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MATHEMATICAL MODELING OF A 3R ROBOT STRUCTURE IN THE NOMINAL CONFIGURATION

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Abstract: *The main purpose of this paper is to develop a study on the direct geometry and kinematics of an RRR-robot (with three degrees of freedom of rotation), in the nominal configuration.*
For this purpose, the algorithm of locating matrices was applied, to determine the direct geometry equations. To compute the velocities and accelerations relatively to $\{0\}$ *fixed system, the algorithm of transfer matrices was used. The results are useful to establish the equations of the motion trajectory.*
Keywords*: robot, direct geometry, kinematics, motion trajectory*

1. INTRODUCTION

The industrial applications where serial robot structures are implemented are becoming an essential part of our lives. Robot manipulators are used for performing manufacturing tasks, such as part handling, welding, or painting. The development of industrial applications demands robot manipulators that ensure an increase in productivity and quality at lower manufacturing costs. As an effect, there is an increasing need to design robot structures that achieve faster and accurate motions [10-12]. These objectives result from improving the design of the robot's mechanical structure and the controllers. The present paper aims to present the geometric and kinematic model applied for three degrees of freedom robot mechanical structure - three rotations around $\bar{x}$, $\bar{y}$ and $\bar{z}$ axes (see *Table 1*). The 3R robot structure, in the nominal configuration, is analyzed. The direct geometrical model (DGM) is obtained by applying an algorithm based on homogenous transformation matrices, while the direct kinematic's equations result from applying the Jacobian matrix [1-5]. The results will be used in future research to determine the inverse kinematic and dynamic model, essential in defining the motion trajectories for the analyzed robot.

2. DIRECT GEOMETRIC MODELING OF AN 3R TYPE ROBOT STRUCTURE

The analysis starts by defining the matrix $M_{vn}^{(0)}$ of nominal geometry for the considered 3R robot structure. This matrix containing the input data corresponding to the direct geometric modeling, and is presented in the table below:

Table 1

Element i=1	*Joint type*	$\bar{p}_i^{(0)T}$			$\bar{k}_i^{(0)T}$		
1	*R*	*0*	*0*	l_0	*0*	*1*	*0*
2	*R*	l_1	*0*	*0*	*1*	*0*	*0*
3	*R*	*0*	l_2	*0*	*0*	*0*	*1*
4	-	*0*	*0*	l_3	*1*	*0*	*0*

The equations that define the direct geometrical model (MGD) were computed using the homogenous transformation matrices. Within Figure 1 is depicted the kinematic diagram of the mechanical structure, in the initial configuration (zero-configuration). The zero configuration is characterized by the fact that all the generalized coordinates initialize to zero:

$$\bar{\theta}^{(0)} = [q_i = 0; \quad i = 1 \rightarrow n]^T . \tag{1}$$

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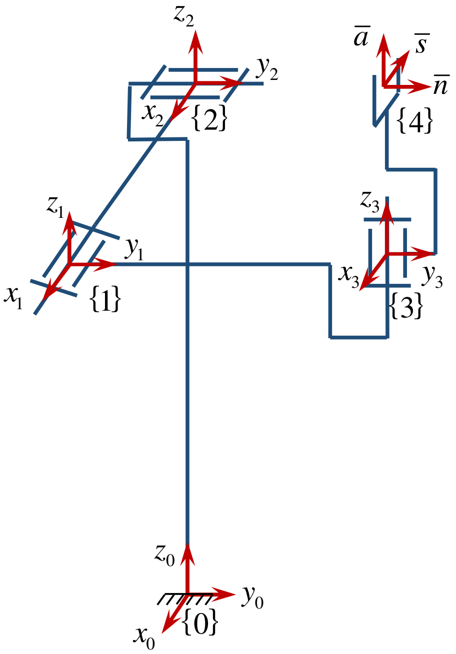

Fig.1. Kinematic diagram of the 3R robot

According to Fig. 1, in the geometrical center of every kinematical joint, is attached a mobile system $\{i\}$ that models the motion of the kinematical link. The homogeneous transformation matrices between the adjoined systems $\{i\}$ and $\{i-1\}$, where $i=1 \rightarrow 4$, are determined according to [1], [6-8] as follows:

$$ {}_1^0[T]=\begin{bmatrix} cq_1 & 0 & sq_1 & 0 \\ 0 & 1 & 0 & 0 \\ -sq_1 & 0 & cq_1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2)$$

$$ {}_2^1[T]=\begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & cq_2 & -sq_2 & 0 \\ 0 & sq_2 & cq_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad (3)$$

$$ {}_3^2[T]=\begin{bmatrix} cq_3 & -sq_3 & 0 & 0 \\ sq_3 & cq_3 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad (4)$$

$$ {}_4^3[T]=\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \qquad (5)$$

The transformations, between the mobile system $\{i\}$ and $\{0\}$ fixed system, for any robot configuration, are computed according to:

$$ {}_2^0[T]=\left[\begin{array}{ccc|c} cq_1 & sq_1 \cdot sq_2 & sq_1 \cdot cq_2 & l_1 \cdot cq_1 \\ 0 & cq_2 & -sq_2 & 0 \\ -sq_1 & cq_1 \cdot sq_2 & cq_1 \cdot cq_2 & l_0 - l_1 \cdot sq_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right], \qquad (6)$$

$$ {}_3^0[T]=\left[\begin{array}{c|c} {}_3^0[R] & \begin{matrix} l_1 \cdot cq_1 + l_2 \cdot sq_1 \cdot sq_2 \\ l_2 \cdot cq_2 \\ l_0 - l_1 \cdot sq_1 + l_2 \cdot cq_1 \cdot sq_2 \end{matrix} \\ \hline 0 \;\; 0 \;\; 0 & 1 \end{array}\right], \qquad (7)$$

${}_3^0[R]$ is the resultant rotation matrix that defines the orientation of the mobile system $\{3\}$ (attached in the geometric center of the third joint) relative to $\{0\}$ fixed system:

$$ {}_3^0[R]=\left[\begin{array}{c|c|c} cq_1 \cdot cq_3 + sq_1 \cdot sq_2 \cdot sq_3 & -cq_1 \cdot sq_3 + sq_1 \cdot sq_2 \cdot cq_3 & sq_1 \cdot cq_2 \\ \hline cq_2 \cdot sq_3 & cq_2 \cdot cq_3 & -sq_2 \\ \hline sq_1 \cdot cq_3 + cq_1 \cdot sq_2 \cdot sq_3 & sq_1 \cdot sq_3 + cq_1 \cdot sq_2 \cdot cq_3 & cq_1 \cdot cq_2 \end{array}\right], \qquad (8)$$

The homogenous transformation between the system attached in the characteristic point of the robot end-effector and $\{0\}$ system is computed:

$$ {}_4^0[T]=\left[\begin{array}{c|c} {}_4^0[R] & \begin{matrix} l_1 \cdot cq_1 + l_2 \cdot sq_1 \cdot sq_2 + l_3 \cdot sq_1 \cdot cq_2 \\ l_2 \cdot cq_2 - l_3 \cdot sq_2 \\ l_0 - l_1 \cdot sq_1 + l_2 \cdot cq_1 \cdot sq_2 + l_3 \cdot cq_1 \cdot cq_2 \end{matrix} \\ \hline 0 \;\; 0 \;\; 0 & 1 \end{array}\right] \qquad (9)$$

In the expression above, ${}_4^0[R]$ is the resultant matrix between the mobile system $\{4\}$ (attached in the characteristic point belonging to robot's end-effector) and the fixed system $\{0\}$:

$$ {}_4^0[R]=\left[\begin{array}{c|c|c} -cq_1 \cdot sq_3 + sq_1 \cdot sq_2 \cdot cq_3 & -cq_1 \cdot cq_3 - sq_1 \cdot sq_2 \cdot sq_3 & sq_1 \cdot cq_2 \\ \hline cq_2 \cdot cq_3 & -cq_2 \cdot sq_3 & -sq_2 \\ \hline sq_1 \cdot sq_3 + cq_1 \cdot sq_2 \cdot cq_3 & sq_1 \cdot cq_3 - cq_1 \cdot sq_2 \cdot sq_3 & cq_1 \cdot cq_2 \end{array}\right] \qquad (10)$$

To define the final orientation of the end-effector, the following matrix identity is used:

$$ {}^0_4[R] = R(\alpha_y - \beta_x - \gamma_z) = $$

$$ = \left[\begin{array}{c|c|c} s\alpha_y \cdot s\beta_x \cdot s\gamma_z + \\ +c\alpha_y \cdot c\gamma_z & s\alpha_y \cdot s\beta_x \cdot c\gamma_z - \\ -c\alpha_y \cdot s\gamma_z & s\alpha_y \cdot c\beta_x \\ \hline c\beta_x \cdot s\gamma_z & c\beta_x \cdot c\gamma_z & -s\beta_x \\ \hline c\alpha_y \cdot s\beta_x \cdot s\gamma_z - \\ -s\alpha_y \cdot c\gamma_z & c\alpha_y \cdot s\beta_x \cdot c\gamma_z + \\ +s\alpha_y \cdot s\gamma_z & c\alpha_y \cdot c\beta_x \end{array}\right] \quad (11) $$

By applying the function $A\tan 2$, the orientation angles, represented by α_y, β_x and γ_z that define the end-effector's final orientation, are obtained. According to [1], it results in the column vector of the orientation angles:

$$ \overline{\overline{\psi}} = \left[q_1 \quad q_2 \quad c\left(\frac{\pi}{2} + q_3\right)\right]^T . \qquad (12) $$

Thus, the DGM equations are written:

$$ {}^{(0)}\overline{X} = \left[\overline{p} \quad \overline{\overline{\psi}} \right]^T = $$

$$ \left[\begin{array}{c} l_1 \cdot cq_1 + l_2 \cdot sq_1 \cdot sq_2 + l_3 \cdot sq_1 \cdot cq_2 \\ l_2 \cdot cq_2 - l_3 \cdot sq_2 \\ l_0 - l_1 \cdot sq_1 + l_2 \cdot cq_1 \cdot sq_2 + l_3 \cdot cq_1 \cdot cq_2 \\ \hline q_1 \\ q_2 \\ c\left(\frac{\pi}{2} + q_3\right) \end{array}\right] \qquad (13) $$

The DGM equations define the end-effector's final position and orientation relative to the fixed system and for any robot configuration.

3. DIRECT KINEMATIC MODEL OF 3R ROBOT USING JACOBIAN MATRIX

The Jacobian matrix is used in robot mechanics to transfer velocities from the space of configurations to the Cartesian space of motion. In kinematical modeling, this matrix corresponds to a specific configuration of the robot in the workspace.
Further on, an algorithm for determining the Jacobian matrix components, based on the transfer matrices method, is applied [4].
The matrix equations for operational velocities and accelerations defining the motion of the robot final effector, projected on the fixed reference system $\{0\}$, are presented in a general form according to the following expressions:

$$ \underset{(6\times1)}{{}^{0}\dot{X}} = \begin{bmatrix} {}^{0}\bar{v}_n \\ {}^{0}\bar{\omega}_n \end{bmatrix} = \begin{bmatrix} V(\bar{\theta}) \\ \Omega(\bar{\theta}) \end{bmatrix} \cdot \dot{\bar{\theta}} = {}^{(0)}J(\bar{\theta}) \cdot \dot{\bar{\theta}}. \qquad (14) $$

$$ \underset{(6\times1)}{{}^{0}\ddot{X}} = \begin{bmatrix} {}^{0}\dot{\bar{v}}_n \\ {}^{0}\dot{\bar{\omega}}_n \end{bmatrix} = \begin{bmatrix} V(\bar{\theta}) \\ \Omega(\bar{\theta}) \end{bmatrix} \cdot [\ddot{\bar{\theta}} \quad \dot{\bar{\theta}}] = $$

$$ = {}^{(0)}J(\bar{\theta}) \cdot \ddot{\bar{\theta}} + {}^{(\dot{0})}J(\bar{\theta}) \cdot \dot{\bar{\theta}} \qquad (15) $$

When required to express the operational velocities and accelerations about the n mobile system, a matrix operator ensuring the transfer from one system to another must be applied:

$$ {}^{(n)}\dot{X} = \begin{bmatrix} {}^{n}\bar{v}_n \\ {}^{n}\bar{\omega}_n \end{bmatrix} = {}^{n}R \cdot {}^{(0)}\dot{X} = {}^{n}J(\bar{\theta}) \cdot \dot{\bar{\theta}}, \qquad (16) $$

$$ {}^{(n)}\ddot{X} = \left[{}^{n}\dot{v}_n \quad {}^{n}\dot{\omega}_n \right]^T = {}^{n}R \cdot {}^{(0)}\ddot{X} = $$

$$ = {}^{n}J(\bar{\theta}) \cdot \ddot{\bar{\theta}} + {}^{\dot{n}}J(\bar{\theta}) \cdot \dot{\bar{\theta}}, \qquad (17) $$

where,

$$ {}^{n}J(\bar{\theta}) = {}^{n}R \cdot {}^{0}J(\bar{\theta}), \qquad (18) $$

$$ {}^{n}\dot{X} = \left[\begin{array}{c} {}^{n}\dot{v}_n \\ \hline {}^{n}\dot{\omega}_n \end{array}\right] = \left[\begin{array}{c|c} {}^{0}_{n}[R]^{-1} & [0] \\ \hline [0] & {}^{0}_{n}[R]^{-1} \end{array}\right] \cdot \left[\begin{array}{c} {}^{0}\dot{v}_n \\ \hline {}^{0}\dot{\omega}_n \end{array}\right] \qquad (19) $$

The following expressions define operational accelerations in the mobile system:

$$ {}^{n}\ddot{X} = \left[{}^{n}\dot{v}_n \quad {}^{n}\dot{\omega}_n \right] = {}^{n}R \cdot \left[{}^{n}\dot{v}_n \quad {}^{n}\dot{\omega}_n \right] = $$

$$ = {}^{n}R \cdot \left[{}^{0}J(\bar{\theta}) \quad {}^{\dot{0}}J(\bar{\theta}) \right] \cdot \begin{bmatrix} \ddot{\bar{\theta}} \\ \dot{\bar{\theta}} \end{bmatrix} \qquad (20) $$

The MCD equations about the $\{n\}$ system, expressed by using the Jacobian matrix, are:

$$ {}^{n}\dot{X} = \begin{bmatrix} {}^{n}\bar{v}_n \\ {}^{n}\bar{\omega}_n \end{bmatrix} = {}^{n}J(\bar{\theta}) \cdot \dot{\bar{\theta}}, \qquad (21) $$

$$ {}^{n}\ddot{X} = \begin{bmatrix} {}^{n}\dot{v}_n \\ {}^{n}\dot{\omega}_n \end{bmatrix} = \left[{}^{n}J(\bar{\theta}) \quad {}^{\dot{n}}J(\bar{\theta}) \right] \cdot \begin{bmatrix} \ddot{\bar{\theta}} \\ \dot{\bar{\theta}} \end{bmatrix}. \qquad (22) $$

For the 3R robot structure represented in Fig. 1, the transfer matrices algorithm from kinematics is applied. The components of the linear velocities transfer matrices, denoted $V(\bar{\theta})$ and linear accelerations $A(\bar{\theta})$ are computed:

$$V(\bar{\theta}) = [V_1 \quad V_2 \quad V_3]^T, \tag{23}$$

$$\dot{V}(\bar{\theta}) = [\dot{V}_1 \quad \dot{V}_2 \quad \dot{V}_3]^T. \tag{24}$$

$$A(\bar{\theta}) = [V(\bar{\theta}) \quad \dot{V}(\bar{\theta})]^T. \tag{25}$$

where the components V_i and $\dot{V}_i$, $i = 1 \to 3$, are:

$$V_1 = \frac{\partial \bar{p}_3}{\partial q_1} = \begin{bmatrix} l_2 \cdot cq_1 \cdot sq_2 - l_1 \cdot sq_1 \\ 0 \\ -l_2 \cdot sq_1 \cdot cq_2 - l_1 \cdot cq_1 \end{bmatrix}, \tag{26}$$

$$V_2 = \frac{\partial \bar{p}_3}{\partial q_2} = \begin{bmatrix} l_2 \cdot cq_2 \cdot sq_1 \\ -l_2 \cdot sq_2 \\ l_2 \cdot cq_1 \cdot cq_2 \end{bmatrix}, \quad V_3 = \frac{\partial \bar{p}_3}{\partial q_3} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \tag{27}$$

$$\dot{V}_1 = \begin{bmatrix} l_2 \cdot \dot{q}_2 \cdot cq_1 \cdot cq_2 - l_1 \cdot \dot{q}_1 \cdot cq_1 - \\ -l_2 \cdot \dot{q}_1 \cdot sq_1 \cdot sq_2 \\ 0 \\ -l_2 \cdot \dot{q}_2 \cdot cq_1 \cdot sq_2 - l_1 \cdot \dot{q}_1 \cdot sq_1 - \\ -l_2 \cdot \dot{q}_1 \cdot sq_1 \cdot cq_2 \end{bmatrix}, \tag{28}$$

$$\dot{V}_2 = \begin{bmatrix} l_2 \cdot \dot{q}_1 \cdot cq_1 \cdot cq_2 - \\ -l_2 \cdot \dot{q}_2 \cdot sq_1 \cdot sq_2 \\ -l_2 \cdot \dot{q}_2 \cdot cq_2 \\ -l_2 \cdot \dot{q}_1 \cdot sq_1 \cdot cq_2 - \\ -l_2 \cdot \dot{q}_2 \cdot cq_1 \cdot sq_2 \end{bmatrix}, \quad \dot{V}_3 - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{29}$$

The components of angular velocities transfer matrices $\Omega(\bar{\theta})$ and angular accelerations transfer matrices $\dot{\Omega}(\bar{\theta})$ are defined according to:

$$\Omega(\bar{\theta}) = [\Omega_1 \quad \Omega_2 \quad \Omega_3]^T, \tag{30}$$

$$\dot{\Omega}(\bar{\theta}) = [\dot{\Omega}_1 \quad \dot{\Omega}_2 \quad \dot{\Omega}_3]^T, \tag{31}$$

$$\Xi(\theta) = [\Omega(\theta) \quad \dot{\Omega}(\theta)], \tag{32}$$

$$\Omega_1 = {}_1^0[R] \cdot \bar{k}_1^{(0)} \cdot \Delta_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \tag{33}$$

$$\Omega_2 = {}_2^0[R] \cdot \bar{k}_2^{(0)} \cdot \Delta_2 = \begin{bmatrix} cq_1 \\ 0 \\ -sq_1 \end{bmatrix}, \tag{34}$$

$$\Omega_3 = {}_3^0[R] \cdot \bar{k}_3^{(0)} \cdot \Delta_3 = \begin{bmatrix} cq_2 \cdot sq_1 \\ -sq_2 \\ cq_1 \cdot cq_2 \end{bmatrix}, \tag{35}$$

$$\dot{\Omega}_1 = \frac{d\Omega_1}{dt} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \dot{\Omega}_2 = \frac{d\Omega_2}{dt} = \begin{bmatrix} -\dot{q}_1 \cdot sq_1 \\ 0 \\ -\dot{q}_1 \cdot cq_1 \end{bmatrix}, \tag{36}$$

The matrices defined with (26) - (29) are substituted in (23) - (24) resulting in the transfer matrices of linear velocities (23) and accelerations (25).

$$\dot{\Omega}_3 = \frac{d\Omega_3}{dt} = \begin{bmatrix} \dot{q}_1 \cdot cq_1 \cdot cq_2 - \dot{q}_2 \cdot sq_1 \cdot sq_2 \\ -\dot{q}_2 \cdot cq_2 \\ -\dot{q}_1 \cdot sq_1 \cdot cq_2 - \dot{q}_2 \cdot cq_1 \cdot sq_2 \end{bmatrix}. \tag{37}$$

The expressions (33) - (37) are substituted into (30) and (31) which results in the transfer matrices of angular velocities (30) and accelerations (32), according to [1-9]. The components of the Jacobian matrix and its derivative, projected on the fixed frame $\{0\}$, are computed by considering the expression (14):

$$^{(0)}J_1 = \begin{bmatrix} V_1 \\ \Omega_1 \end{bmatrix}, \quad ^{(0)}J_2 = \begin{bmatrix} V_2 \\ \Omega_2 \end{bmatrix}, \quad ^{(0)}J_3 = \begin{bmatrix} V_3 \\ \Omega_3 \end{bmatrix}, \tag{38}$$

$$^{(0)}\dot{J}_1 = \begin{bmatrix} \dot{V}_1 \\ \dot{\Omega}_1 \end{bmatrix}, \, ^{(0)}\dot{J}_2 = \begin{bmatrix} \dot{V}_2 \\ \dot{\Omega}_2 \end{bmatrix}, \, ^{(0)}\dot{J}_3 = \begin{bmatrix} \dot{V}_3 \\ \dot{\Omega}_3 \end{bmatrix}. \tag{39}$$

Where, $V_i, \dot{V}_i, \Omega_i$ and $\dot{\Omega}_i$ are defined by (26) - (29) and (33) - (37) respectively. The absolute linear velocities and accelerations that define the motion of the robot end-effector are obtained based on expressions (14) and (15):

$$^{(0)}\bar{\omega}_3 = \begin{bmatrix} \dot{q}_2 \cdot cq_1 + \dot{q}_3 \cdot sq_1 \cdot cq_2 \\ \dot{q}_1 - \dot{q}_3 \cdot sq_2 \\ -\dot{q}_2 \cdot sq_1 + \dot{q}_3 \cdot cq_1 \cdot cq_2 \end{bmatrix}, \tag{40}$$

$$^{(0)}\dot{\bar{\omega}}_3 = \begin{bmatrix} \ddot{q}_1 \cdot cq_1 + \ddot{q}_3 \cdot sq_1 \cdot cq_2 - \dot{q}_1 \cdot \dot{q}_2 \cdot sq_1 - \\ -\dot{q}_2 \cdot \dot{q}_3 \cdot sq_1 \cdot sq_2 + \dot{q}_1 \cdot \dot{q}_3 \cdot cq_1 \cdot cq_2 \\ \ddot{q}_1 - \dot{q}_2 \cdot \dot{q}_3 \cdot cq_2 - \ddot{q}_3 \cdot sq_2 \\ -\ddot{q}_1 \cdot sq_1 + \ddot{q}_3 \cdot cq_1 \cdot cq_2 - \dot{q}_1 \cdot \dot{q}_2 \cdot cq_1 - \\ -\dot{q}_2 \cdot \dot{q}_3 \cdot cq_1 \cdot sq_2 - \dot{q}_1 \cdot \dot{q}_3 \cdot sq_1 \cdot cq_2 \end{bmatrix}, \tag{41}$$

$$
{}^{(0)}\bar{v}_3 = \begin{bmatrix} l_2 \cdot \dot{q}_1 \cdot cq_1 \cdot sq_2 - l_1 \cdot \dot{q}_1 \cdot sq_1 + \\ + l_2 \cdot \dot{q}_2 \cdot sq_1 \cdot cq_2 \\ l_2 \cdot \dot{q}_2 \cdot sq_2 \\ -l_2 \cdot \dot{q}_1 \cdot sq_1 \cdot sq_2 - l_1 \cdot \dot{q}_1 \cdot cq_1 + \\ + l_2 \cdot \dot{q}_2 \cdot cq_1 \cdot cq_2 \end{bmatrix}, \quad (42)
$$

$$
{}^{(0)}\dot{\bar{v}}_3 = \begin{bmatrix} l_2 \cdot \ddot{q}_1 \cdot cq_1 \cdot sq_2 - l_1 \cdot \dot{q}_1^2 \cdot cq_1 - \\ -l_2 \cdot \dot{q}_1^2 \cdot sq_1 \cdot sq_2 - l_2 \cdot \dot{q}_2^2 \cdot sq_1 \cdot sq_2 + \\ + l_2 \cdot \ddot{q}_1 \cdot sq_1 \cdot cq_2 + 2 \cdot l_2 \cdot \dot{q}_1 \cdot \dot{q}_2 \cdot cq_1 \cdot cq_2 \\ -l_2 \cdot \dot{q}_2^2 \cdot cq_2 - l_2 \cdot \ddot{q}_1 \cdot sq_2 \\ -l_2 \cdot \ddot{q}_1 \cdot sq_1 \cdot sq_2 + l_1 \cdot \dot{q}_1^2 \cdot sq_1 - \\ -l_1 \cdot \dot{q}_1^2 \cdot cq_1 - l_2 \cdot \dot{q}_1^2 \cdot cq_1 \cdot sq_2 - \\ -l_2 \cdot \dot{q}_1^2 \cdot cq_1 \cdot sq_2 + l_2 \cdot \ddot{q}_1 \cdot cq_1 \cdot cq_2 + \\ +2 \cdot l_2 \cdot \dot{q}_1 \cdot \dot{q}_2 \cdot sq_1 \cdot cq_2 \end{bmatrix}. \quad (43)
$$

The velocities and accelerations of the end effector relative projected on the mobile reference frame are also determined [1]:

$$
{}^3\bar{\omega}_3 = \begin{bmatrix} \dot{q}_2 \cdot cq_3 + \dot{q}_1 \cdot cq_2 \cdot sq_3 \\ \dot{q}_1 \cdot cq_2 \cdot cq_3 - \dot{q}_2 \cdot sq_3 \\ \dot{q}_3 - \dot{q}_1 \cdot sq_2 \end{bmatrix}, \quad (44)
$$

$$
{}^3\dot{\bar{\omega}}_3 = \begin{bmatrix} \ddot{q}_1 \cdot cq_3 + \ddot{q}_1 \cdot cq_2 \cdot sq_3 - \dot{q}_2 \cdot \dot{q}_3 \cdot sq_3 - \\ \dot{q}_1 \cdot \dot{q}_2 \cdot sq_2 \cdot sq_3 + \dot{q}_1 \cdot \dot{q}_3 \cdot cq_2 \cdot cq_3 \\ \ddot{q}_1 \cdot cq_2 \cdot cq_3 - \ddot{q}_1 \cdot sq_3 - \dot{q}_2 \cdot \dot{q}_3 \cdot cq_3 - \\ \dot{q}_1 \cdot \dot{q}_2 \cdot sq_2 \cdot cq_3 - \dot{q}_1 \cdot \dot{q}_3 \cdot cq_2 \cdot sq_3 \\ \ddot{q}_3 - \dot{q}_1 \cdot \dot{q}_2 \cdot cq_2 - \ddot{q}_1 \cdot sq_2 \end{bmatrix}, \quad (45)
$$

$$
{}^3\bar{v}_3 = \begin{bmatrix} (l_2 \cdot sq_2 \cdot cq_3 - l_1 \cdot sq_2 \cdot sq_3) \cdot \dot{q}_1 \\ (-l_2 \cdot sq_2 \cdot sq_3 - l_1 \cdot sq_2 \cdot cq_3) \cdot \dot{q}_1 \\ l_2 \cdot \dot{q}_2 - l_1 \cdot \dot{q}_1 \cdot cq_2 \end{bmatrix}, \quad (46)
$$

The results from (40) - (43) are included in the column matrix of operational velocities and accelerations ${}^0\dot{X}_{(6\times1)}$ and ${}^0\ddot{X}_{(6\times1)}$, (14) and (15).

Finally, the column matrix of operational velocities and accelerations relative to the mobile system, ${}^n\dot{X}_{(6\times1)}$ and ${}^n\ddot{X}_{(6\times1)}$ results, For this, (21) and (22) were applied.

$$
{}^3\dot{\bar{v}}_3 = \begin{bmatrix} (cq_3 \cdot sq_1 - cq_1 \cdot sq_2 \cdot sq_3) \cdot (l_1 \cdot \ddot{q}_1 \cdot cq_1 - \\ l_1 \cdot \dot{q}_1^2 \cdot sq_1 + l_2 \cdot \dot{q}_1^2 \cdot cq_1 \cdot sq_2 - \\ l_2 \cdot \dot{q}_2^2 \cdot cq_1 \cdot sq_2 - l_2 \cdot \ddot{q}_1 \cdot cq_1 \cdot cq_2 \\ + l_2 \cdot \ddot{q}_1 \cdot sq_1 \cdot sq_2 + 2 \cdot l_2 \cdot \dot{q}_1 \cdot \dot{q}_2 \cdot sq_1 \cdot cq_2) - \\ (cq_1 \cdot cq_3 + sq_1 \cdot sq_2 \cdot sq_3) \cdot (l_1 \cdot \dot{q}_1^2 \cdot cq_1 + \\ l_1 \cdot \ddot{q}_1 \cdot sq_1 + l_2 \cdot \dot{q}_1^2 \cdot sq_1 \cdot sq_2 + \\ l_2 \cdot \dot{q}_2^2 \cdot sq_1 \cdot sq_2 - l_2 \cdot \ddot{q}_1 \cdot cq_1 \cdot sq_2 - \\ l_2 \cdot \ddot{q}_1 \cdot sq_1 \cdot cq_2 - 2 \cdot l_2 \cdot \dot{q}_1 \cdot \dot{q}_2 \cdot cq_1 \cdot cq_2) - \\ cq_2 \cdot sq_3 \cdot (l_2 \cdot \dot{q}_2^2 \cdot cq_2 + l_2 \cdot \ddot{q}_1 \cdot sq_2) \\ (cq_1 \cdot sq_3 - cq_3 \cdot sq_1 \cdot sq_2) \cdot (l_1 \cdot \dot{q}_1^2 \cdot cq_1 + \\ l_1 \cdot \ddot{q}_1 \cdot sq_1 + l_2 \cdot \dot{q}_1^2 \cdot sq_1 \cdot sq_2 + \\ l_2 \cdot \dot{q}_2^2 \cdot sq_1 \cdot sq_2 - l_2 \cdot \ddot{q}_1 \cdot cq_1 \cdot sq_2 - \\ l_2 \cdot \ddot{q}_1 \cdot sq_1 \cdot cq_2 - 2 \cdot l_2 \cdot \dot{q}_1 \cdot \dot{q}_2 \cdot sq_1 \cdot cq_2) - \\ (sq_1 \cdot cq_3 + cq_1 \cdot sq_2 \cdot cq_3) \cdot (l_1 \cdot \ddot{q}_1 \cdot cq_1 - \\ l_1 \cdot \dot{q}_1^2 \cdot sq_1 + l_2 \cdot \dot{q}_1^2 \cdot cq_1 \cdot sq_2 + \\ l_2 \cdot \dot{q}_2^2 \cdot cq_1 \cdot sq_2 - l_2 \cdot \ddot{q}_1 \cdot cq_1 \cdot cq_2 + \\ l_2 \cdot \ddot{q}_1 \cdot sq_1 \cdot sq_2 + 2 \cdot l_2 \cdot \dot{q}_1 \cdot \dot{q}_2 \cdot sq_1 \cdot cq_2) - \\ cq_2 \cdot cq_3 \cdot (l_2 \cdot \dot{q}_2^2 \cdot cq_2 + l_2 \cdot \ddot{q}_1 \cdot sq_2) \\ sq_2 \cdot (l_2 \cdot \dot{q}_2^2 \cdot cq_2 + l_2 \cdot \ddot{q}_1 \cdot sq_2) - \\ cq_1 \cdot cq_2 \cdot (l_1 \cdot \ddot{q}_1 \cdot cq_1 - l_1 \cdot \dot{q}_1^2 \cdot sq_1 + \\ l_2 \cdot \dot{q}_1^2 \cdot cq_1 \cdot sq_2 + l_2 \cdot \dot{q}_2^2 \cdot cq_1 \cdot sq_2 \\ - l_2 \cdot \ddot{q}_1 \cdot cq_1 \cdot cq_2 + l_2 \cdot \ddot{q}_1 \cdot sq_1 \cdot sq_2 \\ + 2 \cdot l_2 \cdot \dot{q}_1 \cdot \dot{q}_2 \cdot sq_1 \cdot cq_2) - \\ sq_1 \cdot cq_2 \cdot (l_1 \cdot \dot{q}_1^2 \cdot cq_1 + l_1 \cdot \ddot{q}_1 \cdot sq_1 + \\ l_2 \cdot \dot{q}_1^2 \cdot sq_1 \cdot sq_2 + l_2 \cdot \dot{q}_2^2 \cdot sq_1 \cdot sq_2 \\ - l_2 \cdot \dot{q}_1^2 \cdot cq_1 \cdot sq_2 - l_2 \cdot \ddot{q}_1 \cdot sq_1 \cdot cq_2 - \\ 2 \cdot l_2 \cdot \dot{q}_1 \cdot \dot{q}_2 \cdot cq_1 \cdot cq_2) \end{bmatrix} \quad (47)
$$

In the expressions (38) and (39), ${}^0J_i,\ i = 1 \rightarrow 3$ and ${}^0\dot{J}_i,\ i = 1 \rightarrow 3$ define the Jacobian matrix and it time derivative corresponding to every robot joint. According to [1], the mathematical connection between the generalized and operational velocities, is done by the Jacobian matrix or the transfer matrix of linear velocities or the matrix of partial derivatives as it is also known.

3. CONCLUSIONS

This paper's objectives include the determination of DGM equations for a 3R robot structure, represented in Fig 1. For this purpose, the algorithm of the locating matrices was applied. Also, the kinematic equations of the direct kinematic model in the nominal configuration were obtained. The expressions for the Jacobian matrix with projections on the fixed and mobile systems were determined based on the transfer matrices.

The Jacobian matrix makes the correlation between the generalized velocities and operational velocities, both defining the forward kinematics equations.

These results will be further use in future research regarding the establishing of motion equations and the study of motion trajectory.

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MODELAREA MATEMATICĂ A UNUI ROBOT DE TIP 3R PENTRU CONFIGURAȚIA NOMINALĂ

Scopul acestei lucrări este de a realiza un studiu asupra geometriei directe și a cinematicii unui robot RRR (cu trei grade de libertate de rotație), în configurația nominală. În acest scop, a fost aplicat algoritmul de matricelor de situare, pentru a determina ecuațiile geometriei directe. Pentru calcularea vitezelor și accelerațiilor în raport cu sistemul fix, a fost utilizat algoritmul matricelor de transfer. Rezultatele sunt utilizate pentru a stabili ecuațiile traiectoriei de mișcare.

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