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THE GEOMETRIC EQUATIONS FOR TWO ROBOTS OF THE TYPE 3TR-2R IN COOPERATION MOVEMENTS

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Abstract: *In this paper are presented the geometric equations for a structure of type 3TR-2R. In the cooperative action between the two structures 3TR and 2R, it is aimed that the piece of revolution, represented by the system {G} and {S}, be processed by the tool {T}, after a specific technological process. Two robots in cooperative movements can be likened to a five-axis machining center.*
Keywords: *cooperation, advanced mechanics, geometric equation, locating matrices.*

1. INTRODUCTION

In the past few years, there was an increasing interest in developing robot structures engaged in cooperative actions. These robotic structures are being studied for several reasons: the tasks are too complex for a single robot to perform, developing simple robot structures can be more convenient in what concerns the costs, flexibility, and error tolerance than a single powerful robot for each separate task.

Cooperative behavior is the ability of a robot to be engaged in cooperative actions due to some support mechanism (cooperation mechanism), which leads to an increase in the workability of the system. The mechanism of cooperation consists of imposing the control or communication structure, task specification, and establishing the dynamic behavior of the robots which are implemented in a cooperative system. The purpose of the present paper is to determine the geometric equations of two robot structures, involved in cooperative actions.

To this end, an algorithm from advanced mechanics - the algorithm of locating matrices, is applied by considering its important computing advantages [1].

The cooperating and general robot 3TR–2R consists of a robot structure whose role is in the positioning and orientating of the TOOL, and a structure designed for the positioning and orientating structure of the PIECE (see Fig. 1).

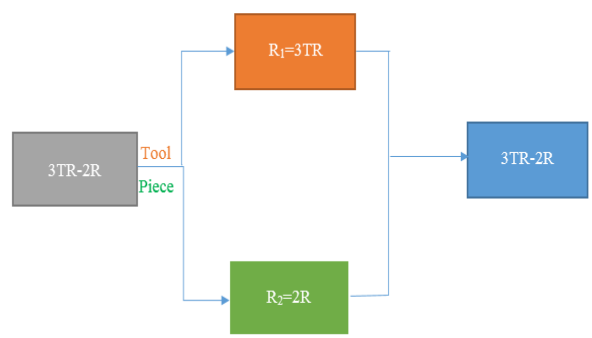

Fig.1. The cooperation of the positioning and orientating structure of the **tool** and the **piece**

2. GEOMETRIC MODELING

For geometric modeling, different geometry-specific algorithms are used [1-9], among which we mention: locating matrices algorithm, the algorithm of composite operators PG, the DH type composite operators algorithm, the algorithm of exponential matrices.

In this paper, we will use the algorithm of locating matrices. The Algorithm of Locating Matrices, due to its computational advantages, can be applied to any robot structure [1].

2.1 DGM equations for 3TR robot structure

For determining the direct geometry model (DGM) equations, the algorithm of locating matrices is applied.

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According to this algorithm [1], the kinematic scheme is first represented in the initial or zero configuration (see Fig.2). This configuration assumes that the generalized coordinate column vector is computed in the following form:

$$\overline{\theta}^{(0)} = \left(q_i^{(0)} = 0; \quad i = 1 \rightarrow 4\right)^T \equiv \equiv \left(q_1^{(0)} = 0 \quad q_2^{(0)} = 0 \quad q_3^{(0)} = 0 \quad q_4^{(0)} = 0\right)^T . \quad (1)$$

In the initial configuration, the following parameters are represented on the kinematic diagram: fixed system, attached to the fixed platform, and symbolized $O_0 x_0 y_0 z_0 \equiv \{0\}$. A fixed system, attached to the fixed base of the 3TR robot and symbolized by $O_0' x_0' y_0' z_0' \equiv \{0'\}$ the system having the same orientation as the system orientation attached to the fixed platform $\{0'\} \equiv \{0\}$. The mobile system, attached to the characteristic point in the final effector's geometric center, $O_T x_T y_T z_T \equiv \{T\}$ represents the tool system located at the end of the mechanical structure of type 3TR, a system with the same orientation as the fixed system's orientation $\{T^{(0)}\} \equiv \{0'\}$. The unit vectors of the four driving axes of the robot $\left(\overline{k}_i^{(0)}\ ; i = 1 \rightarrow 4\right)$ oriented as:

$$\left(\overline{k}_1^{(0)} \equiv \overline{y}_1^{(0)};\ \overline{k}_2^{(0)} \equiv \overline{z}_2^{(0)};\ \overline{k}_3^{(0)} \equiv \overline{x}_3^{(0)};\ \overline{k}_4^{(0)} \equiv \overline{z}_4^{(0)}\right) \quad (2)$$

According to the locating algorithm in geometric modeling, the iteration $(i = 1 \rightarrow 4)$ opens. Using the MuPad symbolic calculation program, matrix transformations between reference systems are obtained $\{i-1\} \rightarrow \{i\}$ and $\{0'\} \rightarrow \{i\}$, where $i = 1 \rightarrow 4$, as well as between $\{4\} \rightarrow \{T\}$ and $\{0'\} \rightarrow \{T\}$:

$${}^{0'}_{1}[T] = \left[\begin{array}{c|c} {}^{0}_{1}[R] \equiv I_3 & \overline{p}_1 \\ \hline 0\,0\,0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & b_0 + q_1 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right], \quad (3)$$

$${}^{1}_{2}[T] = \left[\begin{array}{c|c} {}^{1}_{2}[R] \equiv I_3 & \overline{p}_{21} \\ \hline 0\,0\,0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d_1 - q_2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right], \quad (4)$$

$${}^{0'}_{2}[T] = \left[\begin{array}{c|c} {}^{0}_{2}[R] \equiv I_3 & \overline{p}_2 \\ \hline 0\,0\,0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & b_0 + q_1 \\ 0 & 0 & 1 & -d_1 - q_2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right], \quad (5)$$

$${}^{2}_{3}[T] = \left[\begin{array}{c|c} {}^{2}_{3}[R] \equiv I_3 & \overline{p}_{32} \\ \hline 0\,0\,0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & a_2 + q_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right], \quad (6)$$

$${}^{0'}_{3}[T] = \left[\begin{array}{c|c} {}^{0}_{3}[R] \equiv I_3 & \overline{p}_3 \\ \hline 0\,0\,0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & a_2 + q_3 \\ 0 & 1 & 0 & b_0 + q_1 \\ 0 & 0 & 1 & -d_1 - q_2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right], \quad (7)$$

$${}^{3}_{4}[T] = \left[\begin{array}{c|c} {}^{3}_{4}[R] & \overline{p}_{43} \\ \hline 0\,0\,0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} cq_4 & -sq_4 & 0 & 0 \\ sq_4 & cq_4 & 0 & 0 \\ 0 & 0 & 1 & -d_3 \\ \hline 0 & 0 & 0 & 1 \end{array}\right], \quad (8)$$

$${}^{0'}_{4}[T] = \left[\begin{array}{c|c} {}^{0}_{4}[R] & \overline{p}_4 \\ \hline 0\,0\,0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} cq_4 & -sq_4 & 0 & a_2 + q_3 \\ sq_4 & cq_4 & 0 & b_0 + q_1 \\ 0 & 0 & 1 & -d_1 - q_2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right], \quad (9)$$

$${}^{4}_{T}[T] = \left[\begin{array}{c|c} {}^{4}_{T}[R] \equiv I_3 & \overline{p}_{T4} \\ \hline 0\,0\,0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d_4 \\ \hline 0 & 0 & 0 & 1 \end{array}\right], \quad (10)$$

$${}^{0'}_{T}[T] = \left[\begin{array}{ccc|c} cq_4 & -sq_4 & 0 & a_2 + q_3 \\ sq_4 & cq_4 & 0 & b_0 + q_1 \\ 0 & 0 & 1 & -d_1 - d_3 - d_4 - q_2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right], \quad (11)$$

$${}^{0}_{T}[T] = \left[\begin{array}{ccc|c} cq_4 & -sq_4 & 0 & a_2 + q_3 \\ sq_4 & cq_4 & 0 & b_0 + q_1 \\ 0 & 0 & 1 & d - d_1 - d_3 - d_4 - q_2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]. \quad (12)$$

Next are written the matrix transformations between the fixed system {0}, attached to the fixed platform and the mobile system {T}, the tool system located at the end of the 3TR mechanical structure:

$$ {}^{0}_{T}[T]=\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ \hline 0 & 0 & 0 & 1 \end{array}\right]\cdot\left[\begin{array}{ccc|c} cq_4 & -sq_4 & 0 & a_2+q_3 \\ sq_4 & cq_4 & 0 & b_0+q_1 \\ 0 & 0 & 1 & -d_1-d_3-d_4-q_2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \quad (13) $$

$$ {}^{0}_{T}[T]=\left[\begin{array}{ccc|c} cq_4 & -sq_4 & 0 & a_2+q_3 \\ sq_4 & cq_4 & 0 & b_0+q_1 \\ 0 & 0 & 1 & d-d_1-d_3-d_4-q_2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]. \quad (14) $$

The equations of direct geometry are obtained from the position matrices (10) according to:

$$ {}^{0}\overline{\overline{X}}=\left[\begin{array}{c} \overline{p}_T \\ --- \\ \overline{\psi}_T \end{array}\right]=\left[\begin{array}{c} \left(p_{xT} \quad p_{yT} \quad p_{zT}\right)^T \\ \hline (0 \quad 0 \quad \gamma_{zT})^T \end{array}\right], \quad (15) $$

$$ \overline{p}_T=\begin{pmatrix} p_{xT} \\ p_{yT} \\ p_{zT} \end{pmatrix}=\begin{pmatrix} a_2+q_3 \\ b_0+q_1 \\ (d-d_1-d_3- \\ -d_4-q_2) \end{pmatrix}, \quad \overline{\psi}_T=\begin{pmatrix} 0 \\ 0 \\ \gamma_{zT} \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ q_4 \end{pmatrix}. \quad (16) $$

Equations (12) express the position and orientation of the tool $\{T\}$ through the vectors $\overline{p}_T$ and $\overline{\psi}_T$, about the system attached to the fixed base, assuming known displacements from the joints [10], [11], [14], [15].

2.1 DGM equations for 2R robot structure

In defining the equations of direct geometry, the algorithm of the locating matrices applies. According to this algorithm, an important step is to represent the initial configuration's kinematic scheme (see Fig.2). This configuration assumes that the generalized coordinate column vector is:

$$ \overline{\theta}^{(0)}=\left(q_5^{(0)}=0 \quad q_6^{(0)}=0\right)^T\equiv\left(q_i^{(0)}=0; i=5\rightarrow 6\right)^T. \quad (17) $$

In this initial (zero) configuration, the following parameters are represented on the kinematic diagram: Fixed system, attached to the fixed base of the 2R robot and symbolized by $O_0''x_0''y_0''z_0''\equiv\{0''\}$ a system having the same orientation as the system attached to the fixed platform and denoted with $\{0''\}\equiv\{0\}$.

The mobile system, attached in the characteristic point P from the geometric center of the base of the mobile platform on which the part is located, $O_G x_G y_G z_G\equiv\{G\}$ has the same orientation as the fixed system $\left\{G^{(0)}\right\}\equiv\{0''\}$, the plan $z_G=0$ that is identical to the mobile base plan platform.

The system is fixed in one corner of the part $O_S x_S y_S z_S\equiv\{S\}$, representing the part system located at the end of the mechanical structure of type 2R (see the technical detail in Fig. 2).

The unit vectors of the two driving axes, $\left(\overline{k}_i^{(0)}; i=5\rightarrow 6\right)$ that is $\left(\overline{k}_5^{(0)}\equiv\overline{y}_5^{(0)}; \overline{k}_6^{(0)}\equiv\overline{z}_6^{(0)}\right)$. According to the locating matrices algorithm from the geometrical modeling, the iteration opens $(i=5\rightarrow 6)$. Using the MuPad symbolic calculation program, matrix transformations between reference systems are obtained $\{i-1\}\rightarrow\{i\}$ and $\{0''\}\rightarrow\{i\}$, where $i=5\rightarrow 6$, and between $\{6\}\rightarrow\{G\}$, $\{G\}\rightarrow\{S\}$, $\{0''\}\rightarrow\{G\}$, $\{0\}\rightarrow\{G\}$, $\{0''\}\rightarrow\{S\}$ and $\{0\}\rightarrow\{S\}$.

$$ {}^{0''}_{5}[T]=\left[\begin{array}{c|c} {}^{0''}_{5}[R] & \overline{p}_5 \\ \hline 0\,0\,0 & 1 \end{array}\right]=\left[\begin{array}{ccc|c} cq_5 & 0 & sq_5 & 0 \\ 0 & 1 & 0 & 0 \\ -sq_5 & 0 & cq_5 & d_5 \\ \hline 0 & 0 & 0 & 1 \end{array}\right], \quad (18) $$

$$ {}^{5}_{6}[T]=\left[\begin{array}{c|c} {}^{5}_{6}[R] & \overline{p}_6 \\ \hline 0\,0\,0 & 1 \end{array}\right]=\left[\begin{array}{ccc|c} cq_6 & -sq_5 & 0 & 0 \\ sq_6 & cq_6 & 0 & b_6 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right], \quad (19) $$

$$ {}^{0''}_{6}[T]=\left[\begin{array}{ccc|c} cq_5\cdot cq_6 & -cq_5\cdot sq_6 & sq_5 & 0 \\ sq_6 & cq_6 & 0 & b_6 \\ -sq_5\cdot cq_6 & sq_5\cdot sq_6 & cq_5 & d_5 \\ \hline 0 & 0 & 0 & 1 \end{array}\right], \quad (20) $$

$$ {}^{6}_{G}[T]=\left[\begin{array}{c|c} {}^{6}_{G}[R]\equiv I_3 & \overline{p}_{G6} \\ \hline 0\,0\,0 & 1 \end{array}\right]=\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_7 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]. \quad (21) $$

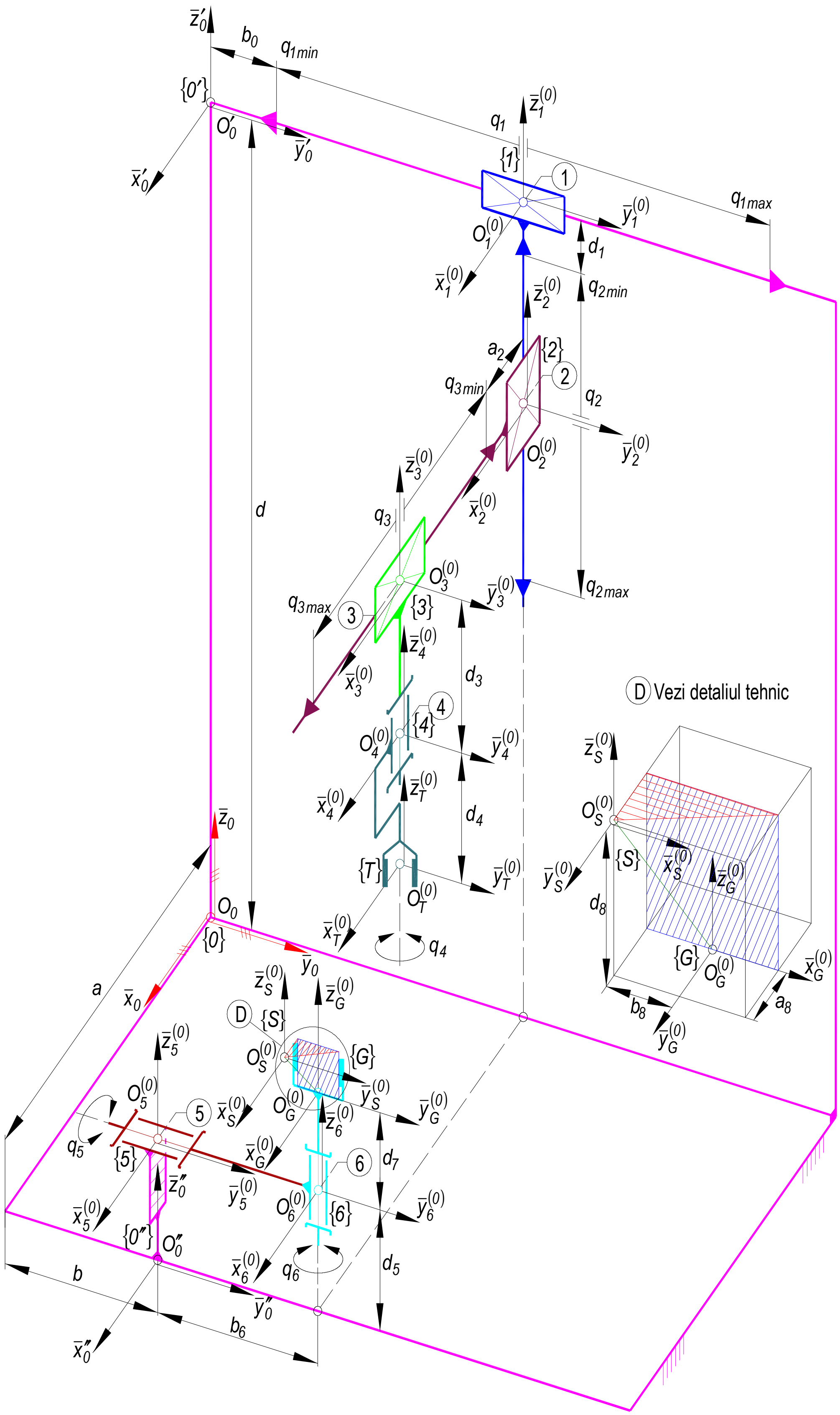

Fig.2 Kinematic scheme of the mechanical structure at zero-configuration 3TR-2R

The locating matrix between the system $\{G\}$, attached to the mobile platform and the fixed system $\{0''\}$ is defined according to:

$$ {}^{0''}_{G}[T] = \left[\begin{array}{c:c} {}^{0''}_{G}[R] & \bar{p}_{G0''} \\ \hdashline 0\,0\,0 & 1 \end{array}\right] = $$

$$ = \left[\begin{array}{ccc:c} cq_5 \cdot cq_6 & -cq_5 \cdot sq_6 & sq_5 & d_7 \cdot sq_5 \\ sq_6 & cq_6 & 0 & b_6 \\ -sq_5 \cdot cq_6 & sq_5 \cdot sq_6 & cq_5 & d_5 + d_7 \cdot cq_5 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]. \quad (22) $$

The system location matrix $\{S\}$ attached to the handled part about the fixed system $\{0''\}$ is:

$$ {}^{G}_{S}[T] = \left[\begin{array}{c:c} {}^{G}_{S}[R] \equiv I_3 & \bar{p}_{SG} \\ \hdashline 0\,0\,0 & 1 \end{array}\right] = \left[\begin{array}{ccc:c} 1 & 0 & 0 & a_8 \\ 0 & 1 & 0 & -b_8 \\ 0 & 0 & 1 & d_8 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right], \quad (23) $$

$$ {}^{6}_{S}[T] = \left[\begin{array}{c:c} {}^{6}_{S}[R] & \bar{p}_{S6} \\ \hdashline 0\,0\,0 & 1 \end{array}\right] = \left[\begin{array}{ccc:c} 1 & 0 & 0 & a_8 \\ 0 & 1 & 0 & -b_8 \\ 0 & 0 & 1 & d_7 + d_8 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right], \quad (24) $$

$$ {}^{0''}_{S}[T] = \left[\begin{array}{c:c} {}^{0''}_{S}[R] & \begin{matrix} (a_8 \cdot cq_6 + b_8 \cdot sq_6) \cdot cq_5 + (d_7 + d_8) \cdot sq_5 \\ b_6 + a_8 \cdot sq_6 - b_8 \cdot cq_6 \\ d_5 - (a_8 \cdot cq_6 + b_8 \cdot sq_6) \cdot sq_5 + (d_7 + d_8) \cdot cq_5 \end{matrix} \\ \hdashline 0\,0\,0 & 1 \end{array}\right] \quad (25) $$

$$ {}^{0''}_{S}[R] = \begin{bmatrix} cq_5 \cdot cq_6 & -cq_5 \cdot sq_6 & sq_5 \\ sq_6 & cq_6 & 0 \\ -sq_5 \cdot cq_6 & sq_5 \cdot sq_6 & cq_5 \end{bmatrix}, \quad (26) $$

where ${}^{0''}_{S}[R]$ represents the resultant rotation matrix between $\{S\}$ and $\{O''\}$ systems.

In the following, the matrix transformations between fixed system $\{0\}$, attached to the platform and mobile system $\{S\}$, attached to the part to be handled are determined:

$$ {}^{0}_{S}[T] = {}^{0}_{0''}[T] \cdot {}^{0''}_{S}[T], \quad (27) $$

$$ {}^{0}_{0''}[T] = \left[\begin{array}{c:c} {}^{0}_{0''}[R] \equiv I_3 & \bar{p}_{0''0} \\ \hdashline 0\,0\,0 & 1 \end{array}\right] = \left[\begin{array}{ccc:c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right], \quad (28) $$

$$ {}^{0}_{S}[T] = \left[\begin{array}{c:c} {}^{0}_{S}[R] & \bar{p}_{S} \\ \hdashline 0\;0\;0 & 1 \end{array}\right], \quad (29) $$

$$ {}^{0}_{S}[R] = \begin{bmatrix} cq_5 \cdot cq_6 & -cq_5 \cdot sq_6 & sq_5 \\ sq_6 & cq_6 & 0 \\ -sq_5 \cdot cq_6 & sq_5 \cdot sq_6 & cq_5 \end{bmatrix}, \quad (30) $$

$$ \bar{p}_S = \begin{bmatrix} a + (a_8 \cdot cq_6 + b_8 \cdot sq_6) \cdot cq_5 + (d_7 + d_8) \cdot sq_5 \\ b + b_6 + a_8 \cdot sq_6 - b_8 \cdot cq_6 \\ d_5 - (a_8 \cdot cq_6 + b_8 \cdot sq_6) \cdot sq_5 + (d_7 + d_8) \cdot cq_5 \end{bmatrix}. \quad (31) $$

Where ${}^{0}_{S}[R]$ is the resultant rotation matrix between $\{S\}$ and $\{O\}$ systems and $\bar{p}_S$ defines the position vector:

$$ \left\{ {}^{0}_{S}[R] \equiv R(\alpha_{yS}; \beta_{xS}; \gamma_{zS}),\ {}^{0}_{G}[R] \equiv R(\alpha_{yG}; \beta_{xG}; \gamma_{zG}) \right\} = $$

$$ = \begin{bmatrix} s\alpha_y \cdot s\beta_x \cdot s\gamma_z + c\alpha_y \cdot c\gamma_z & s\alpha_y \cdot s\beta_x \cdot c\gamma_z - c\alpha_y \cdot s\gamma_z & s\alpha_y \cdot c\beta_x \\ c\beta_x \cdot s\gamma_z & c\beta_x \cdot c\gamma_z & -s\beta_x \\ c\alpha_y \cdot s\beta_x \cdot s\gamma_z - s\alpha_y \cdot c\gamma_z & c\alpha_y \cdot s\beta_x c\gamma_z + s\alpha_y \cdot s\gamma_z & c\alpha_y \cdot c\beta_x \end{bmatrix} $$

To determine the orientation vector $\bar{\psi}_S = (\alpha_{yS} \quad \beta_{xS} \quad \gamma_{zS})^T$, above the matrix identity is applied. The equations of direct geometry are obtained), such as follows:

$$ {}^{0}\bar{\bar{X}} = \begin{bmatrix} \bar{p}_S \\ \hdashline \bar{\psi}_S \end{bmatrix} = \begin{bmatrix} (p_{xS} \quad p_{yS} \quad p_{zS})^T \\ \hdashline (\alpha_{yS} \quad 0 \quad \gamma_{zS})^T \end{bmatrix}, \quad (32) $$

where $\bar{p}_S$ is defined according to (31), and $\bar{\psi}_S$ with the expression below:

$$ \bar{\psi}_S \equiv \bar{\psi}_G = \begin{pmatrix} \alpha_{yS} \\ \beta_{xS} \\ \gamma_{zS} \end{pmatrix} \equiv \begin{pmatrix} \alpha_{yG} \\ \beta_{xG} \\ \gamma_{zG} \end{pmatrix} = \begin{pmatrix} q_5 \\ 0 \\ q_6 \end{pmatrix}. \quad (33) $$

$$ {}^{0}_{G}[T]=\left[\begin{array}{c|c} {}^{0}_{G}[R] & \bar{p}_G \\ \hline 0\,0\,0 & 1 \end{array}\right]= $$

$$ =\left[\begin{array}{ccc|c} cq_5\cdot cq_6 & -cq_5\cdot sq_6 & sq_5 & d_7\cdot sq_5 \\ sq_6 & cq_6 & 0 & b_6 \\ -sq_5\cdot cq_6 & sq_5\cdot sq_6 & cq_5 & d_5+d_7\cdot cq_5 \\ \hline 0 & 0 & 0 & 1 \end{array}\right], \quad (34) $$

The matrix transformations between the fixed system $\{0\}$, attached to the fixed platform and the mobile system $\{G\}$, attached to the mobile platform located at the end of the 2R mechanical structure are defined by (34).

Equations (32) — (34) express the position and orientation of the part $\{G\}$, respectively of the system $\{S\}$ through the vectors $\bar{p}_G$, $\bar{p}_S$ and $\bar{\psi}_S \equiv \bar{\psi}_G$, about the system attached to the fixed base, assuming known displacements (generalized coordinates) from the joints.

2.3 The equations of DGM of the cooperation between 3TR and 2R robots

In the cooperative action between the two structures 3TR and 2R, we aim to ensure the rotation, represented by the system $\{G\}$and $\{S\}$ processed by the tool $\{T\}$, according to a particular technological process.

By considering the location matrices (13) and (27), a series of matrix transformations are written between the two systems $\{T\}\rightarrow\{S\}$, as:

$$ {}^{0}_{S}[R]^{-1}\cdot\bar{p}_T=\left[\begin{array}{c} (a_2+q_3)\cdot cq_5\cdot cq_6+(b_0+q_1)\cdot sq_6- \\ -(d-d_1-d_3-d_4-q_2)\cdot sq_5\cdot cq_6 \\ \hline -(a_2+q_3)\cdot cq_5\cdot sq_6+(b_0+q_1)\cdot cq_6+ \\ +(d-d_1-d_3-d_4-q_2)\cdot sq_5\cdot sq_6 \\ \hline (a_2+q_3)\cdot sq_5+ \\ +(d-d_1-d_3-d_4-q_2)\cdot cq_5 \end{array}\right], \quad (35) $$

$$ -{}^{0}_{S}[R]^{-1}\cdot\bar{p}_S=\left[\begin{array}{c} d_5\cdot cq_6\cdot sq_5-a\cdot cq_6\cdot cq_5- \\ -(b+b_6)\cdot sq_6-a_8 \\ \hline -d_5\cdot sq_6\cdot sq_5+a\cdot sq_6\cdot cq_5- \\ -(b+b_6)\cdot cq_6+b_8 \\ \hline -a\cdot sq_5-d_5\cdot cq_5-d_7-d_8 \end{array}\right]. \quad (36) $$

The location matrix between the systems $\{T\}$ and $\{S\}$ is characterized by the components:

$$ {}^{S}_{T}[T]=\left[\begin{array}{c|c} {}^{S}_{T}[R] & \bar{p}_{TS} \\ \hline 0\,0\,0 & 1 \end{array}\right], \quad (37) $$

$$ {}^{S}_{T}[R]={}^{0}_{S}[R]^{-1}\cdot{}^{0}_{T}[R]= $$

$$ =\left[\begin{array}{c|c|c} cq_4\cdot cq_5\cdot cq_6+ & -sq_4\cdot cq_5\cdot cq_6+ & -sq_5\cdot cq_6 \\ +sq_4\cdot sq_6 & +cq_4\cdot sq_6 & \\ \hline -cq_4\cdot cq_5\cdot sq_6+ & cq_4\cdot cq_5\cdot sq_6+ & sq_5\cdot sq_6 \\ +sq_4\cdot cq_6 & +cq_4\cdot cq_6 & \\ \hline cq_4\cdot sq_5 & -sq_4\cdot sq_5 & cq_5 \end{array}\right]. \quad (38) $$

where ${}^{S}_{T}[R]$is the orientation matrix of the system $\{T\}$ relative to$\{S\}$system. Orientation vector ${}^{S}\bar{\psi}_T$ is obtained from a matrix identity, written by using the algorithm of the orientation functions, as shown in the following equation:

$$ {}^{S}_{T}[R]\equiv R\left({}^{S}\alpha_{zT};\ {}^{S}\beta_{yT};\ {}^{S}\gamma_{zT}\right), \quad (39) $$

$$ {}^{S}_{T}[R]\equiv R\left({}^{S}\alpha_{zT};{}^{S}\beta_{yT};{}^{S}\gamma_{zT}\right)= $$

$$ =\left[\begin{array}{c|c|c} c\alpha_z\cdot c\beta_y\cdot c\gamma_z+ & -c\alpha_z\cdot c\beta_y\cdot s\gamma_z+ & -c\alpha_z\cdot s\beta_y \\ +s\alpha_z\cdot s\gamma_z & +s\alpha_z\cdot c\gamma_z & \\ \hline -s\alpha_z\cdot c\beta_y\cdot c\gamma_z+ & s\alpha_z\cdot c\beta_y\cdot s\gamma_z+ & s\alpha_z\cdot s\beta_y \\ +c\alpha_z\cdot s\gamma_z & +c\alpha_z\cdot c\gamma_z & \\ \hline s\beta_y\cdot c\gamma_z & -s\beta_y\cdot s\gamma_z & c\beta_y \end{array}\right] $$

$$ {}^{S}\bar{\psi}_T=\left({}^{S}\alpha_{zT}\quad {}^{S}\beta_{yT}\quad {}^{S}\gamma_{xT}\right)^T=\left(q_6\quad q_5\quad q_4\right)^T, \quad (40) $$

In computing the position vector, ${}^{S}\bar{p}_{TS}$ the following transfer matrix equations are written:

$$ {}^{S}\bar{p}_{TS}=-{}^{S}_{T}[R]\cdot{}^{S}\bar{p}_{TS}\equiv{}^{0}_{S}[R]^{-1}\cdot\left(\bar{p}_T-\bar{p}_S\right), \quad (41) $$

$$ {}^{S}\bar{p}_{TS}=\left[{}^{S}p_{xTS}\quad {}^{S}p_{yTS}\quad {}^{S}p_{zTS}\right]^T. \quad (42) $$

$$
{}^{S}\overline{p}_{TS}=\begin{bmatrix}-(a-a_2-q_3)\cdot cq_5\cdot cq_6-\\-(b-b_0+b_6-q_1)\cdot sq_6-a_8+\\+(d-d_1-d_3-d_4-d_5-q_2)\cdot sq_5\cdot cq_6\\\hline (a-a_2-q_3)\cdot cq_5\cdot sq_6-\\-(b-b_0+b_6-q_1)\cdot cq_6+b_8+\\+(d-d_1-d_3-d_4-d_5-q_2)\cdot sq_5\cdot sq_6\\\hline -(a-a_2-q_3)\cdot sq_5-d_7-d_8+\\+(d-d_1-d_3-d_4-d_5-q_2)\cdot cq_5\end{bmatrix},(43)
$$

The final position and orientation are defined:

$$
{}^{S}\overline{X}_{TS}=\begin{bmatrix}{}^{S}\overline{p}_{TS}\\\hline {}^{S}\overline{\psi}_{T}\end{bmatrix}=\begin{bmatrix}\left({}^{S}p_{xTS}\quad {}^{S}p_{yTS}\quad {}^{S}p_{zTS}\right)^T\\\hline \left({}^{S}\alpha_{zT}\quad {}^{S}\beta_{yT}\quad {}^{S}\gamma_{zT}\right)^T\end{bmatrix}, (44)
$$

Equations (40) and (43) define the tool system's position and orientation $\{T\}$, attached to the 3TR structure, relative to the part system $\{S\}$, attached to the 2R structure. The position and orientation of the part system $\{S\}$, attached to the 2R structure, relative to the tool system $\{T\}$, attached to 3TR structure, results by applying some matrix transformations:

$$
{}^{0}_{S}[T]={}^{0}_{T}[T]\cdot{}^{T}_{S}[T], \qquad (45)
$$

$$
\left\{\begin{matrix}{}^{T}_{S}[T]={}^{0}_{T}[T]^{-1}\cdot{}^{0}_{S}[T]=\\={}^{0}_{T}[T]^{-1}\cdot\left\{{}^{0}_{6}[T]\cdot{}^{6}_{G}[T]\cdot{}^{G}_{S}[T]\right\}\end{matrix}\right\}, \quad (46)
$$

$$
{}^{T}_{S}[R]=\left[\begin{array}{c|c|c}cq_4\cdot cq_5\cdot cq_6+ & -cq_4\cdot cq_5\cdot sq_6+ & \\ +sq_4\cdot sq_6 & +sq_4\cdot cq_6 & cq_4\cdot sq_5\\\hline -sq_4\cdot cq_5\cdot cq_6+ & sq_4\cdot cq_5\cdot sq_6+ & \\ +cq_4\cdot sq_6 & +cq_4\cdot cq_6 & -sq_4\cdot sq_5\\\hline -sq_5\cdot cq_6 & sq_5\cdot sq_6 & cq_5\end{array}\right](47)
$$

where ${}^{T}_{S}[R]$ is the orientation matrix of the $\{S\}$ system, relative to $\{T\}$. The orientation vector ${}^{T}\overline{\psi}_{S}$ is obtained from the matrix identity below, written according to the algorithm of the orientation functions:

$$
{}^{T}_{S}[R]\equiv R\left({}^{T}\alpha_{zS};{}^{T}\beta_{yS};{}^{T}\gamma_{zS}\right)=R(\alpha_z)^{-1}\cdot R(\beta_y)\cdot R(\gamma_z), (48)
$$

$$
{}^{T}_{S}[R]\equiv R\left({}^{T}\alpha_{zS};\ {}^{T}\beta_{yS};\ {}^{T}\gamma_{zS}\right)=
$$

$$
=\left[\begin{array}{c|c|c}c\alpha_z\cdot c\beta_y\cdot c\gamma_z+ & -c\alpha_z\cdot c\beta_y\cdot s\gamma_z+ & \\ +s\alpha_z\cdot s\gamma_z & +s\alpha_z\cdot c\gamma_z & c\alpha_z\cdot s\beta_y\\\hline -s\alpha_z\cdot c\beta_y\cdot c\gamma_z+ & s\alpha_z\cdot c\beta_y\cdot s\gamma_z+ & \\ +c\alpha_z\cdot s\gamma_z & +c\alpha_z\cdot c\gamma_z & -s\alpha_z\cdot s\beta_y\\\hline -s\beta_y\cdot c\gamma_z & s\beta_y\cdot s\gamma_z & c\beta_y\end{array}\right](49)
$$

The DGM equations are obtained from the matrix (47) and the identity (49), according to:

$$
{}^{T}\overline{\psi}_{S}=\left({}^{T}\alpha_{zS}\quad {}^{T}\beta_{yS}\quad {}^{T}\gamma_{zS}\right)^T=\left[q_4\quad q_5\quad q_6\right]^T, (50)
$$

$$
{}^{T}\overline{p}_{ST}=\begin{bmatrix}(b-b_0+b_6-q_1+a_8\cdot sq_6-b_8\cdot cq_6)\cdot sq_4+\\+[a-a_2-q_3+(d_7+d_8)\cdot sq_5]\cdot cq_4+\\+(a_8\cdot cq_6+b_8\cdot sq_6)\cdot cq_4\cdot cq_5\\\hline (b-b_0+b_6-q_1+a_8\cdot sq_6-b_8\cdot cq_6)\cdot cq_4-\\-\left[a-a_2-q_3+(d_7+d_8)\cdot sq_5\right]\cdot sq_4+\\+(a_8\cdot cq_6+b_8\cdot sq_6)\cdot sq_4\cdot cq_5\\\hline -(a_8\cdot cq_6+b_8\cdot sq_6)\cdot sq_5+(d_7+d_8)\cdot cq_5-\\-(d-d_1-d_3-d_4-d_5-q_2)\end{bmatrix}. (51)
$$

The final position and orientation are defined:

$$
{}^{T}\overline{X}_{ST}=\begin{bmatrix}{}^{T}\overline{p}_{ST}\\\hline {}^{T}\overline{\psi}_{S}\end{bmatrix}=\begin{bmatrix}\left({}^{T}p_{xST}\quad {}^{T}p_{yST}\quad {}^{T}p_{zST}\right)^T\\\hline \left({}^{T}\alpha_{ZS}\quad {}^{T}\beta_{yS}\quad {}^{T}\gamma_{zS}\right)^T\end{bmatrix}. \quad (52)
$$

Equations (50) and (51) express the workpiece system's position and orientation relative to the tool system.

3. CONCLUSIONS

This paper highlights the equations of DGM for the 3TR structure and the 2R structure, the direct geometry of the cooperative action between structure 3TR and structure 2R. For this purpose, the algorithm of the locating matrices was applied.

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ECUAȚIILE GEOMETRICE PENTRU DOI ROBOȚI DE TIPUL 3TR-2R ÎN MIȘCĂRI DE COOPERARE

În această lucrare sunt prezentate ecuațiile geometriei directe pentru o structură de tipul 3TR-2R între care au loc acțiuni de cooperare. Prima structură de robot asigură poziționarea și orientarea sculei de lucru iar cea de-a doua structură, este destinată poziționării și orientării piesei. Cele două structuri de roboți sunt implementate într-un proces de lucru, în care cele două structuri cooperează fiind în conexiune directă cu un centru de prelucrare cu cinci grade de libertate.

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