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ANALYTICAL MODEL FOR PREDICTING THE OPTIMUM HEATING CONDITIONS USED IN HEAT PROCESSING

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Abstract: *The decision to adopt a certain heating conditions must consider the need to avoid dangerous thermal stresses that could cause products deformation, especially those with complex configuration, or even damage to their integrity and at the same time a minimum value of heating time, with direct consequences on technological consumption and productivity, so costs.Starting from the premise that in furnaces with discontinuous operation the heating is most frequently performed in thermal conditions characterized by constant value of heat flux (q =Jc = ct., corresponding to the 2nd oder limit conditions), until the heating chamber attain the required equilibrum temperature ($T_c = T_t + 20 \div 40^oC$) subsequently passes into T_c = ct (3rd order limit conditions), it is particularly important to anticipate the value of the temperature drop during the first stage of heating (q=Jc = ct.), to conclude whether the power stage adopted was correct or not, so what technological measures must be taken so that the heating takes place in optimal conditions.The paper presents the conceptual scheme and the analytical model, with the afferent justifications, containing the necessary steps to be followed in order to predict and optimize the appropriate heating conditions from all points of view.*

Key words: *Heating regime; q=Jc=ct;Tc=Tco+Ct;Tc=ct; Thermophysical Properties; Thermal Gradient; Thermal Stresses.*

INTRODUCTION. Issue type scheme

In the practice of thermal treatments, five thermal regimes are frequently used to heat the metallic parts for thermal processing (fig.1)-[1-7]

a) Slow heating performed in the same time with the thermal aggregate; b) heating in the furnace where the temperature it's kept constant; c) the temperature of the thermal aggregate at the moment of placing of the batch of parts into the thermal aggregate is higher compared to that imposed by the thermal processing and decreases over time; d) the temperature of the thermal aggregate at time of insert the batch of parts in the furnace, it's much higher comparing with that one imposed by the thermal processing and remains constant over time; e) heating with pre-heating, which can be realised in another chamber of the furnace (or even another furnace).

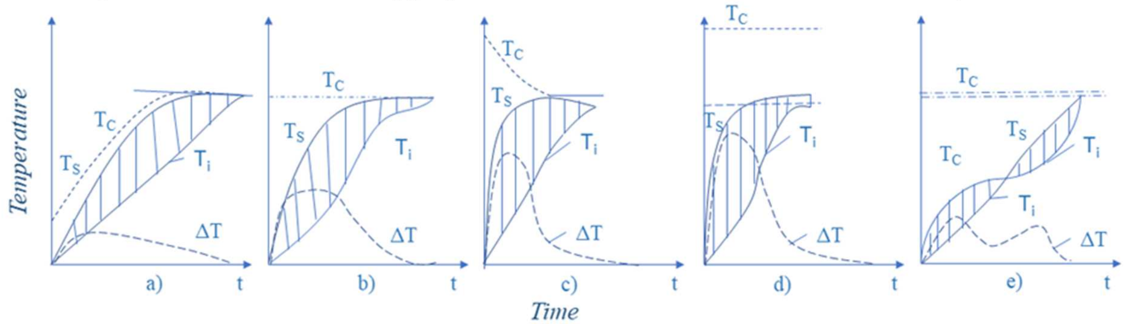


Fig.1 Thermal regimes frequently used in the practice of heat treatments; Tc – furnace temperature (Tm – temperature of the heating environment); Ts – surface temperature; Ti – temperature of the product centre ; ΔT=Ts-Ti – thermal gradient

Note. The thermal regimes presented have as a common feature the fact that they are subordinated to a concrete way of varying the temperature of the heating environment: the heat transfer from the heating medium to the batch of parts is carried out under conditions of moderate values of the heat emission coefficient, α, and the relative transfer coefficient, respectively h =α/λ [m^{-1}]). An exception is the regime d), in which for extremely high values of thermal transmissivity, α and relative transfer coefficient h, respectively, the surface registers an instantaneous temperature jump from the initial value to a value equal to the temperature of the heating medium, and the thermal gradient registers a maximum value from the first moment of the product introduction in the thermal aggregate (immediately after the installation of the regular regime).

The heat treatment furnaces with discontinuous operation – chamber type – are

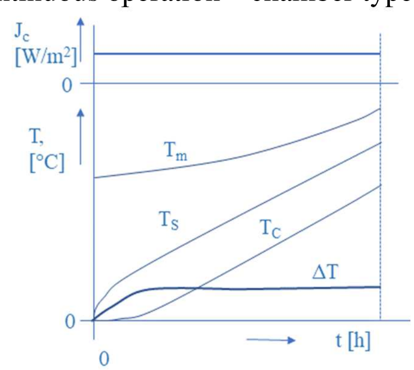
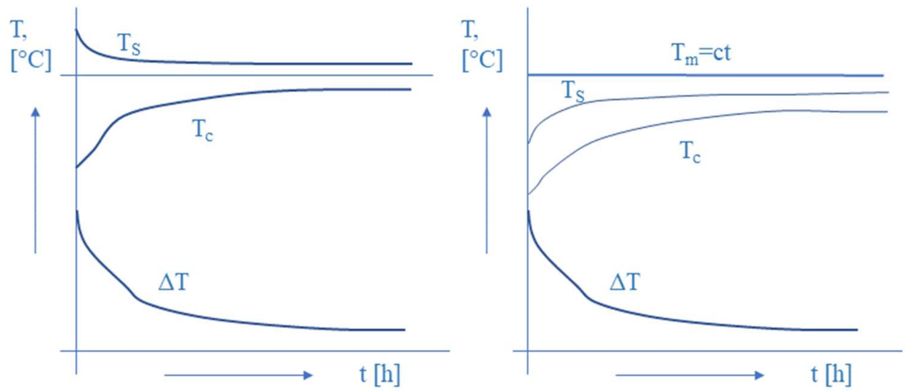


Fig.2 Heating regime with constant heat flow (q=Jc=ct) characteristic of heat treatment furnaces with discontinuous operation; Jc=q_c – heat flow; Tm – temperature of the heating medium (furnace temperature); Tc – temperature of the median area of the product (the centre of the piece)

working frequently with constant heat flow (fig.2), ensuring a swift return of the temperature at the characteristic value of the heat treatment. In case of use a such thermal regime, the value of the thermal gradient established on the section of the product subjected to thermal processing is constant and dependent on the heat flow value, Jc (so, the power level adopted), $\Delta T = \frac{J_C \cdot X}{2 \cdot \lambda}$, where X represent the S or R, depending by the symmetry of the product subjected processing – plane or cylindric, λ represent the coefficient of thermal conductibility, specific to the material from which the product came. As a consequence, this can be rigorously controlled by using the value of the heat flow, meaning the useful furnace power (Jc=q=Pu/A, where Pu represent the useful furnace power part, the one that turns into heat transferred to the parts, A – the corresponding area). Rigorous control of the thermal gradient on the section of the product subjected to thermal processing can be ensured by controlling the power of the thermal aggregate, both when using the linear variation of the temperature of the thermal aggregate and in the thermal regime characterized by constant furnace temperature. In the case of linear variation mode of the environment temperature which takes place at a level corresponding to the thermal transmission (expressed via relative transfer coefficient h =α/λ [m^{-1}]) respective of Biot criteria very high (”massive” product from the point of view of thermotechnical behaviour) the thermal regime comes to be identified with the linear variation of the surface temperature of the product, thus ensuring a value of the temperature drop per section constant in time and controllable by means of the electric power of the thermal aggregate. ***Note***. If the above conditions are not fulfilled, those related to thermotechnical behaviour of the processed products using a linear variation of the medium temperature, the surface temperature evolution will be subordinated to a law of temperature variation other than those related to the variation of environment temperature, established by the presence of a supplementary terms within of the criterial solution, which quantifies the presence and the level of the thermal transmissivity from the heat environment to the product surface and by the conductibility in its volum.

The temperature equalization on the section of heated products using the usual thermal regimes for heating the products in the heat treatment furnaces with discontinuous operation (most frequently, constant heat flow or linear variation of the furnaces temperature) takes place using a null heat flux regime, equivalent to interruption of the power supply (fig.3a) or constant furnace temperature (fig.3b), adopted according to temperature of the heat treatment of the product / batch to be processed: Tc = Tt + (20 ÷ 40) º C (fig.3).



a) b)

Fig.3 Thermal regimes used in order to equalize the temperature on the section heat treated pieces; a) q=Jc=ct=0 W/m^2 ;b) Tm=ct

PROBLEM SOLVING

A particular importance in choosing a certain thermal processing regime it is represented by the way of selecting the heating conditions, in order to avoid the occurrence of dangerous thermal stresses that may lead to irremediably deformations or even integrity destruction of the products subjected to thermal processing. Knowledge how to distribute the temperature against time on the products section or on the batch subjected to thermal processing and therefore of gradients and thermal stresses, will be possible by using the criteria solutions $\Theta, \Delta T, \sigma = f(Fo, Bi, x/X)$ of the differential equation of the heat conductance (Fourier equation), $\frac{\partial T}{\partial t} = a \cdot \frac{\partial^2 T}{\partial x^2}$ or of the graphic expression of them, respectively tabular values (values calculated by Russel[1;2;4]), obtained by solving in a limit conditions of order I, II or III, strictly dependent on the concrete conditions for deployment of the process (heating). In case of using thermal aggregates with discontinuous operation, as a result of drop furnace temperature after removing the heated batches and introducing the next cold batches, to quickly return the thermal aggregate at the temperature imposed by the future thermal treatment (Tc=Tt+20÷40ºC) must ensure a constant value of the heat flow (q=Jc=ct), and the limit conditions under which the differential equation of the thermal conductibility will be resolved analytical and criterial will be those of order II. The criterion solutions used to determine the temperature evolution over time on the product section or batches with different symmetries (plane, cylindrical etc) heated in constant heat flow have the following expressions:

$$\frac{2\lambda[T(x,t)-T_0]}{J_c \cdot S} = \Phi\left(\frac{a \cdot t}{S^2}; \frac{x}{S}\right) \qquad (1)$$

for the products with plane symmetry, respectively:

$$\frac{2\lambda[T(r,t)-To]}{J_c \cdot R} = \Phi\left(\frac{a \cdot t}{R^2}; \frac{r}{R}\right) \qquad (2)$$

for those with cylindrical symmetry.

These functions $\Phi\left(\frac{a \cdot t}{S^2}; \frac{x}{S}\right)$ and $\Phi\left(\frac{a \cdot t}{R^2}; \frac{r}{R}\right)$ representing the relative temperature of the products with plan or cylindrical symmetries, heated in constant heat flow, have graphical expressions to facilitate the temperature establishment, but in regular regime (t>0,3S^2/a, for products with plane symmetry, respectively t>0,25R^2/a for those with cylindrical symmetry) the relations (1) and (2) take simplified forms, very approachable. Thus, first relation becomes:

$$T(x,t) - T_0 = \frac{J_c \cdot S}{2 \cdot \lambda}\left[\frac{2 \cdot a \cdot t}{S^2} + \left(\frac{x}{S}\right)^2 - \frac{1}{3}\right] \qquad (3)$$

and second relation:

$$T(r,t) - T_0 = \frac{J_c \cdot R}{2 \cdot \lambda}\left[\frac{2 \cdot a \cdot t}{R^2} + \left(\frac{r}{R}\right)^2 - \frac{1}{2}\right] \qquad (4)$$

aspect that substantiates the idea of the existence of the constant thermal gradient, (ΔT=ct) on the section of the processed product in constant heat flow conditions. ***Note.***

The simplexes x/S or r/R represent the coordinate functions.

$$\Delta T = T_S - T_i = \frac{J_c X}{2\cdot\lambda} \qquad (5)$$

where X it is S, or R, depending on the type of symmetry of the product under heating. The constant of the thermal gradient on the section of the products subjected to heating in constant thermal flow regime facilitates the determination of the thermal stresses, their values being strictly dependent on the value of the gradient (equation 6, plane symmetry - equation 7, cylindrical symmetry)

$$\sigma = \frac{\beta\cdot E}{1-\nu}\Delta T\left(\frac{1}{3} - \frac{x^2}{S^2}\right) \qquad (6)$$

$$\sigma_z = \frac{\beta\cdot E}{1-\nu}\Delta T\left(\frac{1}{2} - \frac{r^2}{R^2}\right) \qquad (7)$$

where: β represent the linear thermal expansion coefficient; E–modulus of elasticity; v - Poisson's ratio.

Alternatives to the heating methods corresponding to discontinuous furnaces are those that use the mode of continuous, linear variation of the furnace temperature (limit conditions order III) and "in extremis", for high values of thermal transmissivity (high values of Biot criteria), linear variation of the surface temperature of the products (limit conditions order I), respectively heating in environments with constant temperature (Tc = Tt + 20 ÷ 40 ^oC; limit conditions of order III), but with preheated products (or not). Under the conditions of a linear variation of the surface temperature, the maximum thermal gradient per section remains constant in time and equal to:

$$\Delta T \frac{CS^2}{2\cdot a_{max}}$$

respectively (8)

$$\Delta T \frac{CR^2}{4\cdot a_{max}}$$

where C, represent the speed of heating and ***a*** – thermal diffusivity of the material from which the product it is made.

The stability in time of the thermal gradient on the section of thermally processed products will simplify the effort necessary to determine the level of the thermal stresses, their correlation with the thermal gradient having the following mathematical expression for products with plane symmetry.

$$\sigma_y = \sigma_z = \frac{\beta\cdot E}{1-\nu}\cdot\frac{\Delta T_{max}}{3\left[1-3\left(\frac{x}{S}\right)^2\right]} \qquad (9)$$

respectively, for those cylindrical symmetry:

$$\sigma_\theta^S = \sigma_z^S = -\frac{\beta\cdot E}{1-\nu}\cdot\frac{\Delta T_{max}}{2} \qquad (10)$$

for the product surface with cylindrical symmetry, and for their center:

$$\sigma_\theta^c = \sigma_r^c = +\frac{\beta\cdot E}{1-\nu}\cdot\frac{\Delta T_{max}}{4}$$

respectively (11)

$$\sigma_z^c = +\frac{\beta\cdot E}{1-\nu}\cdot\frac{\Delta T_{max}}{2}$$

Note. If in the products subjected to thermal processing there is a thermal gradient at the time of installation of the thermal regime of linear surface temperature variation, the solutions become more complex, requiring new terms that take into account their value. By using the heating method in which the temperature of the furnace environment is constant, (Tm = ct = limit conditions order III), the products being preheated (regime ***e*** in fig.1) to an initial temperature $T_0 = Tp_1$ (Tp_1 - preheating temperature), the evolution over time of the temperature of the characteristic areas of the products (their surface and centre) can be determined by means of the graphical expressions of the criterion solutions or by means of tabular values (values calculated by Russel [1; 2; 4]).

These developments permit the forecast of maximum gradient value on the product section, which corroborated with the value of permissible stress, ensures the posibility of taking some decision against thermal stresses limitation.

In this way, for products with plane symmetry, the criterial solution for determining the temperature criteria Θ and through it the temperature $T_{x,t}$ into a micro-volume placed at x quota after a time interval t, will have the following form:

$$\Theta = \frac{T_{x,t}-T_m}{T_{p1}-T_m} = \Phi\left(\frac{a\cdot t}{S^2};\frac{\alpha\cdot S}{\lambda};\frac{x}{S}\right) \qquad (12)$$

respectively for the products with cylindrical symmetry:

$$\Theta = \frac{T_{r,t}-T_m}{T_{p1}-T_m} = \Phi'\left(\frac{a\cdot t}{R^2};\frac{\alpha\cdot R}{\lambda};\frac{r}{R}\right) \qquad (13)$$

where the functions Φ respectively Φ' can be determined by graphical methods or using tabelar solutions.

Note. If the two-dimensional and three-dimensional temperature distribution field is of interest, the temperature corresponding to the microvolume whose coordinates are x-y-z can be determined by multiplying the function Φ on these three directions ox; oy; oz, so the solution (12) will become:

$$\Theta = \frac{T_{(x,y,z,t)} - Tm}{T_{p1} - Tm} = \Phi_x . \Phi_y . \Phi_z \qquad (14)$$

respectively solution (13) with the form:

$$\Theta = \frac{T_{(r,z,t)} - Tm}{T_{p1} - Tm} = \Phi_r . \Phi_z . \qquad (15)$$

Using solutions (12) and (13), most frequently, the maximum value of the thermal gradient can be estimated, ΔTmax=Ts-Ti, and with this the value of the maximum tension:

$$\frac{\sigma(1-\nu)}{\beta \cdot E} . \frac{1}{T_m - T_0} = f\left(\frac{\alpha \cdot t}{S^2}; \frac{\alpha \cdot S}{\lambda}; \frac{x}{S}\right) \qquad (16)$$

for the products with plane symmetry, respectively:

$$\frac{\sigma(1-\nu)}{\beta \cdot E} . \frac{1}{T_m - T_0} = f'\left(\frac{\alpha \cdot t}{R^2}; \frac{\alpha \cdot R}{\lambda}; \frac{r}{R}\right) \qquad (17)$$

for the products with cylindrical symmetry.

The functions f and f' can be determined with graphical expressions of the criterial solutions (16;17).

In order to simplify the determination of the thermal gradient installed at a given moment on the section of products thermally processed with different geometries, the graphical expressions of the following solutions obtained by criterial solving can be used:

$$\frac{\Delta T}{T_m - To} = \Phi''\left(\frac{a.t}{S^2}; \frac{\alpha \cdot S}{\lambda}\right) \qquad (18)$$

for the products with plane symmetry, respectively:

$$\frac{\Delta T}{T_m - To} = \Phi'''\left(\frac{a \cdot t}{R^2}; \frac{\alpha \cdot R}{\lambda}\right) \qquad (19)$$

for those with cylindrical symmetry.

By continuously comparing the maximum values of thermal stresses, determined by a certain size of the thermal gradient recorded on the section of the product subjected to heat processing, with the level of strength characteristics (yield limit and tensile strength) of the material from which the heat processed product was made, it can be decided on the thermal regime adopted. The logic scheme of the way in which a certain heating variant of the metallic products processed in thermal aggregates with discontinuous operation must be conceived and finally adopted is presented in fig.4-5 This makes it possible to highlight the way in which decisions must be made regarding a certain heating variant, namely, by the permanent control of the thermal gradient and thermal stresses, respectively the ratio in which these quantities are with the resistance characteristics of the metallic material which the heat treated product was manufactured.

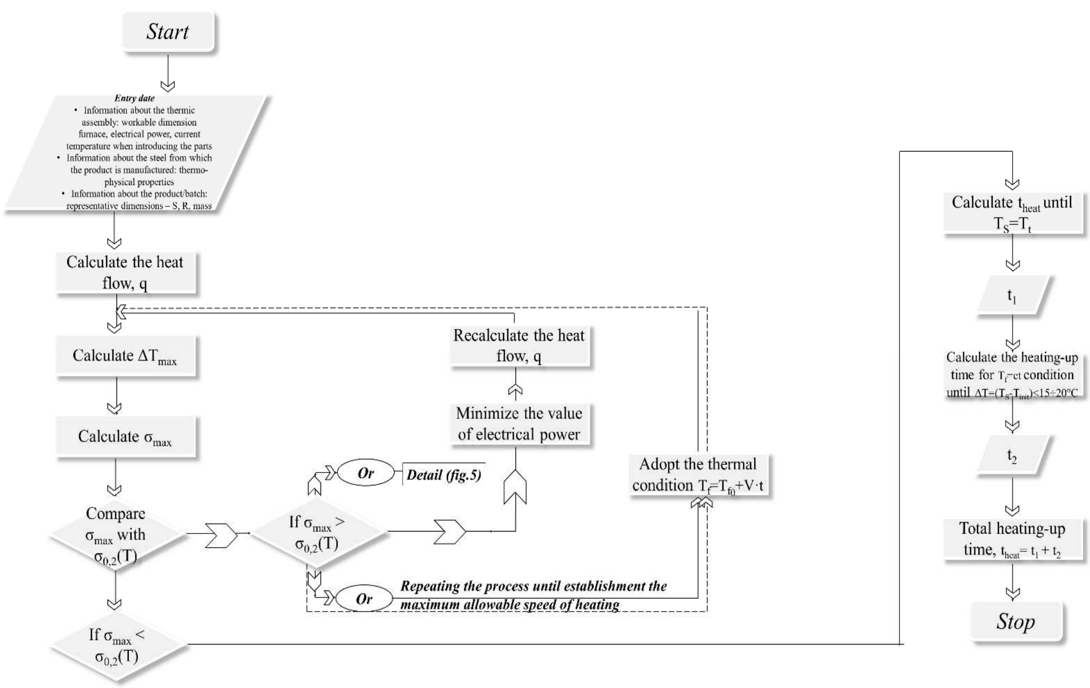


Fig.4 Algorithm to determine optimal heating conditions in heat treatment furnaces with discontinuous operation

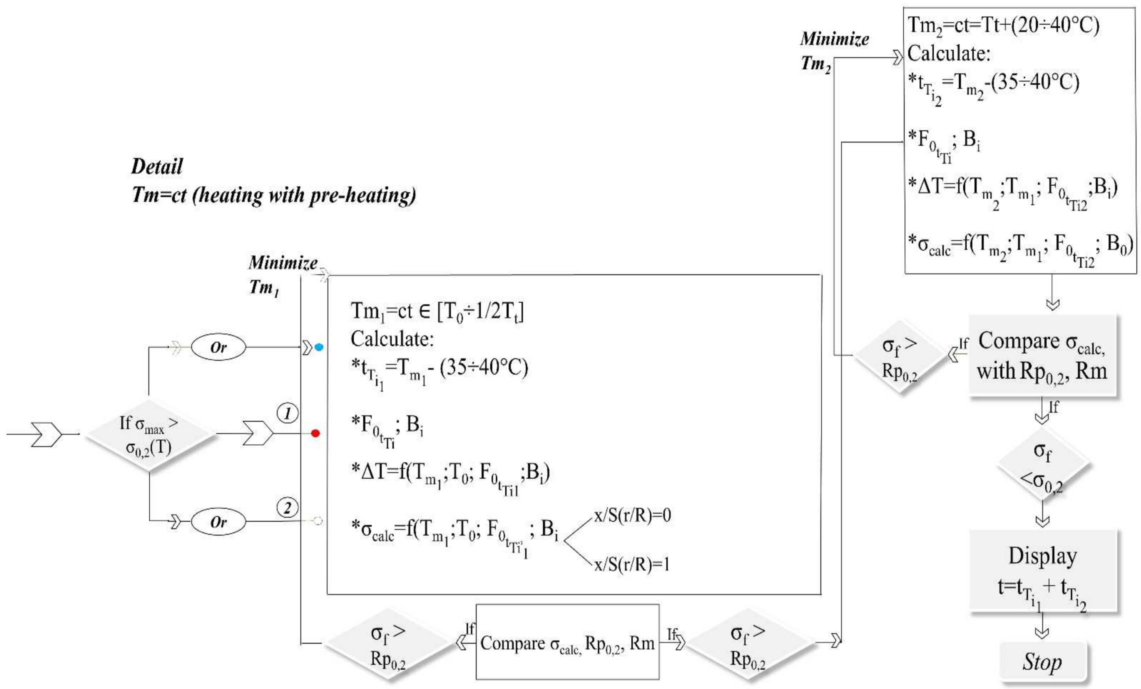


Fig.5 Algorithm to determine optimal heating conditions in heat treatment furnaces with discontinuous operation (DETAIL)

CONCLUSIONS

The proposed algorithm for estimating the temperature evolution on the section of metal products with different symmetries, of the thermal gradient and correlated with it of the maximum level of thermal stresses recorded at a given time, allows the adoption of a certain heating regime - constant thermal flow, linear variation of furnace temperature or the constant temperature of the furnace (with or without preheating), so that the processing in the heat treatment furnaces with discontinuous operation takes place in conditions of maximum safety and profitability. Also, the algorithm allows the development of specialized software to quickly select the regime and optimal conditions for thermal processing of metal products that are made of materials with specified thermophysical characteristics and different symmetries in furnaces with discontinuous operation and more.

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MODEL ANALITIC DE PREDICȚIE A REGIMULUI OPTIM DE INCĂLZIRE UTILIZAT IN PROCESĂRILE TERMICE

Rezumat: Decizia de a adopta anumite condiții de încălzire trebuie să ia în considerare necesitatea de a evita tensiunile termice periculoase care ar putea provoca deformarea produselor, în special a celor cu configurație complexă, sau chiar distrugerea integrității lor și în același timp o valoare minimă a timpului de încălzire, cu consecințe directe asupra consumului tehnologic şi a productivității, deci a costurilor. Pornind de la premisa că în cuptoarele cu funcționare discontinuă încălzirea se efectuează cel mai frecvent în condiții caracterizate prin valoare constantă a fluxului de căldură (q = Jc = ct., corespunzător condițiilor limita de ordinul II), până-n momentul in care cuptorul atinge temperatura impusa (Tc = Tt + 20 ÷ 40^{o}C) după care trece în Tc = ct (condiții limită de ordinul III), este deosebit de important să anticipăm valoarea gradientului termic în timpul primei etapa a încălzirii (q = Jc = ct.), pentru a concluziona dacă treapta de putere adoptată a fost corectă sau nu, deci ce măsuri tehnologice trebuie luate astfel încât încălzirea să aibă loc în condiții optime. Lucrarea prezintă schema conceptuală și modelul analitic, cu justificările aferente, conținând pașii necesari pentru a prezice și optimiza condițiile de încălzire adecvate din toate punctele de vedere.

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