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CONSIDERATIONS ON THE LAGRANGE INTERPOLATION METHOD APPLICABLE IN MECHANICAL ENGINEERING

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Abstract: *The paper presents the theoretical considerations on the Lagrange interpolation method, applicable in mechanical engineering. A mechanical system with a finite number of degrees of freedom is adopted, to which the Lagrange interpolation method is applied step by step. The work comes to the aid of the mechanical engineer, to facilitate his work and the interpretation of difficult mechanical systems.*
Key words: *mechanical systems, theoretical considerations, Lagrange interpolation, practical solution.*

1. INTRODUCTION

The Lagrange interpolation method is used in two distinct situations:

1. The function is known, but it has a complex form, and the calculations to be performed are complicated (derivation, integration), that it is easier to look for a close, but not identical, and that does not differ too much from the graphic representation of the initial function.
2. The function is completely unknown but is defined only by discrete points.

In both situations it can use the Lagrange interpolation function. In mechanical engineering the Lagrange interpolation method has its applicability without restrictions, as will try to prove by this work.

2.THEORETICAL CONSIDERATIONS

It is assumed that there is a function, which depends on the independent parameter **x** by the intermedium of **m+1** polynomial constant [1], denoted with $C_0, C_1, C_2, ..., C_m$.

The situation is presented as in relations:

$$\varphi_i[a,b] \Rightarrow \mathbb{R}, a < b \qquad (1)$$

and

$$\varphi(x, C_0, C_1, C_2, \dots, C_m) \qquad (2)$$

The function depends on **m+1** parameter respective: $C_0, C_1, C_2, \dots, C_m$.

In the closed range **[a, b]** the independent variable is given by the system:

$$x_0 = a, x_1, x_2, \dots\ x_{m-1},\ x_m = b \qquad (3)$$

where

$$x_i < x_{i+1}, \forall i = 0, 1, 2, \dots, m-1 \qquad (4)$$

An initial approximation of the form is defined:

$$C_0 = C_0^0, C_1 = C_1^0, C_2 = C_2^0, \dots, C_m = C_m^0 \qquad (5)$$

So:

$$f(x_i) = \varphi(x_i, C_0^0, C_1^0, C_2^0, \dots, C_m^0),\ i=0, 1, \dots, m \qquad (6)$$

The Interpolation Lagrange function has the expression:

$$L_m(x) := \sum_{i=0}^{m} f(x_i) \prod_{j, j\neq i} \frac{x - x_j}{x_i - x_j} \qquad (7)$$

They are the coefficients in the given function, noted (6). In this situation the polynomial function is:

$$\varphi(x, C_0, C_1, C_2, \dots, C_m) = C_0\, x^m + C_1\, x^{m-1} + \cdots C_m \qquad (8)$$

In which there is the expression [2]

$$\prod_{j\neq i} \frac{x - x_j}{x_i - x_j} f(x_i) \qquad (9)$$

for each numerator, which must be different from the current counter. As:

$$j = 0, 1, 2, \dots, i-1, i+1, \dots, m \Rightarrow$$

For **m=1**, the result is:

$$L_1(x) = f(x_0)\frac{x - x_1}{x_0 - x_1} + f(x_1)\frac{x - x_0}{x_1 - x_0} \qquad (10)$$

For **m=2** the result is:

$$L_2(x) = f(x_0)\frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} + f(x_1)\frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} + f(x_2)\frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} \qquad (11)$$

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Outside the domain defined by the Lagrange interpolation function, is

$$f(x) - L_m(x) = \frac{f^{(m+1)}(\xi)}{(m+1)!}\omega_m(x),$$
$$\xi \in [min(x_0, x), max(x_m, x)],$$
$$\omega_m(x) = \prod_{i=0}^{m}(x - x_i) \qquad (12)$$

and represents the residue of the Lagrange Interpolation function.

3. EXPERIMENTAL APPLICATION

From a mechanical point of view, the Lagrange Interpolation function can be used for the processing of decorative wood. It improves the appearance of mechanically processed surfaces, giving them the temptation of manual processing.

The function, which can be interpolated with the Lagrange Interpolation function, can be expressed by the relationship:

$$f: [-20; 20] \Rightarrow \mathbb{R}, f(x) = |x| \qquad (13)$$

The engraving of the wood is carried out with a special device, at which the coordinate **x** is given by the longitudinal displacement of the piece, and the coordinate **y** is given by the transverse displacement of the cutting tool (cutting knife).

If the machine is ordered according to the relationship (13), with x_1=- 20mm, and x_m= 20mm, then it will be obtained on the processed surface, two bisectors, two straight lines:

1. First bisectors, for moving in the I dial of the coordinates.
2. The second bisector, for moving in the second dial of the coordinate system.

To achieve a processing aspect, as manual, then the mechanical wood processing system will have to be ordered according to the law given by the function of Interpolation Lagrange.

Explain step by step the application of Lagrange Interpolation for grade 3, 4, 5, 6, 7, 8 polynomials, as sub-chapters of the paper.

3.1. Third-degree polynomial

Apply the relationship (7) to the **i** counter by giving the values 0, 1, 2, 3. The polynomial coefficients, thus calculated, are presented in Table 1, and the graphic representation of the third-degree Lagrange Interpolation function is in Figure 1.

Table 1.
Coefficients of Third-degree Lagrange Interpolation Function

Coefficient	Value
C_0	6.813e-19
C_1	0.04329
C_2	-1.2325e-16
C_3	3.8961

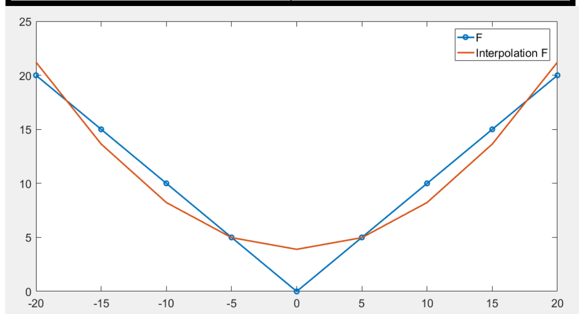

Fig. 1. Third-degree Lagrange Interpolation Function

3.2. Fourth degree polynomial

Apply m = 4, in the relationship (7) to the **i** counter by giving the values 0, 1, …, m+1. The polynomial coefficients, thus calculated, are presented in Table 2, and the graphic representation of the fourth-degree Lagrange Interpolation function is in Figure 2.

Table 2.
Coefficients of Fourth-degree Lagrange Interpolation Function

Coefficient	Value
C_0	-9.324e-05
C_1	-8.0760e-19
C_2	0.08158
C_3	2.6588e-16
C_4	2.0979

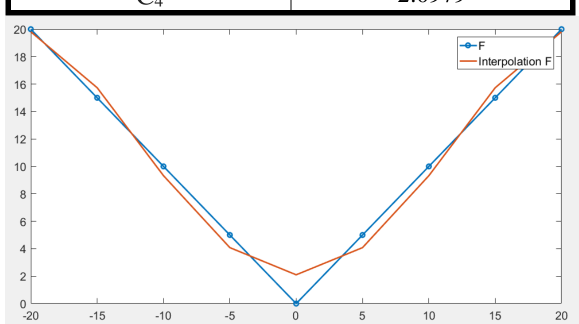

Fig. 2. Fourth-degree Lagrange Interpolation Function

3.3. Fifth degree polynomial

Apply m = 5, in the relationship (7) to the **i** counter by giving the values 0, 1, …, m+1. The polynomial coefficients, thus calculated, are presented in Table 3, and the graphic representation of the fifth-degree Lagrange Interpolation function is in Figure 3.

Table 3.
Coefficients of Fifth-degree Lagrange Interpolation Function

Coefficient	Value
C_0	-2.1281e-21
C_1	-9.324e-05
C_2	1.8188e-18
C_3	0.08158
C_4	-2.7015e-16
C_5	2.0979

3.4. Six-degree polynomial

Apply m = 6, in the relationship (7) to the **i** counter by giving the values 0, 1, …, m+1. The polynomial coefficients, thus calculated, are presented in Table 4, and the graphic representation of the third-degree Lagrange Interpolation function is in Figure 4.

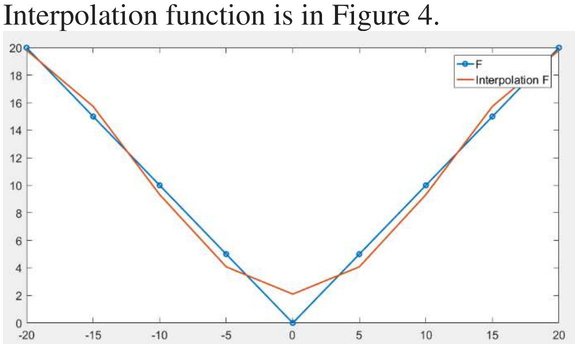

Fig. 3. Fifth-degree Lagrange Interpolation Function

Table 4.
Coefficients of Six-degree Lagrange Interpolation Function

Coefficient	Value
C_0	5.9259e-07
C_1	-8.09741e-21
C_2	-0.00045
C_3	4.88109e-18
C_4	0.13256
C_5	4.64125e-16
C_6	1.0878

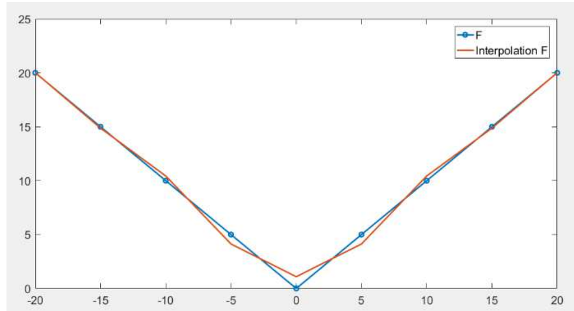

Fig. 4. Six-degree Lagrange Interpolation Function

3.5. Seventh-degree polynomial

Apply m = 7, in the relationship (7) to the **i** counter by giving the values 0, 1, …, m+1. The polynomial coefficients, thus calculated, are presented in Table 5, and the graphic representation of the seventh-degree Lagrange Interpolation function is in Figure 5.

Table 5.
Coefficients of Seventh-degree Lagrange Interpolation Function

Coefficient	Value
C_0	-1.8650e-22
C_1	5.9259e-07
C_2	1.30049e-19
C_3	-0.00045
C_4	-2.4576e-17
C_5	0.13256
C_6	8.8506e-16
C_7	1.0878

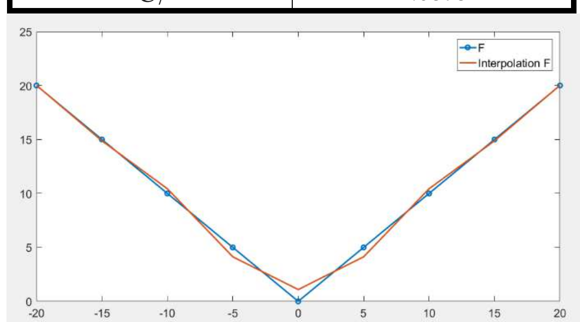

Fig. 5. Seventh-degree Lagrange Interpolation Function

3.6. Eighth-degree polynomial

Apply m = 8, in the relationship (7) to the **i** counter by giving the values 0, 1, …, m+1. The polynomial coefficients, thus calculated, are presented in Table 6, and the graphic representation of the eighth-degree Lagrange Interpolation function is in Figure 6.

Table 5.
Coefficients of Seventh-degree Lagrange Interpolation Function

Coefficient	Value
C_0	-1.00251e-09
C_1	4.11697e-23
C_2	1.34984e-06
C_3	1.34984e-06
C_4	-0.00062
C_5	1.60341e-17
C_6	0.144706
C_7	-1.07307e-15
C_7	0.954773

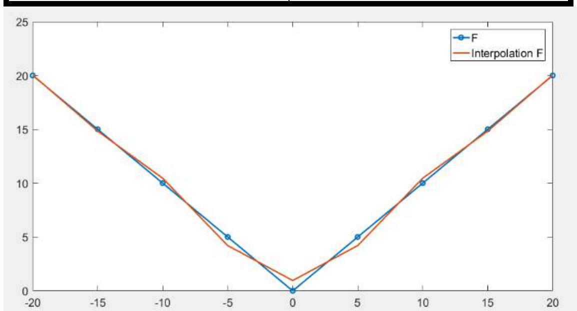

Fig. 6. Eighth-degree Lagrange Interpolation Function

4. DISCUTIONS. CONCLUSIONS

By viewing the representation of the Figure 2 it is found that the function is removed from the theoretical representation of the function. It is found that variable **y** does not exceed the value **20**, which should be the maximum limit.

From Figure 4 compared to the previous figures, it can be said that the six-degree polynomial Lagrange Interpolation approaches to the theoretical representation but does not overlap with it.

Figures 5 and 6, in which the 7th and 8th degree polynomials are represented, approach the bisectors of the two dials, but do not overlap with them. They give the filling of knots in wood, or imperfections in the manual processing of wood.

From what is presented in this paper it follows that the processing of the wood surface according to the law of movement given by the relationship (13) achieves a perfectly symmetrical and uniform control. After each move of go – come the piece one step, so that successive processing does not overlap.

If the processing is done with the law of movement given by the function of Interpolation Lagrange the result of the processing will give the impression of manual processing. Any of the interpolation polynomials can be used, from grade 4 to grade 8. Smaller grades give more ripples, take the larger one's lead to a better visual impression of the processed surface.

The Lagrange Interpolation function can be used for mechanical processing of the wood surface (in plaid), which will give the impression of manual processing.

5. REFERENCES

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Consideratii asupra metodei de interpolare Lagrange cu aplicabilitate in inginerie mecanica

Rezumat: Lucrarea prezinta consideratiile teoretice asupra metodei de interpolare Lagrange, cu aplicabilitate in ingineria mecanica. Se adopta un sistem mecanic cu un numar finit de grade de libertate, caruia i se aplica pas cu pas metoda de interpolare Lagrange. Lucrarea vine in ajutorul inginerului mecanic, pentru a-i facilita munca si modul de interpretare a sistemelor mecanice dificile.

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