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OPTIMAL DESIGN OF A 2K-h PLANETARY GEARBOX INPUT SHAFT

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Abstract: *Designing power transmissions components such as shafts is not an easy task, considering the iterative nature of the process. In this paper a GA was used to automate the complex procedure of shaft mechanical design. The objective of this study is to minimize the mass of the input shaft subassembly. The objective function was described by 7 design variables and was subjected to a set of 19 engineering constraints. The GA conduct to a reduction of mass of 21.6% as compared to traditional design solution.*
Key words: *Optimal design, Genetic Algorithm, 2K-h planetary gearbox, input shaft.*

1. INTRODUCTION

Shafts are key components in mechanical engineering applications usually with circular cross-section which transmit torque and rotational motion from one component i.e. an electric motor, an engine etc. to another one. As a result, they operate under diverse and different conditions therefore they should meet geometrical, functional and strength criterions.
Trial-and-error methods (figure 2) of such machine elements consist in several phases involving computations based on static strength, fatigue strength, bending and torsion stiffness (linear and angular displacements). Also, during this iteratively calculation procedure the mechanical designer should use a set of complex formulas (for strength checking of shafts) of various graphs and tables for selecting diverse geometrical dimensions and different machine parts such as pulley, sealings, bearings etc. which leads to a complex and time-consuming activity. Moreover, the design obtained is not an optimum one.

Automatizing this process by the means of the computer and optimization techniques conduct to optimal design solutions which are compact, efficient and leads to cheaper production.

In the last decades many researchers have introduced computers for designing machine elements. In [1] the authors use the Visual Studio C for developing a computer program to automate the shaft design under different loading conditions. Timerbaev, Sadrtdinov and Safin in [2] use CAD/CAE system for calculation and analysis of shafts to remove the shortcomings of classic process. They use as a case study an input shaft of bevel straight-toothed gear. Crivelli, Ghelichi and Guagliano in [3] give solutions to avoid the failure due to fatigue strength of a shaft from a car lift system. The study is carried out throughout the finite element analysis method. El-Sayed et al. in [4] described a study about the effects of torsional resistance. El-Saeidy [5] presented a FEM for describing the dynamic behaviour of a rotor shaft mounted on radial bearings. Mutasher [6] uses a FEA with ANSYS for a hybrid shaft. The author considers an advanced composite material for the shaft i.e. an aluminium tube wound onto outside by layers of composite material to investigate the shaft's maximum torsion capacity. Kim and Lee in [7] designed a hybrid (composed of aluminium and carbon/epoxy composite) automotive drive shaft which was assembled with the aluminium yoke using the appropriate press fit joint. The one-piece shaft conducts to a considerable mass reduction (i.e. 50%) as opposite to the conventional two-piece steel drive shaft.

In general, a real design problem i.e. such as a *power gear transmission* (figure 1) represents a very complex structure for which designers

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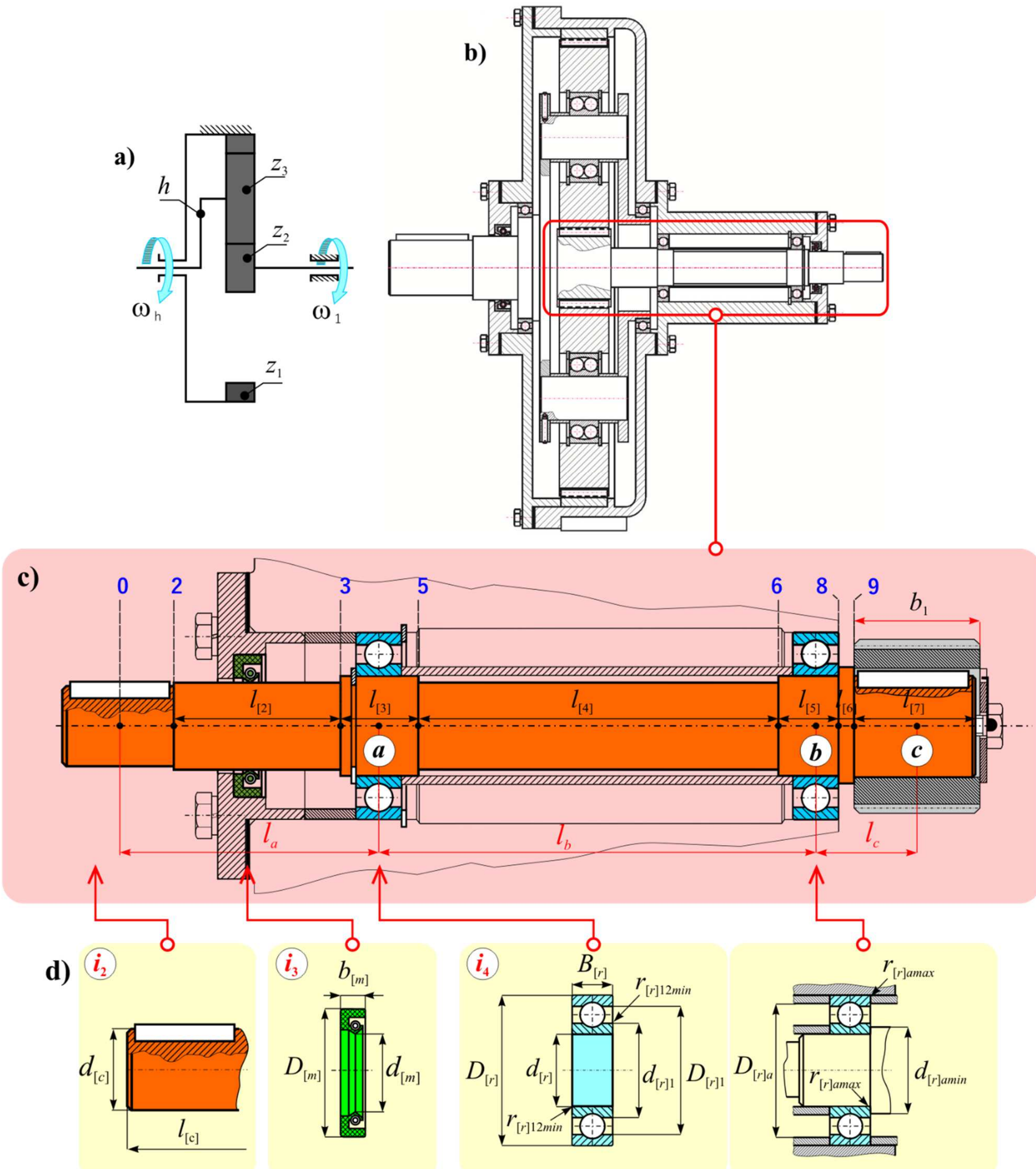

Fig. 1. Planetary gearbox
a) The cinematic scheme of a 2K-h planetary gearbox; **b)** section of planetary gearbox transmission; **c)** the input shaft; **d)** geometric dimensions (for the input end shaft, for the radial shaft sealing and for the radial ball bearing, respectively) described by design variables i_2, i_3 and i_4

should have the proper experience and solving tools (i.e. all sorts of CAD software, different optimization algorithms etc.). Even when these criterions are met, tackling such mechanical design problem is still a complex task considering the interdependencies between various assembly's components (a typically example might be that selecting a larger input end shaft could yield difficulties in mounting the gear). Therefore, it is important to decompose the problem into a set of tractable ones which will be resolved without consuming important resources. Also, this approach allows to take the correct and necessary decisions when the whole problem is considered. This step-by-step strategy was applied by the authors for solving different complex optimization problems (consider in here the optimizations detailed in [8], [9], [10] etc.).

The work described in here relies on this strategy. This optimization of the input shaft subassembly along with the optimal design of the single-row planetary gearbox gearings [11] were the intermediate phases which led to the automated optimal design of the entire planetary gearbox [12]. Into the next Section a brief discussion regarding the current procedure of designing shafts is presented, followed by a short description of the proposed Genetic Algorithm (Section 3). The fourth Section contains the optimal design problem description followed by an effective optimization example. The paper is concluded with some reflections and suggestions regarding the present study.

2. SHAFTS DESIGNING PROCEDURE

The flowchart of the shaft power transmission design is presented in figure 2.

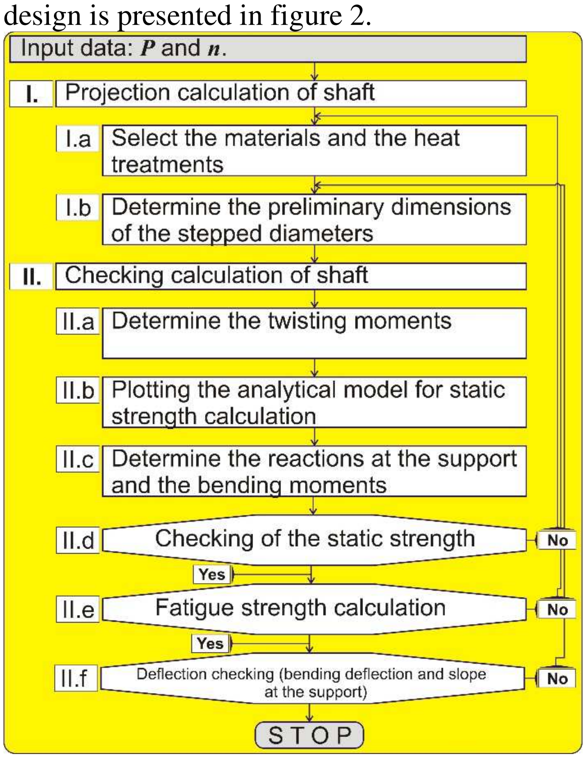

Fig. 2. The flowchart of shaft classical design

As it can be seen from here, this procedure is an iterative one. The design process starts (I) from a data set which contains information about the input power - **P**, (kW) and the rotational frequency of the shaft - **n** (rpm). This means that the mechanical designer knows only the torque **T** (N·m). The moments of deflection **M** will be computed only after the shaft design has been determined [13], when the length (for the stepped diameters) and application sites of the acting loads are recognized **(-II-)**. Next, after the complete embodiment [13] of the shaft, the checking calculation **(-II-)** consisting of static strength **(-II.d-)** (in here are determined the reactions at the support in horizontal and vertical planes, then are plotted the diagrams for the moments in these planes - **II.b**), fatigue strength **(-II.e-)** and deflection at the supporting points [9] **(-II.f-)** it will be performed.

3. GENETIC ALGORITHM OVERVIEW

The GAs were developed by John Holland [14]. They are a subclass of Evolutionary Algorithms (EAs) [15]. Algorithmically, the basic GAs is outlined as bellow: **(-a-)** firstly the initial population is randomly initialized then, each individual's phenotype (for its description, in this design problem were considered a set of 7 design parameters as they are shown in Table 1. From here it could be seen that three design variables are indexes from specific catalogues) is evaluated using the fitness function **(-b-)** i.e. the mass of the shaft (described in subsection 4.2). Next, accordingly to their fitness (obviously the higher the fitness is, the more chances of selection are) two chromosomes are selected (they will become the parents of the upcoming generations) in order to breed the following generations **(-c-)**. The resulted offsprings **(-d-)** (created by applying crossover and mutation operators) are evaluated **(-e-)** are reintegrated into the initial population replacing it partially or totally **(-f-)** [15]. This entire evaluation, reproduction and replacement in loop process continue until an optimal design solution is found or the GA reaches the maximum number of iterations.

Table 1

The 7 design variables used for describing the input shaft optimization problem.

Symbol	Range	Description
i_1	{0…3}	Index of shaft material. Integer values.
i_2	{0…15}	Catalogue index of standardised end for the input shaft. Integer values.
i_3	{0…31}	Catalogue index of radial shaft sealings. Integer values.
i_4	{0…15}	Catalogue index of radial ball bearings. Integer values.
l_a	{4…103}	Distance between the belt wheel installation point and position of relative support of the left radial bearing (figure 1,c). Integer values.
l_b	{80…208}	Distance between relative supports points of the radial bearings (figure 1,c). Integer values.
l_c	{40…103}	Distance between the relative support of the right radial bearing and the point of pinion installation (figure 1,c). Integer values.

4. DESCRIPTION OF THE OPTIMAL DESIGN PROBLEM

4.1 The "genotype" of the input shaft

There is a set of **7 genes** that uniquely described the optimization problem. They are detailed in Table 1. As it could be observed form Table 1 there are **3 genes** (for input end, for radial shaft sealings and for radial ball bearings) that are coded as catalogue indexes. They contain all the geometric dimensions (represented in figure 1,d with i_2, i_3, i_4) as follows: $d_{[c]}$ and $l_{[c]}$ regarding the input end shaft; $d_{[m]}$, $D_{[m]}$ and $b_{[m]}$ about the radial shaft sealing; and $d_{[r]}$, $d_{[r]1}$, $B_{[r]}$, $D_{[r]}$, $D_{[r]1}$, $r_{[r]12\min}$, $r_{[r]a\max}$, $D_{[r]a}$, $d_{[r]a\min}$ for the radial ball bearings.

4.2 The objective function

The main goal of this paper is to minimize the mass of the input shaft, which is computed with the following equation:

$$F\left(\bar{x}\right)=\frac{\pi}{4}\cdot\left(d_{[c]}^{2}\cdot l_{[c]}+d_{[m]}^{2}\cdot l_{[2]}+d_{[r]}^{2}\cdot l_{[3]}+\right.$$
$$+d_{[4]}^{2}\cdot l_{[4]}+d_{[r]}^{2}\cdot l_{[5]}+ \qquad (1)$$
$$\left.+d_{[a\min]}^{2}\cdot l_{[6]}+d_{[7]}^{2}\cdot l_{[7]}\right)\cdot\rho\rightarrow\min$$

where: $\bar{x}$ represents the design vector: $\bar{x}=\left[i_1,i_2,i_3,i_4,l_a,l_b,l_c\right]$ (i_1, i_2, i_3, i_4, l_a, l_b and l_c are shown in Table 1); $l_{[2]}$, $l_{[3]}$, $l_{[4]}$, $l_{[5]}$, $l_{[6]}$ and $l_{[7]}$ are the lengths of the stepped diameters; ρ is the density of steel (i.e. $7.85\cdot10^{-6}$ mm^3/kg) [15].

4.3 The constraints

For this optimization design problem, a set of **19 design constraints** were considered. They were formulated as inequality type [9] i.e. $a_i \le b_i$, with a_i, $b_i > 0$. To achieve a feasible solution all these constraints must be negative or at last zero [8]. These **19 constraints** are:

C1. The input end shaft diameter should be smaller than the radial shaft sealing diameter for correct positioning (and functioning) of the belt wheel.

C2. The inside diameter of the radial ball bearing on the shaft should be less than the mounting diameter of the radial seal.

C3. Constraint regarding the mounting conditions of the radial ball bearings into the transmission housing.

C4. A manufacturability constraint regarding the stepped diameter necessary for axial fixation of the radial ball bearing.

C5-8 A set of constraint regarding the conditions of the shaft stability.

C9 The maximum von Mises equivalent stresses experienced by the shaft must not exceed a certain value.

C10-11. The bending stresses on the shaft in sections **0** and ***c*** (figure 1,c) should be less than the recommended value i.e. $\delta_a = 0.025\cdot$m (*m* is the standardized module of the gearing).

C12. The deflection at the supporting point in sections ***a*** and ***b*** must not exceed the allowable value (figure 1,c).

C13. Constraint regarding the fatigue strength of the shaft ([s] = 1.5-2, [13]).

C14. The torsional strains must be below the allowable value.

C15. The effective service life of the radial ball bearings should be greater than the necessary service life established by the design requirements (i.e. L_h = 8000 h).

C16-19 The shearing stresses on the key and keyway used to assembly the belt wheel and the pinion should be less than the specific allowable value (i.e. 70 MPa for the material of the keys i.e. steel grades E355 [13]).

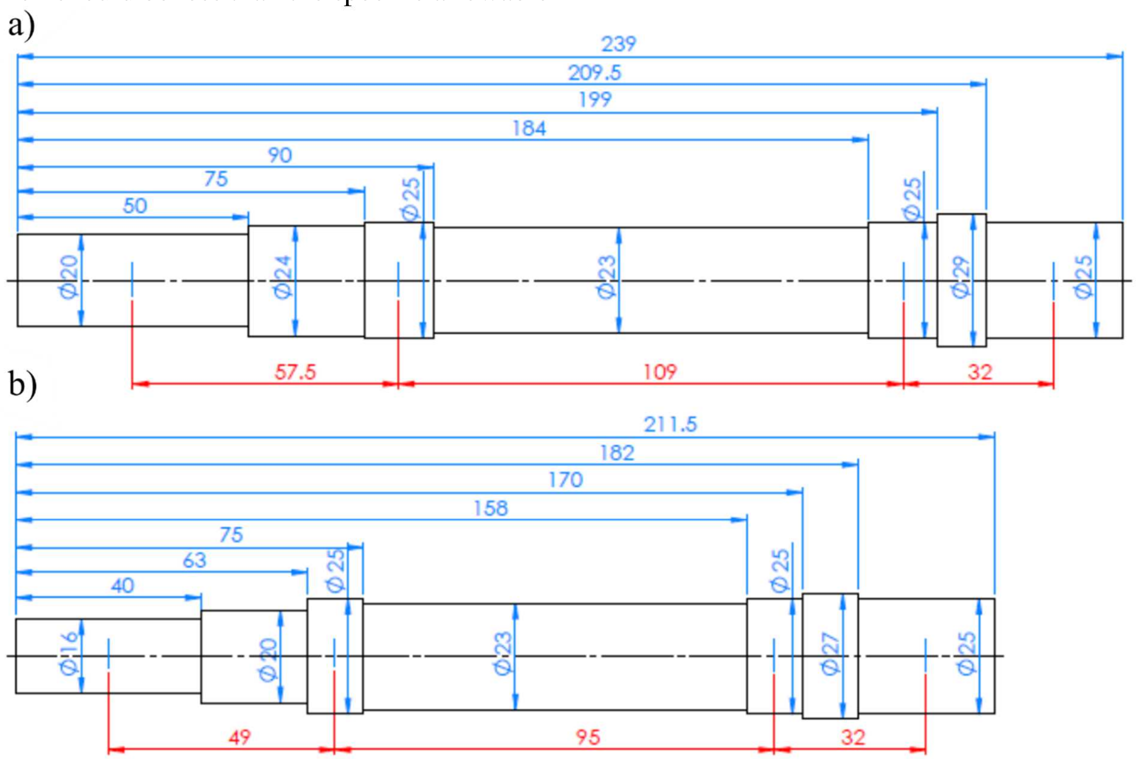

Fig. 3. Design solutions - geometrical dimension of stepped shafts diameters (red dimensions represent the l_a, l_b and l_c design variable - Table 3)
a) Classical design; **b)** optimal design

5. A 2K-h INPUT SHAFT OPTIMAL DESIGN EXAMLE

In order to **minimize the mass** (described by eq. 1) **of the input shaft subassembly** (figure 1) the authors considered the same design data as in the case presented in [11] (the input power - 2.9 kW and the input speed 925 rpm). The proposed GA led to an input shaft weighing **0.74 kg**.

In Table 2 are presented the values for all **7 genes** obtained after the optimization process.

Table 2

The values of the design variables obtained after optimization.

i_1	i_2	i_3	i_4	l_a	l_b	l_c
3	**2**	**11**	**3**	**48**	**98**	**32**

In Table 3 a comparison regarding *the classical* and *optimal design solutions* is presented.

Table 3

Design solutions comparison.

Gene	Classical design	Optimal design
i_1	**1C60**	**1C60**
i_2	**Ø20×50**	**Ø16×40**
i_3	**Ø24×35×7**	**Ø20×30×5**
i_4	**61905**	**61905**
l_a	**57.5**	**49**
l_b	**109**	**98**
l_c	**32**	**32**

6. CONCLUSIONS

Designing mechanical power transmissions components such as shaft is not an easy task, considering the iterative nature of the whole process. For this reason, the authors proposed in here a tool like the GAs to automate the complex design process. The value of the shaft's mass is lighter with **21.6%** as compared to traditional design solution (when the shaft weighted **0.944 kg**). In the case of large series manufacturing process, using such metaheuristic have significant advantages over the classical manual iterative methods. For example, considering only the material costs we can observe that at **≈5 input shaft manufactured, 1 is for free.**

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Proiectarea optimală a arborelui de intrare al unui reductor planetar 2K-h

Proiectarea componentelor transmisiilor mecanice, cum ar fi arborii, nu este o sarcină ușoară, având în vedere natura iterativă a procesului. În această lucrare s-a utilizat un GA pentru a rezolva această problemă complexă de proiectare. Funcția obiectivă, masa subansamblului arborelui de intrare, a fost descrisă de 7 variabile de proiectare și a fost supusă unui set de 19 restricții. Algoritmul genetic utilizat a condus la o reducere a masei cu 21,6% față de soluția clasică.

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