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SINGULARITIES STUDY OF THE 3RTS MANIPULATOR

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Abstract: *This article details how to obtain the type I and II singularities of the 3RTS parallel manipulator. These singularities appear when the determinants of the Jacobian matrices are cancelled. The study of singularities is based on the input-output equations of the manipulator.*

Key words: *3-RTS parallel manipulator, degrees of freedom, Jacobi matrix, parallel manipulator singularities*

1. INTRODUCTION

Figure 1 shows the kinematic scheme of the 3RPS parallel space manipulator which has three degrees of mobility and three identical kinematic chains. The symbolic notation of such a mechanism is related to its characteristics: 3 - the number of degrees of freedom; RTS - the type of successive joints in a kinematic chain starting from the base to the final element (R-revolute; T-translation; S-spherical).

The passive revolute joints from the base have the axes placed along the sides of an equilateral triangle of side "b", the radius corresponding to the circle inscribed to the triangle being "R". The centers of the torques are denoted by B_1, B_2 and B_3, and are placed in the middle of the sides of the equilateral triangle of the base.

The active translation joints are placed on the B_iA_i segments that connect the revolute joints at the base with the spherical joints from the mobile platform.

The passive spherical joints on the last level are placed at the tip A_i of an equilateral triangle of side "a" (the mobile platform of the manipulator), the radius of the circle circumscribed to it being denoted by "r". Such an arrangement of the joints on the kinematic chains leads to a spatial parallel mechanism with 3 degrees of freedom (Fig.1).

In order to study the manipulator, a fixed OXYZ coordinate axis system with origin O was considered in the center of the circle inscribed in the triangle of the base platform. The OXY plane is chosen to contain the fixed platform and the OX axis to be perpendicular to the axis of the first passive joint (passing through B_1). The angles formed by OX with OB_i ($i = 1,2,3$) were denoted by δ_i (according to fig.1).

The oxyz coordinate system with the origin "o" in the center of the circle circumscribed to the triangle of the mobile platform was connected to the mobile platform. The oxy plane is chosen to contain the mobile platform and the ox axis to pass through A_1. The angles formed by ox with oA_i ($i = 1,2,3$) were denoted by δ_i'. The oz axis is along the height "h" of the gripper, perpendicular to the platform.

The generalized coordinates of the mechanism (articular coordinates) are: q_i - the linear displacements from the motor joints, measured from B_i to A_i, $i = 1,2,3$.

The generalized coordinates of the mobile platform (operational coordinates) are: Z_P, Ψ, θ meaning the coordinates of the point P of the center of the gripper with respect to the fixed system OXYZ, the precession angles and

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notation of Euler between the 2 platforms (mobile and fixed).

By varying the coordinates q_i, i = 1,2,3, the object manipulated in space can be positioned according to the phases of the manipulation operation.

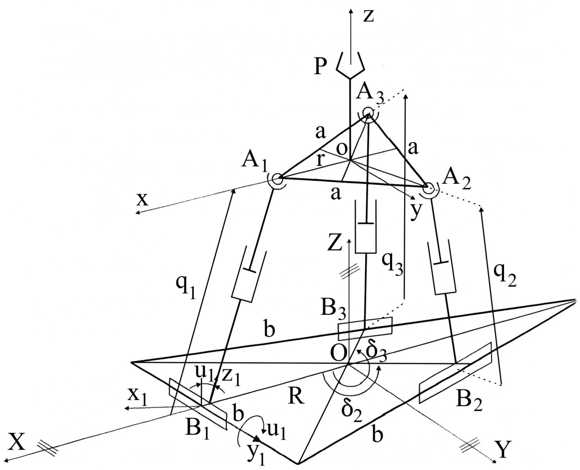

Fig. 1. Kinematic scheme of the 3RTS parallel manipulator

2. COMPUTATION OF JACOBIAN MATRIX EXPRESSIONS FOR THE 3RPS MANIPULATOR

Starting from the input-output equations (1) of the mechanism, defined in [1]:

$$
\begin{aligned}
& q_i^2 - 2[X_{Ai}(\psi,\theta)\cos\delta_i + Y_{Ai}(\psi,\theta) \\
& \sin\delta_i - R]^2 + 2Z_{Ai}^2(Z_p,\psi,\theta) = 0 \equiv \\
& \equiv F_i(q_i, Z_p, \psi, \theta)
\end{aligned} \tag{1}
$$

relations where X_{Ai} , Y_{Ai} şi Z_{Ai} have the expressions defined in eq. (2):

$$
\begin{cases}
X_{Ai} = X_p + \alpha_1 x_{Ai} + \alpha_2 y_{Ai} - \alpha_3 h \\
Y_{Ai} = Y_p + \beta_1 x_{Ai} + \beta_2 y_{Ai} - \beta_3 h \\
Z_{Ai} = Z_p + \gamma_1 x_{Ai} + \gamma_2 y_{Ai} - \gamma_3 h
\end{cases} \tag{2}
$$

In [1] the following relations have been defined:

$$\phi = -\psi \tag{3}$$

$$
\begin{aligned}
& Y_p = r\sin\psi\cos\psi(\cos\theta - 1) - \\
& -h\cos\psi\sin\theta
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
& X_p = \frac{r}{2}(1 - 2\sin^2\psi)(1 - \cos\theta) + \\
& +h\sin\psi\sin\theta
\end{aligned} \tag{5}
$$

The guiding cosines between the mobile oxyz system and the fixed OXYZ system, taking into account the relation (3) are relatively simplified.

In the input-output equations of the mechanism, the variables Ψ, θ, Z_P and q_i are time functions. By deriving eq. (1) we obtain three other equations that can be arranged in matrix form (6):

$$[A]\dot{q} = [B]\dot{q}_p \tag{6}$$

The column matrices of the articular and operational velocities have the expressions (7) and (8).

$$\dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} \quad (7) \qquad \dot{q}_p = \begin{bmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{Z}_p \end{bmatrix} \quad (8)$$

$$[A] = \begin{bmatrix} \dfrac{\partial F_1}{\partial q_1} & 0 & 0 \\ 0 & \dfrac{\partial F_2}{\partial q_2} & 0 \\ 0 & 0 & \dfrac{\partial F_3}{\partial q_3} \end{bmatrix} \tag{9}$$

$$[B] = \begin{bmatrix} \dfrac{\partial F_1}{\partial \psi} & \dfrac{\partial F_1}{\partial \theta} & \dfrac{\partial F_1}{\partial Z_p} \\ \dfrac{\partial F_2}{\partial \psi} & \dfrac{\partial F_2}{\partial \theta} & \dfrac{\partial F_2}{\partial Z_p} \\ \dfrac{\partial F_3}{\partial \psi} & \dfrac{\partial F_3}{\partial \theta} & \dfrac{\partial F_3}{\partial Z_p} \end{bmatrix} \tag{10}$$

Partial derivatives that appear as elements of the matrix [A] are obtained immediately:

$$\frac{\partial F_i}{\partial q_i} = 2q_{i,}\ i = 1,2,3 \tag{11}$$

For the calculation of the elements of the matrix [B] it is necessary to calculate certain partial derivatives whose expressions have been established in [2] and given below:

$$\frac{\partial\alpha_1}{\partial\psi}=2s\psi c\psi(c\theta-1) \tag{12}$$

$$\frac{\partial\alpha_2}{\partial\psi}=(c^2\psi-s^2\psi)(1-c\theta) \tag{13}$$

$$\frac{\partial\alpha_3}{\partial\psi}=s\theta c\psi \tag{14}$$

$$\frac{\partial\beta_1}{\partial\psi}=(1-c\theta)(c^2\psi-s^2\psi) \tag{15}$$

$$\frac{\partial\beta_2}{\partial\psi}=2s\psi c\psi(1-c\theta) \tag{16}$$

$$\frac{\partial\beta_3}{\partial\psi}=s\psi s\theta \tag{17}$$

$$\frac{\partial\gamma_1}{\partial\psi}=-s\theta c\psi \tag{18}$$

$$\frac{\partial\gamma_2}{\partial\psi}=-s\theta c\psi \tag{19}$$

$$\frac{\partial\gamma_3}{\partial\psi}=0 \tag{20}$$

$$\frac{\partial\alpha_1}{\partial\theta}=-s^2\psi s\theta \tag{21}$$

$$\frac{\partial\alpha_2}{\partial\theta}=-s\psi c\psi s\theta \tag{22}$$

$$\frac{\partial\alpha_3}{\partial\theta}=s\psi c\theta \tag{23}$$

$$\frac{\partial\beta_1}{\partial\theta}=c\psi s\psi s\theta \tag{24}$$

$$\frac{\partial\beta_2}{\partial\theta}=-c^2\psi s\theta \tag{25}$$

$$\frac{\partial\beta_3}{\partial\theta}=-c\psi c\theta \tag{26}$$

$$\frac{\partial\gamma_1}{\partial\theta}=-s\psi c\theta \tag{27}$$

$$\frac{\partial\gamma_2}{\partial\theta}=-c\psi c\theta \tag{28}$$

$$\frac{\partial\gamma_3}{\partial\theta}=-s\theta \tag{29}$$

$$\begin{aligned}&\frac{\partial X_P}{\delta\psi}=2rs\psi c\psi(c\theta-1)+hc\psi s\theta=\\&=\frac{r}{2}(1-2s^2\psi)+hs\psi c\theta\end{aligned} \tag{30}$$

$$\begin{aligned}&\frac{\partial Y_P}{\delta\psi}=r(c\theta-1)(c^2\psi-s^2\psi)+hs\psi s\theta=\\&=-rs\psi c\psi s\theta-hc\psi c\theta\end{aligned} \tag{31}$$

$$\begin{cases}\dfrac{\partial X_{Ai}}{\partial\psi}=\dfrac{\partial X_P}{\partial\psi}+rc\delta_i\dfrac{\partial\alpha_1}{\partial\psi}+rs\delta_i\dfrac{\partial\alpha_2}{\partial\psi}-h\dfrac{\partial\alpha_3}{\partial\psi}\\ \dfrac{\partial Y_{Ai}}{\partial\psi}=\dfrac{\partial Y_P}{\partial\psi}+rc\delta_i\dfrac{\partial\beta_1}{\partial\psi}+rs\delta_i\dfrac{\partial\beta_2}{\partial\psi}-h\dfrac{\partial\beta_3}{\partial\psi}\\ \dfrac{\partial Z_{Ai}}{\partial\psi}=\dfrac{\partial Z_P}{\partial\psi}+rc\delta_i\dfrac{\partial\gamma_1}{\partial\psi}+rs\delta_i\dfrac{\partial\gamma_2}{\partial\psi}-h\dfrac{\partial\gamma_3}{\partial\psi}\end{cases} \tag{32}$$

$$\begin{cases}\dfrac{\partial X_{Ai}}{\partial\theta}=\dfrac{\partial X_P}{\partial\theta}+rc\delta_i\dfrac{\partial\alpha_1}{\partial\theta}+rs\delta_i\dfrac{\partial\alpha_2}{\partial\theta}-h\dfrac{\partial\alpha_3}{\partial\theta}\\ \dfrac{\partial Y_{Ai}}{\partial\theta}=\dfrac{\partial Y_P}{\partial\theta}+rc\delta_i\dfrac{\partial\beta_1}{\partial\theta}+rs\delta_i\dfrac{\partial\beta_2}{\partial\theta}-h\dfrac{\partial\beta_3}{\partial\theta}\\ \dfrac{\partial Z_{Ai}}{\partial\theta}=\dfrac{\partial Z_P}{\partial\theta}+rc\delta_i\dfrac{\partial\gamma_1}{\partial\theta}+rs\delta_i\dfrac{\partial\gamma_2}{\partial\theta}-h\dfrac{\partial\gamma_3}{\partial\theta}\end{cases} \tag{33}$$

Thus, the first, second and third columns of the matrix [B] have the expressions (34), (35) and (36):

$$\begin{aligned}&\frac{\partial F_i}{\partial\psi}=-2(X_{Ai}\cos\delta_i+Y_{Ai}\sin\delta_i-R)\cdot\\&\cdot(\frac{\partial x_{Ai}}{\partial\psi}\cos\delta_i+\frac{\partial y_{Ai}}{\partial\psi}\sin\delta_i-2Z_{Ai}\frac{\partial z_{Ai}}{\partial\psi}\end{aligned} \tag{34}$$

$$\begin{aligned}&\frac{\partial F_i}{\partial\theta}=-2(X_{Ai}\cos\delta_i+Y_{Ai}\sin\delta_i-R)\cdot\\&\cdot(\frac{\partial x_{Ai}}{\partial\theta}\cos\delta_i+\frac{\partial y_{Ai}}{\partial\theta}\sin\delta_i-2Z_{Ai}\frac{\partial z_{Ai}}{\partial\theta}\end{aligned} \tag{35}$$

$$\frac{\partial F_i}{\partial Z_p}=2Z_{Ai} \tag{36}$$

Having the expressions of the elements of the matrices [A] and [B], so implicitly of the determinants, the singularities of the mechanism can be analyzed.

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3. TYPE I SINGULARITIES ANALISYS

Type I singularities occur when:

$$\text{Det (A)} = 0 \quad (37)$$

The configurations corresponding to this type of singularity are configurations for which the manipulator is at a limit of the workspace or at an internal limit of his workspace, which he divides into two areas where the number of solutions of the inverse geometric problem is different [2]. In other words, this type of singularity occurs when two branches of the inverse geometric problem meet.

According to equations (6) and (37), because the determinant of the matrix A is zero, it is possible to find zero articular velocity vectors that produce zero (Cartesian) operational velocity vectors. Thus, certain Cartesian velocities cannot be produced at the terminal organ of the manipulator. The end-effector (mobile platform) loses one or more degrees of freedom. According to the principle of kinematic-static duality, in the presence of these singular configurations, the terminal organ can oppose an arbitrary force or torque in a given direction.

The calculations presented below were performed for the following geometric-constructive configuration of the mechanism 3RTS: a = 300 mm, b = 500 mm, h = 50 mm,
$\delta_1 = \delta_1' = 0^0$; $\delta_2 = \delta_2' = 120^0$; $\delta_3 = \delta_3' = 240^0$

With a special program designed in the Fox language, it was verified that from the points of the workspace the value of the determinant of the matrix [A] is zero (or has a very small value). Going through the workspace with the smallest step on all three axes, it is observed that the values of the determinant of the matrix [A] are not in the range (-10^{-2};10^{-2}), therefore there is no question of the appearance of this type of singularity. The program designed in Fox shows that the minimum value of the determinant of the matrix [A] is 221,180 at a certain point of the workspace.

Val.min.det A = 221.180

4. TYPE II SINGULARITIES ANALISYS

Type II singularities occur when the matrix [B] becomes singular:

$$\text{Det (B)} = 0 \quad (38)$$

Contrary to type I singularities, type II singularities occur for configurations located inside the workspace and correspond to the set of configurations where two branches of the forward geometric problem meet [2].

According to equations (6) and (38), non-zero operating speed vectors can be found which correspond to zero actuator speeds. The manipulator is therefore in a position where the terminal organ can undergo an infinitesimal movement even if the actuators are blocked. In this case there is a gain of one or more degrees of freedom and the system loses its rigidity. Therefore, it becomes unable to withstand a force or torque in a given direction on his terminal organ.

A program has also been designed in the Fox language that, traversing the workspace with the smallest step on all three axes, allows the display of the values of the determinant of the matrix B, values that are very high as illustrated in Tab.1. The points where det [B] cancels out determine even the OZ axis of the workspace and another axis parallel to OZ that pierces the XOY plane at the point Xp = -7 mm, Yp = 12 mm, as shown in Tab.1. The minimum value of the determinant of the matrix B is 10,000 at a certain point of the workspace, in the rest of the points of the workspace det [B] having very high values.

5. CONTRIBUTIONS AND CONCLUSIONS

The contributions regarding the study of the singularities of the 3RTS parallel mechanism are:

- Mathematical models and computational algorithms corresponding to computer-assisted solutions of the mechanism singularities problem have been proposed.

• A program for graphical representation of singularity points (type II) within the 3D workspace has been designed, thus obtaining a clear image of the areas of the workspace to be avoided.

The conclusions that emerge from the running of the above-mentioned programs regarding the singularities of the 3RTS mechanism are:

• The study of mechanisms with any 3 DOF is more difficult sometimes because the independent parameters (Zp, Ψ and θ in this case) must be chosen with great care in order to be able to establish analytical relations for the other 3 dependent parameters (Xp, Yp and φ in this case)

• The existence of translational motor joints facilitates the study of the parallel mechanism;

• The expressions of the elements of the Jacobian matrix [B] are more complicated than in the case of the 3RKK mechanism [3] but simpler than in the 3RRS mechanism [4] respectively as in the case of the TRR serial robot [5].

• Type I singularities do not appear, det [A] taking very high values throughout the workspace, instead, type II singularities appear along the OZ axis and on another line parallel to OZ that intersects the XOY plane at the point (X = -7 mm, Y = 12 mm) as shown in tab.1, which is a major disadvantage.

Tab.1

Input data			**Output data**			**Computed data**		
Ψ	**θ**	**Zp**	**q_1**	**q_2**	**q_3**	**Xp**	**Yp**	**Computed value [B]**
(deg)	**(deg)**	**(mm)**	**(mm)**	**(mm)**	**(mm)**	**(mm)**	**(mm)**	**(mm)**
30	-30	160	162.58	30.68	162.58	-7	12	0
30	-30	165	167.51	35.60	167.51	-7	12	0
30	-30	170	172.43	40.53	172.43	-7	12	0
30	-30	175	177.36	45.49	177.36	-7	12	0
30	-30	180	182.30	50.45	182.30	-7	12	0
30	-30	185	187.24	55.42	187.24	-7	12	0
30	-30	190	192.18	60.39	192.18	-7	12	0
30	-30	195	197.13	65.37	197.13	-7	12	0
30	-30	200	202.07	70.35	202.07	-7	12	0
30	-30	205	207.02	75.33	207.02	-7	12	0
30	-30	210	211.97	80.32	211.97	-7	12	0
30	-30	215	216.93	85.30	216.93	-7	12	0
30	-30	220	221.89	90.29	221.89	-7	12	0
30	-30	225	226.84	95.28	226.84	-7	12	0
30	-30	230	231.80	100.27	231.80	-7	12	0
30	-30	235	236.77	105.26	236.77	-7	12	0
30	-30	240	241.73	110.26	241.73	-7	12	0
30	-30	245	246.69	115.25	246.69	-7	12	0
30	-30	250	251.66	120.24	251.66	-7	12	0
30	-30	255	256.63	125.24	256.63	-7	12	0
30	-30	260	261.60	130.23	261.60	-7	12	0
30	-30	265	266.57	135.23	266.57	-7	12	0
30	-30	270	271.54	140.22	271.54	-7	12	0
30	-30	275	276.51	145.22	276.51	-7	12	0
30	-30	280	281.48	150.21	281.48	-7	12	0
30	-30	285	286.46	155.21	286.46	-7	12	0
30	-30	290	291.43	160.21	291.43	-7	12	0
30	-30	295	296.41	165.20	296.41	-7	12	0

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Studiul singularităților manipulatorului 3RTS

Rezumat: *În acest articol se prezintă detaliat modul de obținere a singularităţilor de tipul I şi II ale manipulatorului paralel 3RTS. Aceste singularităţi apar când determinanţii matricelor Jacobiene se anulează. Studiul singularităților are la bază ecuațiile de intrare-ieșire ale manipulatorului.* ***Key words:*** *manipulator paralel 3-RTS,grade de libertate, matrice Jacobiană, singularităţi paralelele*

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