_n^2}+\frac{-2na_2\omega_0+a_1\left(\omega_n^2-\omega_0^2\right)}{\left(\omega_0^2-\omega_n^2\right)^2+4n^2\omega_0^2}\right]\cos\left(\sqrt{\omega_n^2-n^2}\,t\right)+\right.\\&+\frac{1}{\sqrt{\omega_n^2-n^2}}\cdot\left[nq_0+v_0-\frac{na_0}{\omega_n^2}+\frac{-a_2\omega_0\left(2n^2+\omega_0^2-\omega_n^2\right)+na_1\left(\omega_0^2+\omega_n^2\right)}{\left(\omega_0^2-\omega_n^2\right)^2+4n^2\omega_0^2}\right]\cdot\\&\left.\cdot\sin\left(\sqrt{\omega_n^2-n^2}\,t\right)\right\}++\frac{1}{\left(\omega_0^2-\omega_n^2\right)^2+4n^2\omega_0^2}\cdot\left\{\left[2na_2\omega_0-a_1\left(\omega_n^2-\omega_0^2\right)\right]\cos(\omega_0 t)-\right.\\&\left.-\left[2na_1\omega_0+a_2\left(\omega_n^2-\omega_0^2\right)\right]\sin(\omega_0 t)\right\}.\end{aligned} \qquad (2.2^{(4)})$$

## 2.4 Graphical representation of relative displacements

In the case of the gearing mechanism chosen for our study for which the provided parameters are: $a_0$=40142,857, $a_1$=40142.844, $a_2$=31,966, n=2007,142, $\omega_n = 28334,733\left[s^{-1}\right]$, $\omega_0 = 10\left[s^{-1}\right]$ under initial homogeneous conditions the time functions ($2.1^{(4)}$), ($2.2^{(4)}$) respectively ($2.3^{(3)}$) do have the representations illustrated in figures 2, 3, 4.

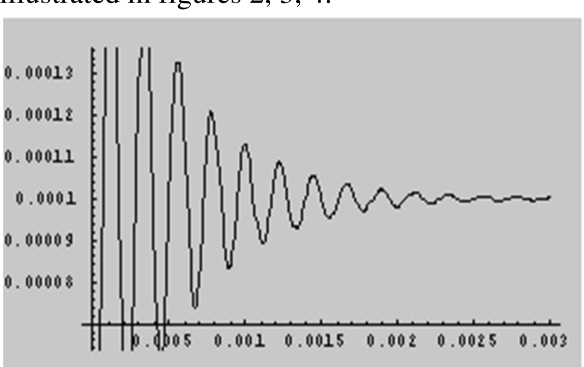


**Fig.2.** Variation of the relative displacement q=q(t) of the two cog wheels onto the direction of the gearing line while the teeth are making contact upon the active side

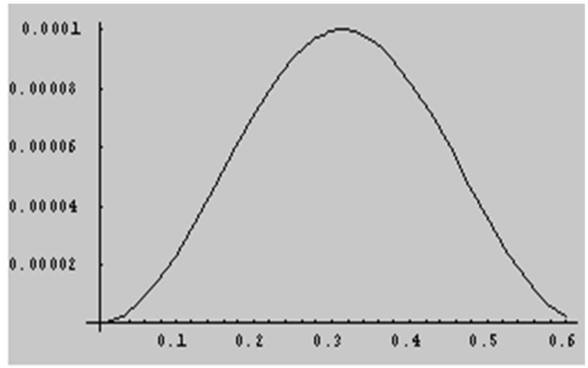


**Fig. 3.** Variation of the relative displacement q=q(t) of the two cog wheels onto the direction of the gearing line while the teeth are making contact upon the unactive side

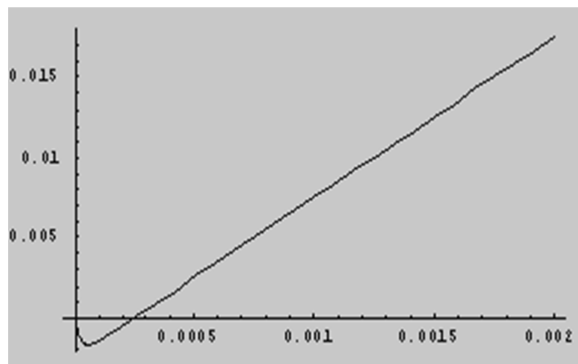


**Fig. 4.** Variation of the relative displacement q=q(t) of the two cog wheels during the lost touch intervals (gearing discontinuities)

Let us remark the fact that for the same time interval the relative displacements of the two cog wheels which do occur during the contact upon the unactive side of the tooth are much smaller in respect to the ones which do occur during the contact upon the active side of the tooth. In the case of a contact upon the active side of the tooth its upmost deformation should come to be stabilized at the value of 0,1[mm].

## 3. PRESENTATION OF MOVEMENT EQUASIONS AND OF THE MODALITY THROUGH WHICH THEY COULD BE SOLVED NUMERICALLY

Let us present below another modality of approaching the study of vibrations for a gearing mechanism.

### 3.1.Mathematical model of the movement

As a prime hypothesis the kinetics energy of the teeth does come to be neglected in respect to the one of the body of the gearing mechanism. Therefore we will take into consideration the static contributions brought in by the elastic basis of Pasternak only. Yet this hypothesis which is a classical one for the dynamics of a gearing mechanism does overlook the respective influences held by the vibration modalities exerted by each of the teeth (which do usually present high frequencies) but instead does favour the more extended modalities which do involve the shaft, the bearings and the body of the gearing mechanism which are submitted to smaller frequencies. Let us denote by $q_j,\ j=\overline{1,n}$ the Lagrange coordinates while n is the number of freedom degrees. Then let us denote by N the number of modalities which have been selected for the modalities'analysis of the concerned sub-structure and – finally – by $N_1$, $N_2$ the number of discretizing segments which do exist for the elastic basis of the pinion respectively for the one of the driven cog wheel at a given moment in time t. The applying of the Lagrange equations to the various elements of the above defined modellings does lead us towards the movement equations which do rule over the whole aggregate of the concerned mechanical system. They could be expressed under the form:

$$\begin{bmatrix}[0] & [0]\\ [0] & [M_{xq}]\end{bmatrix}\begin{Bmatrix}\begin{Bmatrix}\{\ddot{\omega}_1\}\\ \{\ddot{\omega}_2\}\end{Bmatrix}\\ \begin{Bmatrix}\{\ddot{x}\}\\ \{\ddot{q}\}\end{Bmatrix}\end{Bmatrix}+\begin{bmatrix}[0] & [0]\\ [0] & [C_{xq}]\end{bmatrix}\begin{Bmatrix}\begin{Bmatrix}\{\dot{\omega}_1\}\\ \{\dot{\omega}_2\}\end{Bmatrix}\\ \begin{Bmatrix}\{\dot{x}\}\\ \{\dot{q}\}\end{Bmatrix}\end{Bmatrix}+$$

$$+\begin{bmatrix}[K_{\omega_1\omega_2}] & [K_{\omega_1\omega_2 xq}]\\ [K_{qx\omega_2\omega_1}] & [K_{xq}]\end{bmatrix} g \begin{Bmatrix}\begin{Bmatrix}\{\omega_1\}\\ \{\omega_2\}\end{Bmatrix}\\ \begin{Bmatrix}\{x\}\\ \{q\}\end{Bmatrix}\end{Bmatrix}=\begin{Bmatrix}\{F_{\omega_1\omega_2}\}\\ \{F_{xq}\}\end{Bmatrix} \tag{3.1}$$

where:

$M_{xq}$ - the global matrix of the masses - the dimension of which is $[n,n]$ and which is defined through:

$$M_{xq}=\begin{bmatrix}[M_x] & [0]\\ [0] & [M_{red}]\end{bmatrix}, \tag{3.2}$$

with:

$[M_x]$ - the matrix of the masses for the propelling part which does correspond to the respective pinion and to the concerned conditions of leaning (shaft and engine with the dimension $[24,24]$);

$[M_{red}]$ - the matrix of the modal mass reduced in the case of the respective transmission which does represent the whole of its receiving part (with the dimension $[N,N]$);

$\begin{bmatrix}[K_{\omega_1\omega_2}] & [K_{\omega_1\omega_2 xq}]\\ [K_{qx\omega_2\omega_1}] & [K_{xq}]\end{bmatrix}$ - the matrix of global rigidity which is dependent upon time while the displacements are considered )to be generated by the teeth and through the general coordinates of the system (with the dimension $[N_1+N_2+n,\ N_1+N_2+n]$);

$\{F_{xq}\}$ - the vector of the exterior generalized forces which do depend upon time and which do regroup the terms of the static efforts as well as the ones of the excitations which are introduced through the setting errors (with the dimension $[n,1]$);

$\{F_{\omega_1\omega_2}\}$ - the vector depending upon time which does introduce the effect of the geometrical deviations (with the dimension $[N_1+N_2, 1]$);

$\{x\}$ - the vector of the state variables which do consist in the generalized coordinates held by the propelling part of our transmission which is modelled through four knots, each of which disposing of six degrees of freedom (with the dimension $[24,1]$);

$\{q\}$ - the vector constituted of the modal degrees of freedom of our sub-structure which does constitute the receiving part of our transmission (with the dimension $[N,1]$);

$\{\omega_1\}$ and $\{\omega_2\}$ - are respectively the displacements which are seen to be respectively generated by the elastic bases of the pinion (with the dimension $[N_1,1]$) and by the ones of the guided wheel (with the dimension $[N_2,1]$);

$[C_{xq}]$ - the matrix of the global cushioning (with the dimension $[n,n]$).

Let us suppose the fact that for a system of a mean rigidity the global cushioning matrix is orthogonal in respect to its own vibrating modalities. We could therefore write:

$$c^{\phi}_{jk} = 2\xi_j\sqrt{k^{\phi}_{jk}m^{\phi}_{jk}}\ , \tag{3.3}$$

where:

$$c^{\phi}_{jk} = 0, j \neq k \tag{3.4}$$

$c^{\phi}_{jk}$ - are the components of the matrix $[C_{\phi}]$;

$k^{\phi}_{jk}$ - are the diagonal components of the rigidity modal matrix:

$[K_{\phi}] = [\phi]^T g[K_{qq}]g[\phi]$;

$m^{\phi}_{jk}$ - are the diagonal components of the mass modal matrix:

$[M_{\phi}] = [\phi]^T g[m_{qq}]g[\phi]$;

$[\phi]$ - is the matrix of its own vibrating modalities for a system of a mean rigidity;

$\xi_j$ - is the relative modal cushioning factor for the module j.

As we have to take into consideration the diversity and the large number of the modes involved by this model our goal is to estimate the modal cushioning indices of the teeth through a ponderated mean which would rely upon the percentages of the deforming energy attributed to the teeth on one side and the ones of the rest of the structure on the other side. Let then be:

$$\xi_j = \xi_{angrenaj}\rho_j + (1-\rho_j)\xi_{structură}$$

where:

$\rho_j$ - is the modal relative deformation percentage of the teeth for the mode j;

$\xi_{angrenaj}$ - does hold a value which is typical for the cushioning indices of the teeth and does rely upon the experimental works of d'Umezawa.

$$0,03 < \xi_{angrenaj} < 0,07$$

$\xi_{structură}$ - is in a similar way representing the cushioning upon the parts of the structure for which a value of the order 0,01 would be pretty reasonable.

The matrix of the cushioning is expressed through the reversed transformation:

$$[C_{qq}] = [\phi]^{-T} g[C_{\varphi}]g[\phi]^{-1}. \tag{3.5}$$

In order to reduce the number of unknown sizes the movement equation (3.1) could be decomposed under the following form:

$$\begin{cases} \left[K_{\omega_1\omega_2}\right]\begin{Bmatrix}\{\omega_1\}\\ \{\omega_2\}\end{Bmatrix} + \left[K_{\omega_1\omega_2 xq}\right]\begin{Bmatrix}\{x\}\\ \{q\}\end{Bmatrix} = \{F_{\omega_1\omega_2}\} \\ \left[M_{xq}\right]\begin{Bmatrix}\{\ddot{x}\}\\ \{\ddot{q}\}\end{Bmatrix} + \left[C_{xq}\right]\begin{Bmatrix}\{\dot{x}\}\\ \{\dot{q}\}\end{Bmatrix} + \left[K_{xq\omega_1\omega_2}\right]\begin{Bmatrix}\{\omega_1\}\\ \{\omega_2\}\end{Bmatrix} + \left[K_{xq}\right]\begin{Bmatrix}\{x\}\\ \{q\}\end{Bmatrix} = \{F_{xq}\} \end{cases} \tag{3.6}$$

The first equation of the system (3.6) does lead us to the equation:

$$\begin{Bmatrix}\{\omega_1\}\\ \{\omega_2\}\end{Bmatrix} = \left[K_{\omega_1\omega_2}\right]^{-1}\left(\{F_{\omega_1\omega_2}\} - \left[K_{\omega_1\omega_2 xq}\right]\begin{Bmatrix}\{x\}\\ \{q\}\end{Bmatrix}\right) \tag{3.7}$$

which, once introduced into the second equation of this system of differential equations should finally lead us to the equation:

$$\left[M_{xq}\right]\begin{Bmatrix}\{\ddot{x}\}\\ \{\ddot{q}\}\end{Bmatrix} + \left[C_{xq}\right]\begin{Bmatrix}\{\dot{x}\}\\ \{\dot{q}\}\end{Bmatrix} + \begin{Bmatrix}\{x\}\\ \{q\}\end{Bmatrix}\left(\left[K_{xq}\right] - \left[K_{xq\omega_1\omega_2}\right]\left[K_{\omega_1\omega_2}\right]^{-1}\left[K_{\omega_1\omega_2 xq}\right]\right) = \{F_{xq}\} - \left[K_{xq\omega_1\omega_2}\right]\left[K_{\omega_1\omega_2}\right]^{-1}\{F_{\omega_1\omega_2}\}$$

or wich

$$\left[K(t,\omega_1,\omega_2,x,q)\right] = \left[K_{xq}\right] - \left[K_{xq\omega_1\omega_2}\right]\left[K_{\omega_1\omega_2}\right]^{-1}\left[K_{\omega_1\omega_2 xq}\right]$$
$$\{K(t,\omega_1,\omega_2,x,q)\} = \{F_{xq}\} - \left[K_{xq\omega_1\omega_2}\right]\left[K_{\omega_1\omega_2}\right]^{-1}\{F_{\omega_1\omega_2}\}, \tag{3.8}$$

to the equation:

$$\left[M_{xq}\right]\begin{Bmatrix}\{\ddot{x}\}\\ \{\ddot{q}\}\end{Bmatrix} + \left[C_{xq}\right]\begin{Bmatrix}\{\dot{x}\}\\ \{\dot{q}\}\end{Bmatrix} + \left[K(t,\omega_1,\omega_2,x,q)\right]\begin{Bmatrix}\{x\}\\ \{q\}\end{Bmatrix} = \{K(t,\omega_1,\omega_2,x,q)\}. \tag{3.9}$$

## 4. NUMERICAL SOLVING OF MOVEMENT EQUATIONS

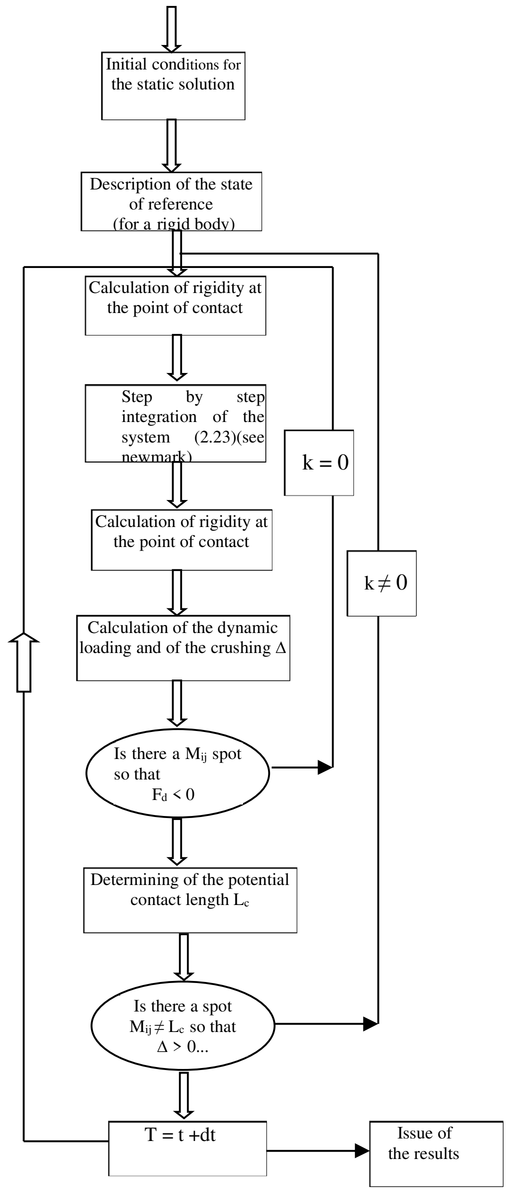


**Fig.5.** Resolving scheme created by Pasternak coupled with a contact algorithm

The obtained result has been a system of differential equations of the second order with parametrical excitations which might as well be non-linear ones. The numerical resolution of the second order system of differential equations (3.9) could be done with the help of the implicit scheme of Newark coupled with a normal unilateral contact algorithm (see figure 5). In order to integrate the mathematical models of a linear type we have made use of among others of the method that does consist in applying the integrated Laplace transformation. Through an original procedure this method has been as well extended to some non-linear mathematical models. We did not neglect as well the newest trend of the systems'theory. In what does follow we will present an original method able to integrate the previously presented matrix equation (3.9) which does call for the rendering discrete the time interval during which the movement is watched upon.

***Stage 1***

Let be

$$[0;t_n]=\bigcup_{j=0}^{n-1}[t_j;t_{j+1}]$$

a division of the time interval during which the movement is watched so that for:

$\forall t \in [t_j;t_{j+1}]$

the elements of the global rigidity matrix may be considered as being constant. Therefore - keeping as valid the matrix writing - they have the values:

$$\begin{bmatrix} [K_{\omega_1\omega_2 j}] & [K_{\omega_1\omega_2 xqj}] \\ [K_{qx\omega_2\omega_1 j}] & [K_{xqj}] \end{bmatrix}.$$

***Stage 2***

To the matrix equation (3.9) we are applying the unilateral Laplace transformation in respect to time for:

$\forall t \in [t_j;t_{j+1}]$

the result being the mathematical model of the vibrations sustained by the gearing mechanism expressed through the Laplace images of the unknowns:

$\{\tilde{x}_j(s)\}$si $\{\tilde{q}_j(s)\}$

under the form:

$\{\widetilde{K}(s)\}=$

$$s^2[M_{xq}]\begin{Bmatrix}\{\tilde{x}_j(s)\}\\\{\tilde{q}_j(s)\}\end{Bmatrix}-s[M_{xq}]\begin{Bmatrix}\{x(t_j)\}\\\{q(t_j)\}\end{Bmatrix}-[M_{xq}]\begin{Bmatrix}\{\dot{x}(t_j)\}\\\{\dot{q}(t_j)\}\end{Bmatrix}+$$

$$+s[C_{xq}]\begin{Bmatrix}\{\tilde{x}_j(s)\}\\\{\tilde{q}_j(s)\}\end{Bmatrix}-[C_{xq}]\begin{Bmatrix}\{x(t_j)\}\\\{q(t_j)\}\end{Bmatrix}+[K_j]\begin{Bmatrix}\{\tilde{x}_j(s)\}\\\{\tilde{q}_j(s)\}\end{Bmatrix}$$

or, by grouping the terms:

$$\left(s^2[M_{xq}]+s[C_{xq}]+[K_j]\right)\begin{Bmatrix}\{\tilde{x}_j(s)\}\\\{\tilde{q}_j(s)\}\end{Bmatrix}=$$

$$=\{\widetilde{K}(s)\}+s[M_{xq}]\begin{Bmatrix}\{x(t_j)\}\\\{q(t_j)\}\end{Bmatrix}+[C_{xq}]\begin{Bmatrix}\{x(t_j)\}\\\{q(t_j)\}\end{Bmatrix}+[M_{xq}]\begin{Bmatrix}\{\dot{x}(t_j)\}\\\{\dot{q}(t_j)\}\end{Bmatrix}. \tag{4.1}$$

***Stage 3***

Let us solve the algebraic system (4.1) in order to obtain, expressed within a formal writing, the dynamic answer through the Laplace images of the unknowns as in what follows:

$$\begin{Bmatrix}\{\tilde{x}_j(s)\}\\\{\tilde{q}_j(s)\}\end{Bmatrix}=$$

$$=\frac{adj\left[s^2[M_{xq}]+s[C_{xq}]+[K_j]\right]\left(s[M_{xq}]\begin{Bmatrix}\{x(t_j)\}\\\{q(t_j)\}\end{Bmatrix}+[M_{xq}]\begin{Bmatrix}\{\dot{x}(t_j)\}\\\{\dot{q}(t_j)\}\end{Bmatrix}+[C_{xq}]\begin{Bmatrix}\{x(t_j)\}\\\{q(t_j)\}\end{Bmatrix}\right)}{P(s)}+ \tag{4.2}$$

$$+\frac{adj\left[s^2[M_{xq}]+s[C_{xq}]+[K_j]\right]}{P(s)}\{\widetilde{K}(s)\},$$

where:

$$P(s)=\det\left[s^2[M_{xq}]+s[C_{xq}]+[K_j]\right].$$

***Stage 4***

The reversing of the Laplace transformation in (4.2) is done through calling for the reversing theorems while for the term which does contain the vector $\{\widetilde{K}(s)\}$ the convolution theorem is going to be applied. The reversing does lead to the determining of the dynamic response for $\forall t \in \left[t_j; t_{j+1}\right]$ under the form of the time functions:

$$\begin{Bmatrix} \{x_j\} \\ \{q_j\} \end{Bmatrix} = \begin{Bmatrix} \{x_j(t)\} \\ \{q_j(t)\} \end{Bmatrix}. \tag{4.3}$$

***Stage 5***

In the end, by making use of the Heaviside function:

$$H(t-a) = \begin{cases} 0; t < a \\ 1; t > a, \end{cases}$$

we are obtaining the solution of the equation (3.9) while the time functions which do represent the dynamic response of the system are expressed under the forms of some series as do illustrate the expressions below:

$$\begin{Bmatrix} \{x(t)\} \\ \{q(t)\} \end{Bmatrix} = \begin{Bmatrix} \left\{ x_0(t) + \sum_{j=0}^{n-1} \left[ x_{j+1}(t) - x_j(t) \right] H\left(t - t_{j+1}\right) \right\} \\ \left\{ q_0(t) + \sum_{j=0}^{n-1} \left[ q_{j+1}(t) - q_j(t) \right] H\left(t - t_{j+1}\right) \right\} \end{Bmatrix}. \tag{4.4}$$

## 5. CONCLUSION

As a conclusion we may say that the developments we have presented in the present work have allowed us, through an original method, to predict the static and dynamic behaviors of the transmission through gearing mechanisms which do make use of elements chosen from some flexible structures. The developed models do represent a hybrid approach which does consist in the combination between the finite elements of a classical girder and the elastic bases for the contact among the teeth and for the structural flexible parts of the sub-structures which do pertain to the models with finite tri-dimensional elements. The shaft and the body of the pinion (excluding the teeth) have been modelled out of elements of a girder which have been submitted to the processes of bending, torsion and traction-compression. The contact between two teeth has been assimilated with two Pasternak elastic bases having different features which are connected through some independent contact rigidities. This fact has allowed us to take into consideration the elastic coupling among the contact points. The resolving scheme by Pasternak coupled with a contact algorithm has been presented in parallel with our own method of solving a system of non-linear differential equations, an original method able to integrate the above mentioned matrix equation which does make use of the procedure of rendering discrete the time interval during which the movement is watched upon. Let us also remark the fact that the applying of the unilateral Laplace transformations in respect to time does render algebraic the concerned problem which this way is hugely simplified.

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## O metoda de analiza a vibratiilor unor sisteme mecanice care contin roti dintate

**Rezumat.** Pornind de la modelul matematic pentru deplasarea relativă a două roţi dinţate ale unui angrenaj (model dat sub forma unui sistem de ecuaţii diferenţiale de ordinul doi cu excitaţii parametrice, eventual neliniare), în această lucrare vom prezenta o metodă originală de integrare a ecuaţiei matriceale de mişcare care apelează la discretizarea intervalului de timp de observare a mişcării. Aplicarea transformatelor integrale Laplace, unilaterale în raport cu timpul, va conduce la algebrizarea modelului, ceea ce simplfică enorm problema. În plus, dăm şi reprezentările grafice ale deplasărilor relative pentru angrenajul considerat.

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