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DYNAMIC CONTROL FUNCTIONS FOR A TRANSLATIONAL AXIS OF A SERIAL ROBOT STRUCTURE

Claudiu SCHONSTEIN, Adriana Corina DOBOCAN

Abstract: *The study of dynamic behavior for a robot structure, aims to determine the laws of motion, which define the control of the movements of the components of a multibody mechanical system. The process of mathematical modeling is based on simplifying hypotheses, in order to obtain workable and easy-to-use dynamic equations. Dynamic modeling of a structure requires on the other hand the knowledge of parameters such as mass, center of mass and moments of inertia for each of the component bodies of the mechanical system under study. In advanced mechanics, some of the fundamental theorems play an essential role in determining the matrix equations of dynamics known as generalized motor functions. Theorems are based on fundamental notions specific to the dynamics of mechanical systems, including kinetic energy.*

Key words: *robot, Dynamic Control functions, serial structure, generalized forces.*

1. INTRODUCTION

The dynamic study of mechanical systems involves the dynamic analysis of mechanical structures and has as central objective the determination of the motion expressions responsible for the motion of each component of the system. Hence, a dynamic model consists of equations of motion for each component of the mechanical system [1]. The establishing of the equations of motion, or the dynamic control functions, is based on several simplifying hypotheses.

A simplifying hypothesis is the rigidity of the elements in the component of the handling structure; another is the negligible size of the dry friction forces, which imposes a series of constructive and proper assembly conditions. A third simplifying hypothesis is the negligible clearances, which are included in the precision of the mechanical assembly and imposes constructive restrictions in the synthesis process.

In the paper, there will be considered a translation axis, belonging to a mechanical robot structure, as presented in Figure 1 for which, based on specialty literature, there will be determined the dynamic control functions [2].

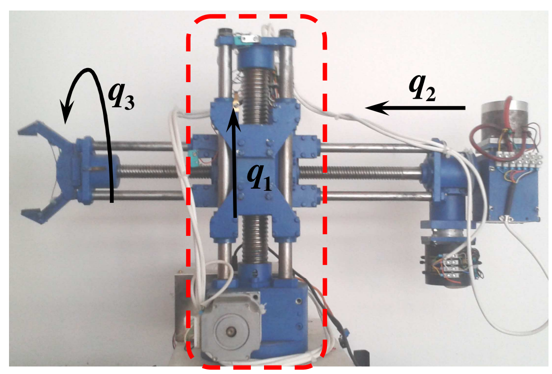

Fig. 1 Mechanical Robot Structure

First there will be presented the notions that are conducting to motion expressions, as mass properties, kinetic energy, and generalized forces.

2. NOTIONS REGARDING RIGID BODY MASS PROPERTIES

There is considered a mechanical robot structure, considered as a rigid body, having n d.o.f., consisting in n kinetic elements,

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connected by (n) driving links of i=R (rotation) or i=T (translation) type.

In concordance to [1], [2], [4], the mass distribution parameters are represented by: the mass of the rigid, the position of mass center, respectively the axial tensor corresponding to each $i = 1 \rightarrow n$ kinetic assembly (element).

To highlight these parameters, according to Figure 2, there is considered a sequence of element belonging to a robot, having $i = 1 \rightarrow n$ kinetic elements, each element having an irregular geometric shape, for which the integral mass calculation, specific to mechanical moments of inertia, cannot be applied.

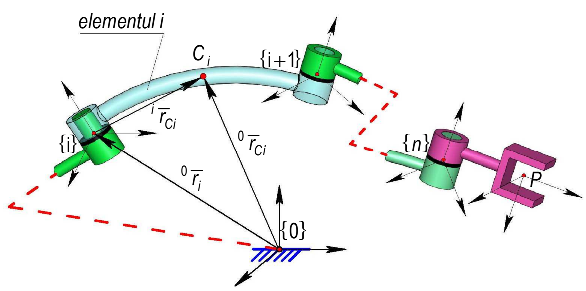

Fig. 2 Subassembly of o robot structure

To each kinematic element (i) is restrictively attached a reference system $\{i\}$ whose origin is at the center of each driving joint. Considering the element (i) for which the position vector $^{0}\overline{r}_{i}$ of the system $\{i\}$ with respect to the fixed reference system is known, the position of the of mass center C_i, of the element (i) with respect to the fixed reference frame $\{0\}$, is defined according to the following expression:

$$^{0}\overline{r}_{C_i} = {^{0}\overline{r}_{i}} + {^{0}_{i}[R]} \cdot {^{i}\overline{r}_{C_i}} \tag{1}$$

where $^{0}_{i}[R]$ represents the rotation matrix between reference systems $\{i\}$ and $\{0\}$.

According to [1], [5], the axial-centrifugal inertial tensor, for each element (i) is established as:

$$^{i}I_i = \begin{pmatrix} ^{i}I_x & ^{i}I_{xy} & ^{i}I_{xz} \\ ^{i}I_{yx} & ^{i}I_y & ^{i}I_{yz} \\ ^{i}I_{zx} & ^{i}I_{zy} & ^{i}I_z \end{pmatrix}; \tag{2}$$

Due to the fact that in the study of dynamics the expression axial-centrifugal inertial tensor of the kinetic element (i) applied in C_i is needed, they are determined with the expressions:

$$^{i}I_i^{*} \equiv {^{i}I_i} - {^{i}I_{C_i}} \tag{3}$$

where:

$$^{i}I_{C_i} = m_i \cdot \begin{bmatrix} ^{i}y_{C_i}^2 + {^{i}z_{C_i}^2} & -{^{i}x_{C_i}} \cdot {^{i}y_{C_i}} & -{^{i}x_{C_i}} \cdot {^{i}z_{C_i}} \\ -{^{i}y_{C_i}} \cdot {^{i}x_{C_i}} & ^{i}z_{C_i}^2 + {^{i}x_{C_i}^2} & -{^{i}y_{C_i}} \cdot {^{i}z_{C_i}} \\ -{^{i}z_{C_i}} \cdot {^{i}x_{C_i}} & -{^{i}z_{C_i}} \cdot {^{i}y_{C_i}} & ^{i}x_{C_i}^2 + {^{i}y_{C_i}^2} \end{bmatrix} \tag{4}$$

Due to the complex geometry and the large number of components of the structures, the determinations of mass distribution parameters by analytical modeling lead to certain approximations. With the development of computerized computing systems, as well as specialized software packages, the specific parameters of mass distribution can be determined more rigorously, which leads to a considerable reduction in possible computational errors and erroneous determination of geometric characteristics of mechanical systems components.

3. FUNDAMENTAL THEOREMS IN RIGID BODY DYNAMICS

In study of rigid body behavior, some of the fundamental theorems are essential in determining the equations of dynamics known as generalized motor functions or dynamic control functions. The theorems are based on the fundamental notions specific to the dynamics of mechanical systems, including kinetic energy or energy of accelerations [1]. According to [1], the defining expression for kinetic energy is:

$$E_C^i = (-1)^{\Delta_M} \cdot \frac{1-\Delta_M}{1+3\cdot\Delta_M} \cdot \left\{\frac{1}{2} \cdot M_i \cdot {^{i}v_{C_i}^T} \cdot {^{i}v_{C_i}}\right\} + \Delta_M^2 \cdot \frac{1}{2} \cdot {^{i}\omega_i^T} \cdot {^{i}I_i^{*}} \cdot {^{i}\omega_i}; \tag{5}$$

where, the parameters ${}^{i}\bar{v}_i = {}^{i}\bar{v}_{C_i} + {}^{i}\bar{\omega}_i \times {}^{i}\bar{r}_{Ci}$, ${}^{i}\bar{v}_{C_i}$ and ${}^{i}\bar{\omega}_i$ represents the linear and angular velocity of mass center, ie the kinematic parameters specific to the general motion of the same kinetic assembly (i), considered a rigid body. In the same expression there is considered:

$$\Delta_M = \begin{cases} -1 & \text{for general motion} \\ 0 & \text{for translation motion} \\ 1 & \text{for rotation motion} \end{cases}; \qquad (6)$$

4. GENERALIZED FORCES IN DYNAMICS OF THE RIGID BODY

According to [1], the dynamic model of a mechanical robot structure can be expressed mathematically by equations of the form:

$$\bar{Q}_m(t) = f^{-1}(\bar{\theta}(t)) \qquad (7)$$

where $\bar{Q}_m(t)$ is a column vector of the generalized dynamic forces of the kinematic joints of a robot, having the form $\bar{Q}_m(t) = \left[\bar{Q}_m^i(t),\ i = 1 \to n\right]^T$, and $\bar{\theta}(t)$ is another column vector of the generalized variables belonging to driving joints of the mechanical system. In literature, the expression (7) is known as the inverse dynamic model, according to which the time functions responsible for the motion of each driving joint are considered known, being expressed according to the following expressions:

$$\left\{\begin{array}{c} \bar{\theta} = f^{-1}({}^{0}\bar{X}); \\ \dot{\bar{\theta}} = {}^{0}J(\bar{\theta})^{-1} \cdot ({}^{0}\dot{\bar{X}}); \\ \ddot{\bar{\theta}} = {}^{0}J(\bar{\theta})^{-1} \cdot ({}^{0}\ddot{\bar{X}}) - {}^{0}J(\bar{\theta})^{-1} \cdot {}^{0}\dot{J}(\dot{\bar{\theta}}) \end{array}\right\} \qquad (8)$$

where the unknowns are the generalized forces in the driving joints of the structure, whether rotational or translational, which are mechanically considered scleronomes and olonomes, so that the equations of dynamics will lead to the determination of time functions of driving moments.

To approach the inverse dynamic model, according to [1], [2] the algorithm of generalized forces in robot dynamics is used. Due to the fact that the Lagrange-Euler formalism is specific to non-conservative mechanical systems and with holonomic links, based on the considerations in [1] [7], the equations of motion of the mechanical structure of the robot under study will be determined for $i = 1 \to n$ taking into account the following differential expression [6],[7]:

$$\frac{d}{dt}\left[\frac{\partial E_C(\bar{\theta};\dot{\bar{\theta}})}{\partial \dot{q}_i}\right] - \frac{\partial E_C(\bar{\theta};\dot{\bar{\theta}})}{\partial q_i} + $$
$$+ Q_g^i + Q_{SU}^i = Q_m^i(\bar{\theta};\dot{\bar{\theta}};\ddot{\bar{\theta}}); \qquad (9)$$

In previous expressions, there are included $\{Q_{i\mathcal{F}}^i; Q_g^i; Q_{SU}^i\}$, representing the generalized inertia, gravitational and manipulation forces.

According to [1], the generalized inertia forces can be obtained as:

$$Q_{iF}^i \equiv \frac{d}{dt}\left[\frac{\partial E_C(\bar{\theta};\dot{\bar{\theta}})}{\partial \dot{q}_i}\right] - \frac{\partial E_C(\bar{\theta};\dot{\bar{\theta}})}{\partial q_i} \qquad (10)$$

Remarks: The generalized inertia forces, can be established with the following expressions:

$$Q_{iF}(\bar{\theta}) = \left[Q_{iF}^i = {}^{0}J_i^T \cdot {}^{0}\bar{F}_{X_i}^*, i = 1 \to n\right]^T \qquad (11)$$

where ${}^{0}J(\bar{\theta})^T$ is the transpose of Jacobian matrix [1], and ${}^{0}\bar{\mathcal{F}}_{X_i}^*$ is the force-moment resultant vector of inertia forces, established as:

$${}^{(n)0}\bar{\mathcal{F}}_{X_i}^* = \left\{\begin{array}{c} {}^{(n)0}\bar{F}_{X_i}^* \\ \cdots\cdots \\ {}^{(n)0}\bar{N}_{X_i}^* \end{array}\right\} =$$
$$= \left\{\begin{array}{c} \sum_{j=i}^{n} {}^{(n)0}_{j}[R] \cdot {}^{j}\bar{F}_j \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \sum_{j=i}^{n} \left[\begin{array}{l} {}^{(0)n}_{n}[R]^T \left({}^{0}\bar{r}_{C_j} - \bar{p}_n\right) \times {}^{(n)0}_{j}[R] \cdot {}^{j}\bar{F}_j + \\ + {}^{(n)0}_{j}[R] \cdot {}^{j}\bar{N}_j \end{array}\right] \end{array}\right\} \qquad (12)$$

In the previous expressions there are introduced:

$${}^{j}\bar{F}_j = M_j \cdot \left\{ {}^{j}\dot{\bar{v}}_j + {}^{j}\dot{\bar{\omega}}_j \times {}^{j}\bar{r}_{C_j} + {}^{j}\bar{\omega}_j \times {}^{j}\bar{\omega}_j \times {}^{j}\bar{r}_{C_j} \right\} \qquad (13)$$

the mass center motion theorem, and:

$$ {}^{j}\overline{N}_{j} = \left\{ {}^{i}I_{j}^{*} \cdot {}^{j}\dot{\overline{\omega}}_{j} + {}^{j}\overline{\omega}_{j} \times {}^{j}I_{j}^{*} \cdot {}^{j}\overline{\omega}_{j} \right\} \quad (14) $$

angular momentum theorem relative to the center of the masses.

The generalized gravitational forces [], are established as:

$$ Q_{g}^{i}\left(\overline{\theta}\right) = {}^{0}J\left(\overline{\theta}\right)^{T} \cdot {}^{0}\mathscr{F}_{X_i}\left(\overline{\theta}\right) \quad (15) $$

where ${}^{0}J\left(\overline{\theta}\right)^{T}$ is the transpose of Jacobian matrix [1], and ${}^{0}\mathscr{F}_{X_i}$ is the force-moment resultant vector of gravitational loads.

The force-moment resultant vector of gravitational loads expressed with respect to the frame attached to the geometric center of the last driving joint is:

$$ \underset{(i=1\rightarrow n)}{{}^{0}\mathscr{F}_{X_i}} = \begin{Bmatrix} {}^{0}\overline{F}_{X_i} \\ \cdots\cdots\cdots \\ {}^{0}\overline{N}_{X_i} \end{Bmatrix} = $$

$$ = \begin{Bmatrix} \sum_{j=i}^{n} M_j \cdot {}^{(0)n}_{\ \ n}[R]^{T} \cdot \overline{g} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \sum_{j=i}^{n} M_j \cdot {}^{(0)n}_{\ \ n}[R]^{T} \cdot \left[\left({}^{0}\overline{r}_{C_j} - \overline{p}_n\right) \times \overline{g}\right] \end{Bmatrix} \quad (16) $$

Remarks: In order to establish the generalized gravitational forces there is considered [1]:

$$ \begin{gathered} \overline{g} = \tau \cdot g \cdot \overline{k}_0; \\ \overline{k}_0 = \{\overline{x}_0; \overline{y}_0; \overline{z}_0\}; \\ \overline{k}_g = \frac{{}^{0}\overline{g}}{\left|{}^{0}\overline{g}\right|}; \\ \tau = -\overline{k}_g^{T} \cdot \overline{k}_0 = \begin{Bmatrix} 1; & \overline{k}_g^{T} \cdot \overline{k}_0 = -1 \\ -1; & \overline{k}_g^{T} \cdot \overline{k}_0 = 1 \end{Bmatrix} \end{gathered} \quad (17) $$

Considering that the structure is mounted on a floor, results that:

$$ {}^{0}\overline{g} = \left(g = \tau \cdot g \cdot k_0^{T}\right) \equiv \begin{pmatrix} 0 & 0 & g \end{pmatrix}^{T}; \quad (18) $$

The handling load m_{SU} applied to the end-effectors of the robot induces in each motor axis a generalized force with implications in the study of the dynamic behavior of the mechanical structure (see Figure 3).

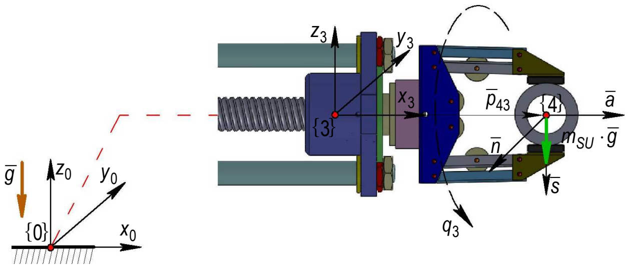

Fig. 3 Handling load

Based on [1], the expression generalized handling force is expressed as follows:

$$ Q_{SU}^{i} = {}^{0}J_{i}^{T} \cdot {}^{0}\overline{F}_{X}, \quad i = 1 \rightarrow n \quad (19) $$

In the above expression, the resulting force-momentum vector of the handling load is defined by:

$$ {}^{0}\overline{F}_{X} = \begin{Bmatrix} {}^{0}\overline{F}_{X} \\ \cdots\cdots \\ {}^{0}\overline{N}_{X} \end{Bmatrix} = $$

$$ = \begin{Bmatrix} {}^{\ \ i}_{n+1}[R] \cdot {}^{n+1}\overline{f}_{n+1} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \left(\overline{p}_{n+1} - \overline{p}_n\right) \times {}^{\ \ i}_{n+1}[R] \cdot {}^{n+1}\overline{f}_{n+1} + {}^{\ \ i}_{n+1}[R] \cdot {}^{n+1}\overline{n}_{n+1} \end{Bmatrix} \quad (20) $$

where:

$$ {}^{i}\overline{f}_{i} = M_i \cdot {}^{0}_{i}[R]^{T} \cdot \overline{g} + {}^{\ \ i}_{i+1}[R] \cdot {}^{i+1}\overline{f}_{i+1}; \quad (21) $$

$$ \begin{aligned} {}^{i}\overline{n}_{i} &= {}^{i}\overline{r}_{C_i} \times {}^{0}_{i}[R]^{T} \cdot \overline{g} \cdot M_i + \\ &+ {}^{i}\overline{r}_{i+1} \times {}^{\ \ i}_{i+1}[R] \cdot {}^{i+1}\overline{f}_{i+1} + {}^{\ \ i}_{i+1}[R] \cdot {}^{i+1}\overline{n}_{i+1} \end{aligned} \quad (22) $$

Of all these forces, the generalized force of inertia $\left(Q_{\mathscr{F}}^{i}\right)$ has the greatest impact, being able to influence the movement of the structure, in certain stages of the work process in which the robot is implemented.

5. ANALITICAL RESULTS FOR A SERIAL TRANSLATIONAL AXIS

On the basis of previous considerations further there will be presented for the considered translational robot axis presented in

Figure 4, the driving moment on the motor shaft.

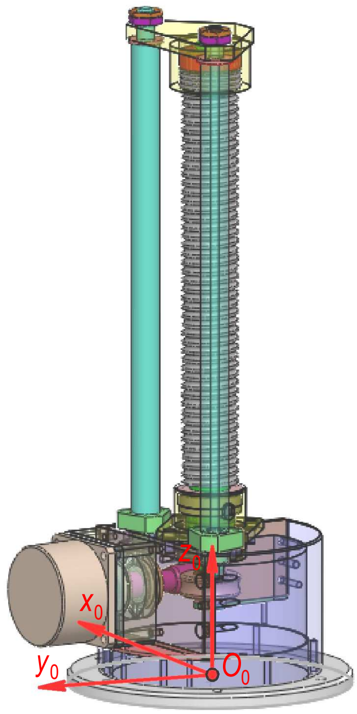

Fig. 4 Translation axis of the serial structure

According to (10) the generalized inertia force for the axis is:

$$Q_{iF}^{1}(\bar{\theta}) = (1{,}9724 \cdot 10^{-3} \cdot cos\, q_3 - 5{,}2096 \cdot 10^{-3} \cdot sin\, q_3) \cdot \dot{q}_3^2 +$$
$$7{,}4636 \cdot \ddot{q}_1 + (5{,}2096 \cdot 10^{-3} \cdot cos\, q_3 + 1{,}9724 \cdot 10^{-3} \cdot sin\, q_3) \cdot \ddot{q}_3 \qquad (23)$$

As important remark, the generalized inertia force for the above axis, in [2], was determined with (10) and (11), the result being the same (23).

The generalized gravitational force, according to (15), is established as:

$$Q_g^1(\bar{\theta}) = 73{,}191; \qquad (24)$$

The handling force applied to the end with profound implications in the study of the dynamic behavior of the analized axis, is:

$$Q_{SU}^{i} = 9{,}8066 \cdot \cos(q_3) \cdot m_{SU} \qquad (25)$$

The previously determined generalized forces (gravitational, handling, inertia) and the corresponding moments in the driving axes, respectively, are components of the generalized driving forces, having the opposite direction to the direction of motion. Determining as accurately as possible these generalized forces and moments has an essential role in the correct design of a kinematic axis. Therefore, substituting the expressions for the generalized gravitational, manipulative, and inertial forces previously obtained in expression (9), the driving moments necessary for the motion of the analised kinematic axeis of the serial structure, has the following final form, according to the expression below [2]:

$$Q_m^1 = 9{,}8066 \cdot sin\, q_3 \cdot m_{SU} +$$
$$+(1{,}9724 \cdot 10^{-3} \cdot cos\, q_3 - 5{,}2096 \cdot 10^{-3} \cdot sin\, q_3) \cdot \dot{q}_3^2 +$$
$$+(1{,}9724 \cdot 10^{-3} \cdot sin\, q_3 + 5{,}2096 \cdot 10^{-3} \cdot cos\, q_3) \cdot \ddot{q}_3 +$$
$$+7{,}4636 \cdot \ddot{q}_1 + 73.1906 +$$
$$+7{,}8 \cdot 10^{-6} \cdot \{[(5{,}2096 \cdot 10^{-3} \cdot cos\, q_3 + 1{,}9724 \cdot 10^{-3} \cdot sin\, q_3) \cdot \dot{q}_3^2 +$$
$$+(5{,}2096 \cdot 10^{-3} \cdot sin\, q_3 - 1{,}9724 \cdot 10^{-3} \cdot cos\, q_3) \cdot \ddot{q}_3 + $$
$$+9{,}8066 \cdot sin\, q_3 \cdot m_{SU}]^2 +$$
$$13{,}1667 \cdot \dot{q}_2^2\} \qquad (26)$$

6. CONCLUSIONS

The dynamic study of mechanical systems, known as rigid ones, involves the determination of equations of motion. The equations determined in dynamic modeling, serve depending on the type of unknowns, to determine the generalized driving forces and operational variables, as well as to calculate the generalized variables that express the motion of the mechanical system in the space of configurations.

For the implementation of dynamic control algorithms, simplifying hypotheses are required, such as the rigidity of the components of the mechanical structure, the neglect of dry and viscous friction forces, the neglect of play and the elasticity of the transmission elements, which can induce geometric errors.

The methods for determining the dynamic model are based on the fundamental notions of the dynamics of mechanical systems, such as: momentum, kinetic moment, mechanical work, kinetic energy and energy of accelerations. knowledge, or when creating a control algorithm that requires a fast calculation algorithm.

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Funcții de control dinamic pentru o axa de translație a unei structuri seriale de robot

Studiul comportamentului dinamic pentru o structura de robot, are ca scop determinarea legilor de mişcare, ce definesc comanda mişcărilor elementelor componente ale unui sistem mecanic multicorp. Procesul de modelare matematica, se face pe baza unor ipoteze simplificatoare, în vederea obţinerii unor ecuaţii dinamice prelucrabile şi uşor de folosit. Modelarea dinamică a unei structuri impune pe de altă parte cunoaşterea unor parametri precum masa, centrul de masă şi momentele de inerție pentru fiecare dintre corpurile componente ale sistemului mecanic luat în studiu. În mecanica avansată, o parte dintre teoremele fundamentale au un rol esențial în determinarea ecuațiilor matriceale ale dinamicii cunoscute sub denumirea de funcții generalizate motoare. Teoremele au la baza noțiunile fundamentale specifice dinamicii sistemelor mecanice, printre care *energia cinetică.*

SCHONSTEIN Claudiu, Dr. Eng, Lecturer, Technical University of Cluj-Napoca, Department of Mechanical Systems, schonstein_claudiu@yahoo.com, 103-105 Muncii blvd, office: C06, 400641, Cluj-Napoca, Romania.

DOBOCAN Corina Adriana, Dr. Eng, Lecturer, Technical University of Cluj-Napoca, Department of Design Engineering and Robotics, corina.dobocan@muri.utcluj.ro, 0264-401664, 103-105 Muncii blvd, office: C06, 400641, Cluj-Napoca, Romania