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ANALYSIS OF THE DISPLACEMENT AMPLIFICATION RATIO FOR A PIEZOELECTRIC DRIVEN COMPLIANT MECHANISM

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Abstract: *Piezoelectric elements are widely used in many precision systems and devices. Although they have many advantages, piezo elements offer limited displacements and have hysteresis nonlinearity. When a larger displacement is needed, a very common approach is to combine a piezoelectric element with a mechanical displacement amplifier usually based on a compliant mechanism. An important feature of these mechanisms is the displacement amplification ratio. In the paper the amplification ratio of a compliant mechanism driven by a piezoelectric stack, was determined using theoretical and experimental methods.*
Key words: *Piezoelectric stack, compliant mechanisms, amplification ratio, hysteresis*

1. INTRODUCTION

Nowadays, piezoelectric stacks are used in many applications such as energy harvesting [1], microgrippers [2] or micro/nano positioning systems [3] [4]. They benefit of low volume, high resolution and bandwidth, high stiffness, and fast response. But the piezoelectric elements can provide only a limited displacement and have hysteresis nonlinearity.

Usually the deformation range of a piezoelectric stack is about 0.1% of its length [5]. Therefore, in most applications where greater displacement is required, compliant mechanisms are used to amplify the stroke of piezoelectric elements.

Compliant mechanisms can be generally divided in two categories: with flexure hinges or with flexure elements. For the first category, displacements are due the deformation of the flexure hinges which are parallel to the axis of the actuating force. In the second category, the displacements are caused by the deformation of the elements. A survey of various amplification mechanisms is presented in [6].

When designing these amplification mechanisms, some accurate theoretical models are required in generally to predict the output displacements caused by the elongation of the piezoelectric stack. An overview of kinetostatic and dynamic modeling approaches for compliant mechanisms is presented in [7]. In addition to the finite element analysis, the most used methods for kinetostatic modeling are based on: Castigliano's second theorem, elastic beam theory and compliant matrix method [7],[8]. The ideal displacement amplification ratio can also be derived using kinematic theory [5], [9].

In this paper the amplification ratio of a compliant mechanism with flexure elements driven by a piezoelectric stack, is determined using theoretical and experimental methods.

2. ANALYSIS OF THE AMPLIFICATION MECHANISM

2.1 Kinematic analysis

Figure 1a) shows a schematic representation of a compliant mechanism with flexure elements. The vertical output displacement δy is due to the force F_{Pz} generated by a piezoelectric stack. The actuating force is on the longitudinal direction and forms an angle with element AB.

From the point of view of kinematic analysis, these mechanisms can be simplified into ideal mechanisms with rigid elements and revolute joints as presented in figure 1b). As can be seen, the mechanism has a double symmetrical structure, so it is enough if we analyze only a quarter of its structure.

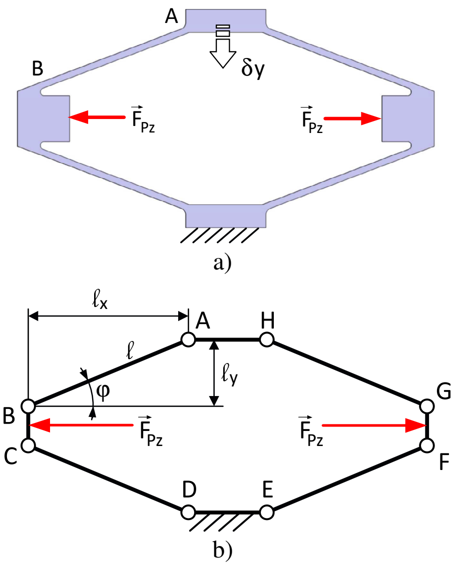


Fig.1 Schematic representation of a compliant mechanism

Figure 2 shows the simplified kinematic model of element AB. In this simplifying hypothesis, the horizontal displacement ∂x of joint B, produces a vertical output displacement ∂y for joint A, which modifies the angle φ of the arm AB. The ratio between output displacement and input displacement represents the ideal amplification ratio.

In [5] and [9] several methods to obtain the amplification ratio using kinematics theory, are presented.

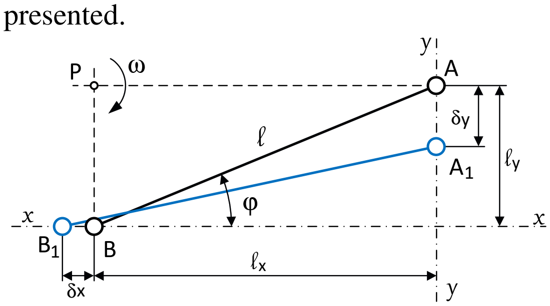


Fig.2 Simplified kinematic model of element AB

The instantaneous center of rotation of element AB is located at point P, therefore the instantaneous velocities of joints A and B can be written as follows:

$$\begin{cases} v_B = \omega \cdot l_x \\ v_A = \omega \cdot l_y \end{cases} \quad (1)$$

where:

$$\begin{cases} v_B = \dfrac{dx}{dt} \\ v_A = \dfrac{dy}{dt} \end{cases} \quad (2)$$

The amplification ratio can be written as:

$$A = \frac{dy}{dx} = \frac{v_A}{v_B} \quad (3)$$

Considering the equations (1) and (2), it results:

$$A = \frac{\omega \cdot l_y}{\omega \cdot l_x} = \frac{l_y}{l_x} = ctg\ \varphi \quad (4)$$

The amplification ratio can also be obtained by writing the projection equations of the element AB on the *x* and *y* axis. The result is a 2nd degree equation with the unknown ∂y. In this way the amplification ratio can be expressed as [9],[10]:

$$A = \frac{l \sin\varphi + \sqrt{l^2 \sin^2\varphi - \partial x^2 - 2l\partial x \cos\varphi}}{\partial x} \quad (5)$$

where φ can be derived as [11]:

$$\varphi = arccos\left(\frac{l_x + \partial x}{\sqrt{l_y^2 - l_x^2}}\right) - arctan\left(\frac{l_y}{l_x}\right) \quad (6)$$

From the kinematic analysis it results that the amplification factor depends only on the value of the angle φ. In fact, the output displacement is due to the bending deformation of the arms and it is greatly influenced by the material properties.

2.2 The elastic beam method

The simplified static model is presented in Figure 3.

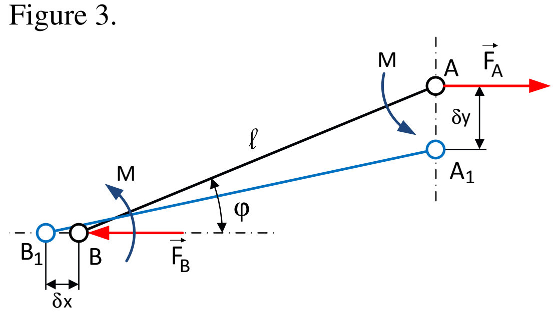


Fig.3 Static model of element AB

Considering the element AB in static equilibrium, the following equations can be written:

$$F_B = F_A = F_{Pz} = F \\ 2M = Flsin\varphi \quad (7)$$

The analytical modeling of the displacement amplification ratio based on elastic beam theory is presented in detail in [7] [8]. According to this method the relationship between de input displacement and the force generated by the piezoelectric stack can be expressed as follows:

$$\delta x = \left(\frac{cos^2\varphi}{K_l} + \frac{l^2 sin^2\varphi}{12K_\varphi}\right)F \quad (8)$$

where: K_l and K_φ represents the translational and rotational stiffness.

Figure 4 shows that the output displacement δy_N for a point N on the element AB is in fact, the normal component of the perpendicular deflection η_N.

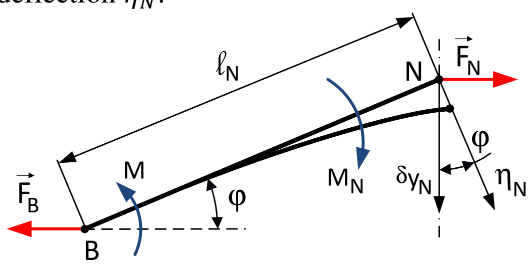


Fig.4 Deflection of element AB

From [8] the perpendicular deflection can be written as:

$$\eta = \frac{l^2 F sin\varphi}{12K_\varphi} \quad (9)$$

therefore, the output displacement is:

$$\delta y = \eta cos\varphi = \frac{l^2 sin\varphi cos\varphi}{12K_\varphi}F \quad (10)$$

Combining equations (8) and (10) the amplification ratio can be obtained as follows:

$$A = \frac{\delta y}{\delta x} = \frac{K_l l^2 sin\varphi cos\varphi}{12K_\varphi cos^2\varphi + K_l l^2 sin^2\varphi} \quad (11)$$

If w and h define the cross-section of the flexure element, the expressions for translational and rotational stiffness can be written as [12]:

$$K_l = \frac{Ewh}{l} \\ K_\varphi = \frac{Ewh^3}{12l} \quad (12)$$

where: E represents the Young's modulus and l is the length of the flexure element.

2.3 Finite element analysis

To estimate the output displacement of the flexure mechanism we performed a finite element analysis (FEA) using CATIA®.

The amplification mechanism is considered made of an aluminum alloy with Young's modulus $E = 69 \cdot 10^9 N/m^2$, yield strength $\tau = 9.5 \cdot 10^7 N/m^2$ and with the overall dimensions presented in figure 5. The flexure elements form the angle $\varphi = 6^o$ with the longitudinal direction.

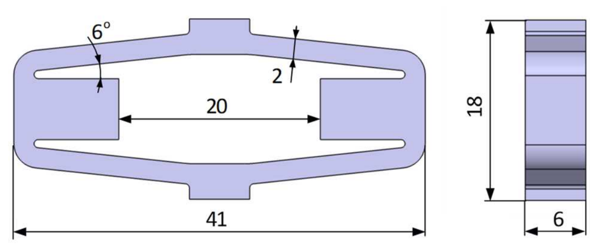
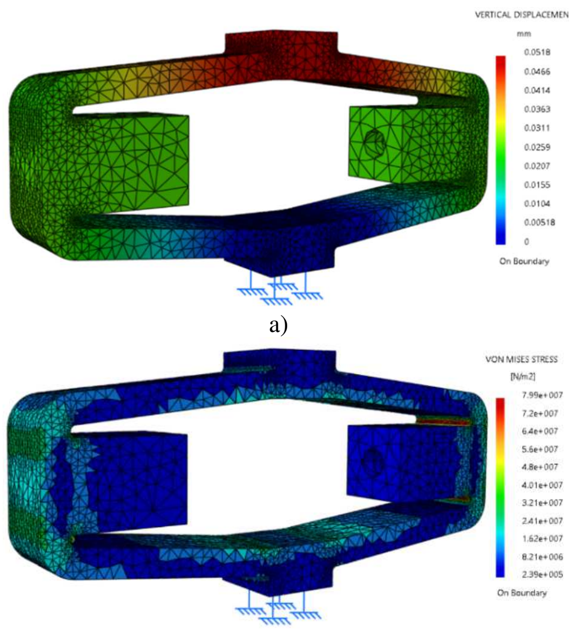


Fig.5 CAD model of the amplification mechanism

The maximum elongation of the piezoelectric stack was considered equal to 16µm. The results of FEA are presented in figure 6. Maximum value for vertical displacement, according to figure 6a) is 0.0518 mm in -y direction. For this value, a maximum theoretical amplification ratio can be obtained as:

$$A_{FEA} = \frac{\delta y}{\delta x} = \frac{0.0518}{0.0160} = 3.237 \quad (13)$$

Figure 6b) shows the stress distribution in the structure of amplification mechanism due to the deformation of the piezoelectric stack. The FEA error was less than 1%.



a)

b)

Fig.6 FEA results

3. EXPERIMENTAL RESULTS

The amplification mechanism was designed to be driven by a PZS001 co-fired piezoelectric stack form Thorlabs, whose specifications are listed in Table 1. The mechanism was manufactured of a 6 mm aluminum alloy plate, according to the dimensions presented in figure 5. To eliminate the unwanted backlash, the piezoelectric stack was mounted pre-stressed.

Table 1

Specifications of the piezoelectric stack

Overall dimensions:	6.0 mm x 7.0 mm x 20.0 mm
Piezo material:	lead zirconium titanate (PZT)
Piezo cross-section:	5.0 mm x 5.0 mm
Blocking Force:	850 N
Drive Voltage:	150 V
Displacement:	17.4µm ±2µm
Resonant Frequency:	69kHz

The piezoelectric actuator has attached four strain gauges in a Wheatstone bridge circuit configuration. Thus, it can be used in a closed-loop system, in which the strain gauge signal is used to determine the piezo control drive voltage and to compensate hysteresis nonlinearities [10].

In order to validate the previously presented theoretical models, we configured an experimental stand presented in figure 7.

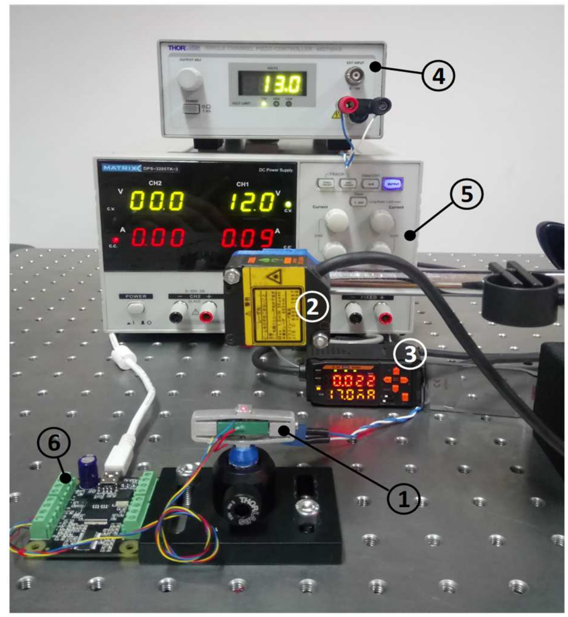
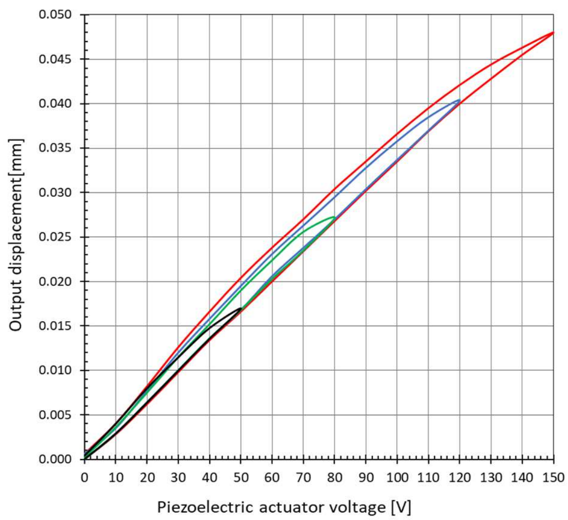


Fig.7 Experimental stand

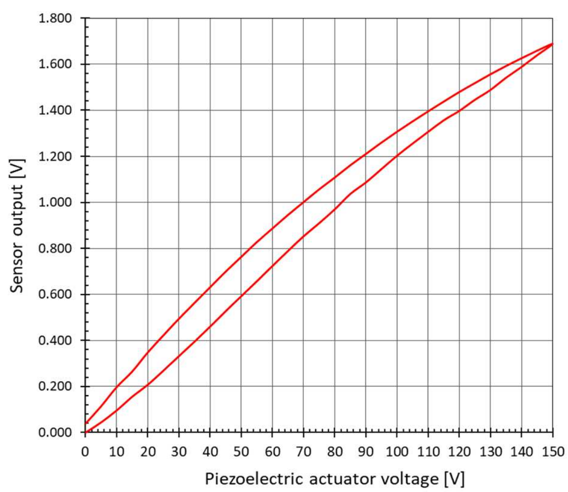
The designed amplification mechanism (1) was mounted through a clamping device on an anti-vibration table. The vertical displacement is measured with an external laser displacement sensor (2) form OMRON. On the controller (3) is displayed the measured value of displacement with an accuracy of 1 µm. The controller also has an analog output port (4…20mA) through which the values can be read with an external DAQ device. An MDT694 single channel piezo controller (4) from Thorlabs, was used to drive the piezoelectric stack with a voltage between 0 and 150V. The voltages required to supply the controller (3) and the laser sensor (2) are provided by the power source (5). To measure the signal from the strain gauge sensors we use an USB based DAQ device (6) with a resolution of 24bit and a sampling time of 8ms.

Based on the average values of a several series of measurements, the graphs presented in figure 8 were obtained.

Therefore, in figure 8 a) the relationship between the output displacement of the bridge-type mechanism and the input voltage of the piezoelectric stack is presented. Starting from the maximum range (0 ... 150V) we decreased the amplitude of the input voltage for the piezo actuator in four steps. Experimental measurements revealed a hysteresis error of about 12%. Maximum value of the measured displacement was 0.0480 mm in -y direction.



a)



b)

Fig.8 Hysteresis loops of the actuator

The relationship between the signal from the strain bridge sensor and the input voltage of the piezoelectric stack in presented in figure 8b).

3.1 Comparison of results

The theoretical and experimental output displacements of the amplification mechanism for different values of piezo elongation are shown in figure 9. Are presented also the values for the maximum amplification ratio. For the kinematic model, the values are calculated using equation (5). The relationship between the flexure angle φ and the piezoelectric stack elongation is based on equation (6). The calculated values are presented in figure 10. The results for the elastic beam method are based on equation (11).

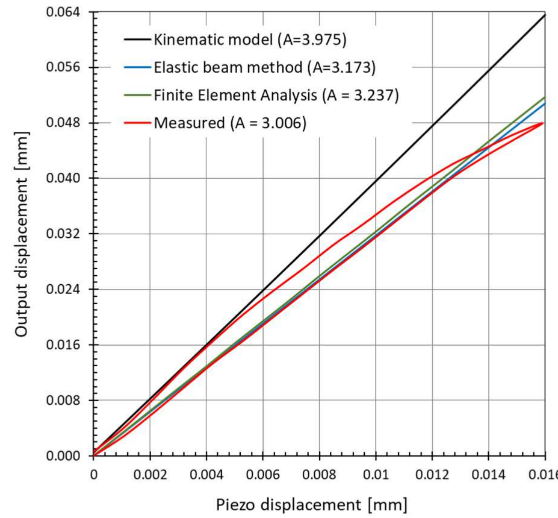


Fig.9 Theoretical and experimental results comparison

As can be seen from Figure 10, the results obtained using the elastic beam method and those obtained by FEA are quite similar. The difference between the two models is about 1.77%. The kinematic model has the largest error compared to the FEA, of about 18,7%.

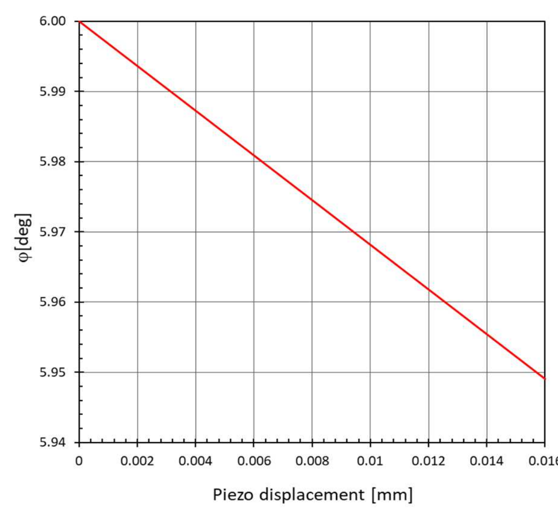


Fig.10 Flexure angle vs piezo displacement

The difference between the experimental values and those obtained by the FEA or by elastic beam method is mainly due to the prestressed mounting of the piezoelectric stack. It must also be considered, the manufacturing errors of the amplification mechanism, the fact that the real material parameters are different from those used in the theoretical models and last but not least the measurement errors

4. CONCLUSION

In this paper, the amplification ratio of a compliant mechanism with flexure elements driven by a piezoelectric stack, was determined using theoretical methods. The first method based on kinematic theory, provides the most inaccurate results. The second method based on elastic beam theory is the closest to the experimental results and those obtained by FEA.

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ANALIZA FACTORULUI DE AMPLIFICARE AL DEPLASARII PENTRU UN MECANISM COMPLIANT ACTIONAT PIEZOELECTRIC

Rezumat: Elementele piezoelectrice sunt utilizate pe scară largă în multe sisteme și dispozitive de precizie. Deși au numeroase avantaje, elementele piezoelectrice au curse limitate și caracteristici neliniare (histerezis). Când este necesară o deplasare mai mare, o abordare obișnuită este combinarea elementului piezoelectric cu un amplificator de deplasare mecanică, de obicei bazat pe un mecanism compliant. O caracteristică importantă a acestor mecanisme este raportul de amplificare a deplasării. În lucrare, a fost determinat folosind metode teoretice și experimentale, raportul de amplificare al unui mecanism compliant acționat cu un element piezoelectric

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