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REDESIGN OF AN EQUIPMENT REGARDING CORIOLIS EFFECT

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Abstract: *The paper wants to present the redesign of laboratory equipment that demonstrates the appearance of the Coriolis force. Redesigning, to smaller dimensions, the goal being its low weight, implicitly its equipment mobility. In the analysis, we will start from the choice of the length of the elastic rod that will deform under the effect of the Coriolis force, a length that must be able to be included in the diameters of the frame imposed by the designer.*

Keywords: *redesign, laboratory equipment, elastic rod, Coriolis Effect.*

1. INTRODUCTION

It is indisputable that in nature, there is presence of the Coriolis force. As effects of the Coriolis force, in general, could be consider the rotation of the Earth around its own axis, because the Earth's surface rotates at a higher velocity near the Equator than at the poles so that the Coriolis force that arises is weaker at the equator and increase to the poles. A visible effect of the Coriolis force in the northern hemisphere is giving by the flowing waters, which cause erosion of the right and left banks (in the southern hemisphere). The same could be observe for the railways so that, in the northern hemisphere the right rail has a slightly more accentuated wear, in the southern one the reverse phenomenon takes place.

If the Coriolis force is defining according to the literature [8], [10], an apparent, inertial force acts on a body when it is located in rotational motion with respect to a chosen reference system. From a mechanical or physical point of view, it is a consequence of the conservation of the kinetic moment of the rotational motion. In the following, it wants to show the fact that, in technique, if there is relative movement, this phenomenon is also find in mechanisms. In this case, its neglect leading to wear of the parts or even their breakage.

The paper aims, in the following, to contribute, in terms of redesigning a laboratory equipment, to sizes and weights smaller and to maintain the effectiveness of the Coriolis Effect.

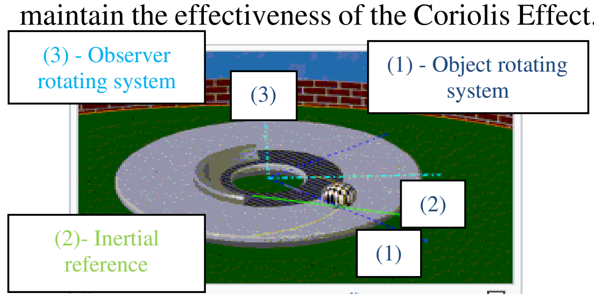

Fig. 1 Coriolis Effects [10].

2. GENERALITIES ON THE EFFECT PRODUCED BY THE CORIOLIS FORCE

Therefore, in the technique, the appearance of the Coriolis force [5] could be observe only in the relative motion, respectively, if we consider that, a body that rotates together with another body, which in turn moves radially towards it, without friction and all this without the effect of any external force. It could be see that, a body is subject to a deviation from the radial rectilinear motion, in the opposite direction to the rotational motion.

The effect of this Coriolis force is a sag (*f*), which deviates from the radial direction of one of the bodies and, is the product between the body mass and the Coriolis acceleration that produces the force [6], according to Newton's Law II [2].

The vector formula of the Coriolis force [2] is:

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$$\bar{F}_{jc} = -m \cdot \bar{a}_c = -2m \cdot (\bar{\omega} \times \bar{v}_r) \quad (1)$$

where: $\bar{F}_{jc} - Coriolis\ force\ [N]$

$\bar{a}_c - Coriolis\ acceleration\ [\frac{m}{s^2}]$

$\bar{\omega} - angular\ velocity\ [\frac{rad}{s}]$

$\bar{v}_r - relative\ velocity\ [\frac{m}{s}]$.

Coriolis force has perpendicular direction on plane ($\bar{\omega}$, $\bar{v}_r$), in the sense of rotating $\bar{\omega}$ over $\bar{v}_r$.

Other applications that demonstrate the effects of Coriolis force are:

Application 1 _ The Foucault pendulum [7] was a famous experiment put into practice by Jean Bernard Léon Foucault. The experiment confirmed the rotation of the Earth around its axis.

*Application 2*_ A vibrating structure gyroscope [7], defined by the IEEE as a Coriolis vibrating gyroscope (CVG), is a gyroscope that uses a vibrating structure to determine the velocity of rotation.

*Application 3*_ Laboratory study, experiments to highlight the presence of a Coriolis acceleration [2] in mechanisms, respectively the appearance of the Coriolis force in the technique. The study starts from an existing equipment [2] and aims to redesign it, to small dimensions explained in the top. The equipment must consist of the following main elements and some auxiliary ones, as shown in the figure 1.

Other practical applications for the device could be its use, as a component of a navigation instrument or gyroscope in the building of micro robots [4].

3. STUDIES ON EQUIPMENT REMODELING

The paper aims to redesign an existing equipment that highlights the Coriolis effecting. The redesign to the smallest dimensions through studies of strength, size, design, this includes the choice of new components such as engine, rod etc.

The basic elements of the existing equipment (Fig.2) and the shape, it wants to keep as closely as possible, and these are:

- the frame that allows the engine to be fixed and offers the rotation space (1);
- the elastic rotation rod (3) fixed to the engine shaft (5) which will deviate with the sag (*f*);
- the sliding element that moves on the elastic rod (4) and which is released by an electromagnet (6);
- the scale that allows the measurement of the sag (*f*) of the elastic rod, under the action of the Coriolis effect (2).

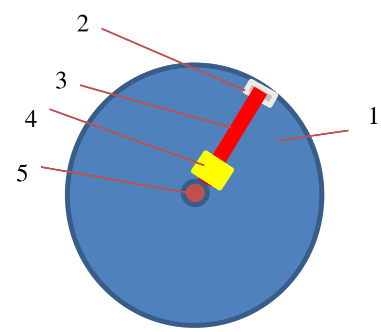

Fig. 1 Top view of equipment [2].

It starts the study from the existence of an equipment that demonstrates the Coriolis Effect and from the desire for mobility of this equipment, it's remodeling to the smallest dimensions to keep the effect. For this purpose, three constructive variants will be present it (Table 1); one of them keeps the size of the existing equipment (which is excluded due to large size) and the choice for redesign of one of the two remaining ones.

Table 1

Some characteristics of constructive variants

Options	**D [mm] (outer diameter)**	**H [mm] (height)**	**m [kg] (equipment mass)**	**n [rot/min] (rotation)**	**Material rod**	**Engine transmission**	**Frame material**
Option 1	130	110	120	350	OLC 30X	belt	E295
Option 2	80	100	65	350	OLC 30X	belt	E295
Option 3	**estimate 60cm, of calculation 32cm**	100	25-30	600	OLC 30X (elastic steel)	belt	E295

Table 1 presents only examples of miniaturization process of the equipment at the dimensions desired by the designer, only in terms of its size and mass.

In this case, in the following step, it presents the some design and dimensions of variants constructive equipment proposed through redesign.

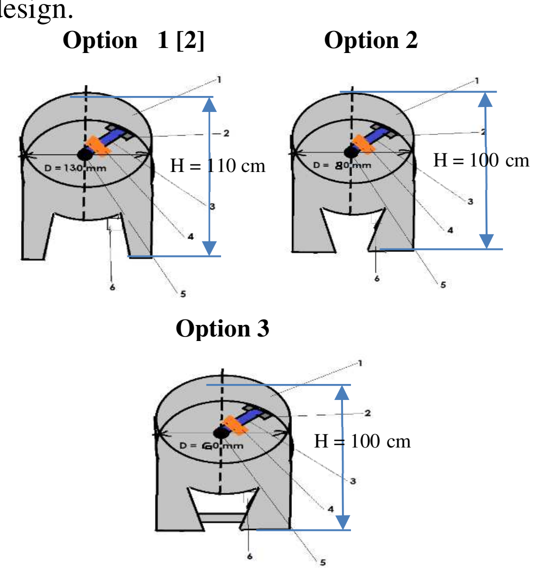

Fig. 2 Constructive variants.

As previously mentioned, the important differences between the 3 variants in table 1, in general, are differences in size, and among the 3 variants, variant 1 exists and is functional, but it will be constructively chosen the variant 2, because the equipment's reduction is desired by decreasing sizing. However, a too drastic reduction in diameter D = 60 mm (variant 3) would not allow a sufficient length of the elastic rod (3), so that one sag can be observed under the action of the Coriolis force, and this study will be demonstrated in the following step. In fact, when adapting the external size, it is the minor modification of the frame shape, for the stability of the equipment.

With all these remodels, in the end, the result must demonstrate the Coriolis Effect, which starts from the appearance of acceleration, respectively Coriolis force in relative motion.

$$m \cdot \bar{a}_r = \bar{R} + \bar{R}_l + \bar{F}_{jt} + \bar{F}_{jc} \qquad (2)$$

where: m = pill mass [kg];
$\bar{a}_r$ − relative acceleration $[\frac{m}{s^2}]$;
$\bar{R}$ − resultant vector of external forces [N];
$\bar{R}_l$ − resultant vector of bending forces $[N]$;
$\bar{F}_{jt}$ = inertial transport force $[N]$;
$\bar{F}_{jc}$ = inertial Coriolis force (rel. 1) $[N]$.

Following some mathematical calculations will result the theoretical sag [2] in a first form:

$$f_{theoretic} = \frac{N_2 \cdot l_1^2}{E \cdot l_2} \cdot \left(\frac{l_1}{3} + \frac{l_2}{2}\right) [mm] \qquad (3)$$

where: N_2 = normal reaction component N regarding pill (4) situates on the road (3) [N];
l_1 = distance between the engine shaft till pill (4) displacement on the road (3) [mm];
l_2 = distance between the engine shaft till pill (4) displacement on the end road (3) [mm];
E = 2.1· 10^{11} longitudinal coefficient of elasticity regarding road (3) [N/m^2].

But, this sag must theoretically be able to be determined and experimentally, on a small scale, by a laboratory experiment, in one of the variants of the equipment (Fig. 3) and practically demonstrated, the fact that, in the relative motion, the Coriolis force appears and deforms rod (3) with an experimental sag, practically visible on the ruler (4). Finally, theoretical and experimental values for sag, it could be comparative into error.

4. SELECTION THE COMPONENTS THAT ARE NOT DESIGN IN ORDER TO REDESIGNED OF THE EQUIPMENT

4.1 Principle of operation of the equipment

The operating principle of the equipment (Fig.1) consists in the fact that a relative movement occurs, composed of:
- the rotational movement of the rod (3) around the engine shaft (absolute rotational movement with respect to the frame with absolute angular velocity ω_a);
- the relative movement is produced by releasing the pill (4) from the electric engine, this making a rectilinear movement along the rod (3) (with relative velocity v_r);
- the transport movement, it is after a circular trajectory with angular velocity ω_t.

4.2 Choosing the engine like as size and power

The remodeling and redesign study was started with the choice of the type of engine purchased commercially, all this because the engine rod at the outlet of the housing must be long enough, to incorporate into it the elastic rod (3), and its mounting is perpendicular to the engine shaft. It was decided to supply the engine at 220V (Fig.3), single-phase and asynchronous engine, made of cast iron, with mounting on the sole. This sole will stick to the frame of the car.

These engines are use where velocity is not required to vary with load it. Its technical and overall characteristics are:

Power 1.1 Kw;
Rotation: 1500 rpm;
Axial diameter: 24 mm;
Outer disk diameter: 85 mm;
Channel diameter: 16 mm;
Length of the ledge: 18 cm;
Width ledge: 20 cm;
Engine mass: 1.5 kg;

Fig. 3 Monophasic engine type ECOTIS.

In order to bring the engine velocity to the one the explorer request, a vary velocity would be connect to it, which allows us to set the velocity of the equipment in operating mode, to obtain the desired result.

4.3 Checking the phenomena, Coriolis by establishing the sag

Schematically, the sizes to be determine in the study of the sag is presents in the figure 4.
It starts in the calculation of the sag from the inverse phenomenon; the level of the sag is impose, so that we can determine the length of the elastic rod (3) and respectively the size of the frame. Thus, starting from the relation (3) and taking account from the values of the sizes included in it, the length (l_1) of rod (3) [2], will be calculate according to the relation (4).

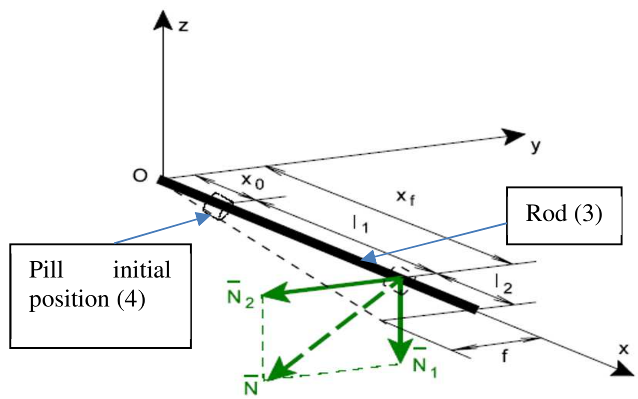

Fig. 4 Scheme of exploration equipment [2].

$$f_{theoretic} = \frac{16 \cdot \pi}{675} \cdot \frac{n^2 m l_1^2 \sqrt{x^2 - x_0^2} \cdot (2l_1 + 3l_2)}{E \cdot d^4} [\mathrm{m}] \quad (4)$$

Where the *imposed constructive values* are:
$f = 0.015$ *m (15mm)* - rod (3) **impose sag**;
d_rod = 0.005 m (0.5cm) – rod diameter;
n = 600 RPM - engine rotation: (reduce from 1500 RPM);
m = 10 gr - mass (brass) of rod released by the engine;
$E = 2.1 * 10^{11}$ N/m^2 – longitudinal elasticity coefficient for steel;
$\boldsymbol{l_1}$ **= requests it**;
$l_2 = 1$ cm (as long as it remains from the bar after the pill meets the stop);
$x_0 = 4$ cm; $x_f = 18$ cm requests result must be:
$l_1 = 14$ cm;
$x = x_f + x_0 = 20$ cm – 4 cm = 16 cm.

It solve the equation of degree five, in Matlab, and we will obtain the previous values, then we choose the constructive elements at their absolute value, respectively:

$l_1 = 14$ cm $l = l_1 + l_2$ (see Fig. 4);
$l = 14$ cm + 1 cm = 15 cm;
R-equipment frame= 16 cm.

Note: Because constructive equipment frame with diameter *D* = 32 cm, it would be too small, although the rod ($l = 15$ cm) would check the imposed sag. It will optimize diameter upwards, so to obtain *D* – equipment frame = 50 cm, it means that *l_rod* = 24 cm with the same road diameter $d = 0.5$ cm, and we will probably obtain a sag (f) larger than 15 mm in comparison with the previously.

5. CALCULUS OF STRENGTH AND ROD STRENGTH

All calculations are performing respecting the units of measurement in the international system.

a) Calculations of elastic rod resistance (3) - subjected to bending

Fig. 5 Schematic rod model.

Studying the bending tensions for rod *l/f* at a median axis of rod (3) is obtaining it:

$$\frac{1}{f} = \frac{M}{EI_y} \qquad (5)$$

where: f – sag after Ox axis [m];

$M = M(x)$ – bending moment [1];

E – elastically longitudinal coefficient for steel (E_steel = 2.1 x 10^{11} N/m^2);

I_y – geometric inertial moment [m^4].

To product EI_y – $ct.$, differential equation of 4-th order at bending is:

$$\frac{d^4w}{dx^4} = \frac{p}{EI_y} \qquad (6)$$

where: $w = w(x)$ – the equation of the deformed axis in the system xOz;

$p = F_{jc}/(l_1 + l_2)$ (see Fig. 4) - rod loads [N] (7)

If the basis of the continuity hypothesis and the small elastic deformations of the rod / bar are kept, the deformed axis will be a smooth curve (without jumps), having a unique left-right tangent. These conditions are meeting, if: $w(x)$ and $\varphi = \frac{dw}{dx} = w'(x)$ are continues functions along of the rod's axis.

In the case of the recessed beam (the case under study), these boundary conditions are:

$$w(x) = 0 \text{ and } \varphi = \frac{dw}{dx} = w'(x) = 0 \qquad (8)$$

In the case of the recessed beam subjected to bending, the maximum normal stresses ($\sigma_{(max,\ min)}$) take place in the points in the section furthest from the neutral axis [1] and have the expressions in the case of rectangular sections:

$$\sigma_{max,min} = \frac{N}{A}\left(1 \pm \frac{6e}{h}\right) \quad [\text{N/m}^2] \qquad (9)$$

In this case: $h = l\_rod = 15$ cm.

where: N – normal force [N];

$A = d \cdot l = 0.075$ – section surface [m^2];

$e = l_l = 14$ cm – misalignment of application of normal force N [mm].

There is a risk that the rod / beam will be deformed above the yield strength, in which case it will break.

In the present case $e > h/6$ (ex. 14 > 15/6) from which result that: σ_{min} and σ_{max} have different signs, and:

$$N_1 = G = 0.01\,N;\ N_2 = 2m\omega^2\sqrt{x^2 - x_0^2} = \ = 2 \cdot 0.01 \cdot (20 \cdot \pi)^2\sqrt{0.16^2 - 0.04^2} = 12.21\,N \qquad (10)$$

and $$\omega = \frac{\pi n}{30} = \frac{\pi \cdot 600}{30} = 20 \cdot \pi \quad [\frac{rad}{s}] \qquad (11)$$

$$N = \sqrt{N_1^2 + N_2^2} \sim 12.21\,N \qquad (12)$$

Relations (12) become:

$$\sigma_{min} = \frac{N}{A}\left(1 - \frac{6l_1}{l}\right) = -75.0\ \frac{N}{m^2} \qquad (13)$$

$$\sigma_{max} = \frac{N}{A}\left(1 + \frac{6l_1}{l}\right) = 107.4.5\ \frac{N}{m^2} \qquad (14)$$

In general, the normal tensions (strength) depending on Young's modulus that is: $\sigma = E \cdot \varepsilon$.

Where: $\varepsilon = \frac{l - l_0}{l_0}$ – relative elongation (15)

Therefore, $$\sigma = 525 \cdot 10^5\ \frac{N}{m^2} \qquad (16)$$

and, $$\sigma_{adm} = \frac{N}{A} = \frac{12.21}{0.075} = 162.8\ \frac{N}{m^2} \qquad (17)$$

$\sigma_{max} < \sigma_{adm}$, as can be seen, if we compare the relationships (17) with (13-14) the bent rod/beam resists to the dimensions chosen in the study.

6. CONCLUSIONS

The paper achieved its objective, from a theoretical point of view, by redesigning and calculating a small equipment, which would preserve and show the effect of the Coriolis force, as well as all the calculations related to this remodeling.

The paper can also, be considered a starting point in the miniaturization of equipment that demonstrates the Coriolis Effect and which equipment is integrated into larger structures, such as robots. The paper presents only a part of the remodeling of the equipment, and other papers will present other defining calculations in this regard.

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Reproiectarea unui echipament privind efectul Coriolis

Abstract *Lucrarea vrea să prezinte reproiectarea unui echipament de laborator care să demonstreze apariția forței Coriolis. Reproiectarea, la dimensiuni mai mici, scopul fiind greutatea sa redusă, implicit mobilitatea acesteia. În analiză, vom pleca de la alegerea lungimii tijei elastice care se va deforma sub efectul forței Coriolis, lungime care trebuie să poată fi inclusă în diametrele (cuvei) batiului, impusă de proiectant.*

Cuvinte cheie: reproiectare, echipament de laborator, tija elastica, efect Coriolis.

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