## TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

## ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering<br>Vol. 64, Issue IV, November, 2021

# ANALYTICAL AND NUMERICAL STUDY OF THE STATE OF EFFORTS IN THE PIPES BURIED IN THE MEMBRANE THEORY 

Marius Șerban FETEA, Ancuța Coca ABRUDAN, Constantin CILIBIU


#### Abstract

The paper presents a numerical study of the stresses that occur in a pipe buried in membrane theory, focusing on the static analysis of the stresses that occur in the pipe walls for several types of loads, namely loads generated by transported liquid, loads generated by soil weight and road traffic loads. Key words: pipe, sectional efforts, traffic loads, theory of thin cylindrical coatings, beam theory


## 1. INTRODUCTION

At present, the pipes are certified after several international standards. ASTM, AWWA, ISO and EN are applicable in a multitude of applications, including wastewater transmission systems domestic and industrial. In the case of the pipeline flexible considered in this paper, they can withstand large deformations. The vertical loads being given by land, traffic and canvas groundwater, they cause a deformation of the pipe that is depending on the degree of compaction of the soil in around it and the annular rigidity of the pipe. In the paper, the simplifying hypothesis was considered according to which the groundwater loads were considered to be negligible. According to the bibliographic data at present, the static calculation of the pipes is done according to the manual AWWA M45 and ATV 127.

Theoretical studies indicate the possibility of determining the states of tension and stress on the thickness of the pipe ring using the membrane theory of thin cylindrical shells, as well as the possibility of applying the theory of beams for pipes with their ratio between length and radius or equal to 8 . At present, no determined data are known for the calculation of buried pipes using the theory of thin cylindrical coatings, this being verified only from a theoretical point of view.

In this paper we aimed to perform the analytical and numerical study for the particular
case of a buried pipe. In the calculation performed, the results obtained by other authors were taken into account through previous studies performed by other authors. Due to the multitude of parameters involved in the calculation performed by the authors, performing a rigorous calculation with extremely accurate results is difficult to achieve. But, by applying some simplifying hypotheses that take into account the soil and loading conditions of the pipeline, covering results can be obtained from the point of view of their accuracy. The results that are aimed to be determined in the present paper are related to the analysis of the state of efforts and tensions that appear in the pipe walls. The calculation and study of these sectional efforts was performed in the median plane of the pipe. The loads that were taken into account in the static analysis of the pipeline are the following:

- loading by the weight of the liquid;
- the load from the weight of the soil located above the upper generator of the pipeline;
- loading from road traffic.

In the membrane theory taken into account for performing the static analysis, because the stresses show a constant variation on the thickness of the pipe wall (actually having a uniformly distributed variation) the sectional efforts mentioned above can be determined using the relations [5], [6]:

$$
\begin{equation*}
N_{x}=\sigma_{x} \cdot g_{C} \tag{1}
\end{equation*}
$$

$$
\begin{gather*}
N_{\theta}=\sigma_{\theta} \cdot g_{C}  \tag{2}\\
N_{x \theta}=N_{\theta x}=\tau_{x \theta} \cdot g_{C} \tag{3}
\end{gather*}
$$

The expression of the loads at the level of an arbitrary point considered in the median plane being [2]:

$$
\begin{align*}
F_{Z}^{T O T} & =\gamma_{L} \cdot r_{\text {med }} \cdot(1-\cos \theta)-\gamma_{P} \\
& h \cdot \cos \theta-q_{T} \cdot L_{\text {TRAFIC }} \tag{4}
\end{align*}
$$

## 2. MATERIALS AND METHODS

The pipe analyzed under the study was considered to be related to a system of triorthogonal XOYZ axes, and the angle at the center of the circular cross section of the pipe was denoted by $\theta$.
The analysis of the state of tension and efforts was performed at a certain current point in the median plane of the pipe taking into account the following data related to the dimensional geometry and the load of the pipe:

- the specific gravity of the liquid in the pipe $\gamma_{L}=11\left[\frac{K N}{m^{3}}\right][6] ;$
- specific weight of the soil $\gamma_{P}=19\left[\frac{K N}{m^{3}}\right][6]$;
- the angle that the coordinate axis Z makes with the radius corresponding to the chosen arbitrary point $\theta$;
- radius of the pipe $r=609,5[\mathrm{~mm}]$, [7];
- average radius of the pipe $r_{\text {med }}=$ 603,95[mm];
- the length of the pipe considered when performing the mechanical calculation $\mathrm{L}=5$ [m], [7];
- the outer diameter of the pipe $D_{\text {ext }}=$ 1219 [ mm ], [7];
- the average diameter of the pipe $D_{\text {med }}=$ 1207,9[mm], [7];
- pipe wall thickness $g_{C}=13[\mathrm{~mm}]$;
- $\quad D_{t}=0,8$ coefficient for flexible paving;
- traffic pressure, $p_{T}=0.0826$. ( $h^{-1.25}$ ), [7];
- loads from heavy road traffic $q_{T=} a_{T} \cdot D_{T}$. $p_{T}=1.367\left[\frac{\mathrm{~N}}{\mathrm{~mm}}\right]$
- $h$, distance from the ground surface to the upper pipe generator $h=500[\mathrm{~mm}]$.
- $\quad F_{Z}^{T O T}$, total load.

The expression of the loads at the level of an arbitrary point considered in the median plane being:

$$
\begin{gather*}
F_{Z}^{T O T}=\gamma_{L} \cdot r_{m e d} \cdot(1-\cos \theta)-\gamma_{P} \cdot h \cdot \cos \theta- \\
q_{T} \cdot L_{T R A F I C} \tag{5}
\end{gather*}
$$

### 2.1 Determination of expressions of efforts from liquid pipe loading

The considered liquid load is:

$$
\begin{align*}
& \mathrm{F}_{\mathrm{Z}}^{\mathrm{L}}=\gamma_{\mathrm{L}} \cdot \mathrm{r}_{\text {med }} \cdot(1-\cos \theta)-\gamma_{\mathrm{P}} \cdot \mathrm{~h} \cdot \cos \theta- \\
& \mathrm{q}_{\mathrm{T}} \cdot \text { Ltrafic } \tag{6}
\end{align*}
$$

The static scheme of liquid loading leads to the following expressions of efforts:

$$
\begin{gather*}
N_{\theta}^{L}=\gamma_{L} \cdot r^{2}{ }_{\text {med }} \cdot(1-\cos \theta)  \tag{7}\\
N_{\theta X}^{L}=-x \cdot \sin \theta \cdot \gamma_{L} \cdot r_{\text {med }}+C_{1}(\theta)  \tag{8}\\
N_{X}^{L}=\frac{x^{2}}{2} \cdot \gamma_{L} \cdot \cos \theta-\frac{1}{r m e d} \cdot \frac{\partial C_{1}}{\partial \theta}+C_{2}(\theta) \tag{9}
\end{gather*}
$$

By imposing the conditions at the limits [1], [3], [6], the integration constants are determined $C_{1}(\theta)$ and $C_{2}(\theta)$. The length of the pipe was considered $L=5[m]$, being simply supported at its ends and loaded with evenly distributed force $F_{Z}$. The origin of the axis system to which the pipe was reported is considered to be in the middle of the pipe opening in the cross section of the pipe, in the respective section for the independent parameter $x=0$, it follows that the tangential stresses are $\tau_{x \theta}=0$, because the stress $N_{X \theta}=0$.

$$
\begin{gather*}
C_{1}(\theta)=0  \tag{10}\\
C_{2}=-\frac{\gamma_{L} L^{2}}{8} \cdot \cos \theta \tag{11}
\end{gather*}
$$

Taking into account the integration constants for the efforts $N_{X \theta}$ si $N_{X}$ the following expressions were determined:

$$
\begin{align*}
& N_{\theta}^{L}=\gamma_{L} \cdot r^{2}{ }_{\text {med }} \cdot(1-\cos \theta)  \tag{12}\\
& N_{X \theta}^{L}=-x \cdot \sin \theta \cdot \gamma_{L} \cdot r_{\text {med }}  \tag{13}\\
& N_{X}^{L}=\frac{x^{2}}{2} \cdot \gamma_{L} \cdot \cos \theta-\frac{\gamma_{L} L^{2}}{8} \cdot \cos \theta \tag{14}
\end{align*}
$$

On the contour of the pipe, taking into account the variation of the angle at the center $\theta$, the following expressions were determined for the stress expressions.

In the most entailed section of the pipeline for $x=0$, depending on the variation of the angle $\theta$ the following expressions of the sectional efforts at the level of the pipe wall are obtained:

- for $\theta=0, N_{\theta}=0, N_{X \theta}=0$,

$$
\begin{aligned}
& \quad N_{X}=\frac{x^{2}}{2} \cdot \gamma_{L} \cdot \frac{\gamma_{L} \cdot L^{2}}{8} \\
& - \text { for } \theta=\frac{\pi}{2}, N_{\theta}=\gamma_{L} \cdot r^{2}{ }_{\text {med }}, N_{X}=0, \\
& N_{X \theta}=-x \cdot \gamma_{L} \cdot r_{m e d} \\
& - \text { for } \theta=\pi, N_{\theta}=2 \gamma_{L} \cdot r^{2}{ }_{\text {med }}, N_{X \theta}=0, \\
& \\
& N_{X}=-\frac{x^{2}}{2} \cdot \gamma_{L} \cdot+\frac{\gamma_{L} \cdot L^{2}}{8} \\
& - \text { for } \theta=\frac{3 \pi}{2}, N_{\theta}=\gamma_{L} \cdot r^{2}{ }_{\text {med }}, \\
& N_{X \theta}=x \cdot \gamma_{L} \cdot r_{\text {med }}, N_{X}=0
\end{aligned}
$$

Taking into account the bar theory, the following expressions of efforts are obtained:

$$
\begin{align*}
& M_{X}^{L}=\frac{\gamma_{L} \cdot L^{2} \cdot \pi \cdot r_{\text {med }}^{2}}{8}  \tag{15}\\
& T_{X}^{L}=\frac{\gamma_{L} L \cdot \pi \cdot r_{\text {med }}^{2}}{2} \tag{16}
\end{align*}
$$

### 2.2 Determination of expressions of efforts from loading the pipeline by weight of the soil

The analysis of the efforts on the pipe circumference leads to the following results:

- on account of $\theta=0, N_{\theta}^{P}=-\gamma_{P} \cdot h \cdot r_{\text {med }}, N_{X \theta}^{P}=$ $0, N_{X}^{P}=-\frac{L^{2}}{4 r} \cdot \gamma_{L} \cdot h\left(1-\frac{4 x^{2}}{L^{2}}\right)$
- on account of $\theta=\frac{\pi}{2}, N_{\theta}^{P}=0, N_{X}^{P}=0$,
$N_{X \theta}^{P}=-2 x \cdot \gamma_{P} . h$
- on account of $\theta=\pi, N_{\theta}^{P}=\gamma_{P} \cdot h \cdot r_{\text {med }}$,
$N_{X \theta}^{P}=0, N_{X}^{P}=\frac{L^{2}}{4 r} \cdot \gamma_{P} \cdot h\left(1-\frac{4 x^{2}}{L^{2}}\right)$
- on account of $\theta=\frac{3 \pi}{2}, N_{\theta}^{P}=0, N_{X \theta}^{P}=2 x \cdot \gamma_{P} . h$, $N_{X}^{P}=0$

In bar theory, the following expressions of efforts and tensions were obtained:

$$
\begin{align*}
& M_{\text {max }}^{P}=p i \cdot r_{\text {med }} \cdot \gamma_{P} \cdot g_{C} \cdot \frac{L^{2}}{4}  \tag{17}\\
& T_{\text {max }}^{P}=p i \cdot r_{\text {med }} \cdot \gamma_{P} \cdot g_{C} \cdot L \cdot L \tag{18}
\end{align*}
$$

### 2.3 Determination of effort expressions from road traffic loads

Taking into account the mode of action of the intensity force $q_{T}$ due to road traffic, it will have two projections along the coordinate axes, namely:

- external loading along the axis $Z$ will be

$$
\begin{equation*}
Z^{T}=-q_{T} \cdot \cos \theta \tag{19}
\end{equation*}
$$

- external loading along the axis Y will be

$$
\begin{equation*}
Y^{T}=q_{T} \cdot \sin \theta \tag{20}
\end{equation*}
$$

The following expressions were obtained for the sectional efforts:

$$
\begin{gather*}
N_{\theta}^{T}=-q_{T} \cdot r \cdot \cos \theta  \tag{21}\\
N_{X \theta}^{T}=-2 \mathrm{x} \cdot q_{T} \cdot \sin \theta  \tag{22}\\
N_{X}^{T}=-\frac{\mathrm{L}^{2}}{4 \mathrm{r}} \cdot q_{T} \cdot \cos \theta\left(1-\frac{4 \mathrm{x}^{2}}{\mathrm{~L}^{2}}\right) \tag{23}
\end{gather*}
$$

The variation of the efforts on the thickness of the ring on its circumference is:

- on account of $\theta=0, N_{\theta}^{T}=-q_{T} \cdot r_{\text {med }}$,
$N_{X \theta}^{T}=0, N_{X}^{T}=-\frac{\mathrm{L}^{2}}{4 \mathrm{r}} \cdot q_{T}\left(1-\frac{4 \mathrm{x}^{2}}{\mathrm{~L}^{2}}\right)$
- on account of $\theta=\frac{\pi}{2}, N_{\theta}^{T}=0, N_{X}^{T}=0$,
$N_{X \theta}^{T}=-2 x \cdot q_{T}$
- on account of $\theta=\pi, N_{\theta}^{T}=q_{T} \cdot r_{\text {med }}, N_{x \theta}^{T}=0$, $N_{X}^{T}=\frac{\mathrm{L}^{2}}{4 \mathrm{r}} \cdot q_{T}\left(1-\frac{4 \mathrm{x}^{2}}{\mathrm{~L}^{2}}\right)$
- on account of $\theta=\frac{3 \pi}{2}, N_{\theta}^{T}=0, N_{X \theta}^{T}=2 \mathrm{x} \cdot q_{T}$, $N_{X}^{T}=0$

In bar theory, the expressions of maximum moments and maximum shear forces were obtained:

$$
\begin{align*}
& M_{\text {max }}^{T}=\frac{\pi \cdot r_{\text {med }} \cdot \mathrm{q}_{\mathrm{T}} \cdot L^{2}}{4}  \tag{24}\\
& T_{\text {max }}^{T}=\pi \cdot r_{\text {med }} \cdot \mathrm{q}_{\mathrm{T}} \cdot L \tag{25}
\end{align*}
$$

2.4 Analytical study of the state of efforts and stresses in the main sections of the pipe by applying the method of overlapping the effects and numerical analysis of their variation

Having determined the efforts for the three loads of the pipe considered separate, by applying the principle of overlapping the effects, the following results were obtained:

- on account of $\theta=0, \quad N_{\theta}^{\text {ToT }}(0)=N_{\theta}^{L}(0)+$
$N_{\theta}^{P}(0)+N_{\theta}^{T}(0)=-\gamma_{p} \cdot h \cdot r_{\text {med }}-q_{T} \cdot r_{\text {med }}-$
$r_{\text {med }} \cdot\left(\gamma_{P} \cdot h+q_{T}\right), N_{X \theta}^{T o T}(0)=N_{X \theta}^{L}(0)+N_{X \theta}^{P}(0)+$
$N_{\theta X}^{T(0)}=0, N_{X}^{T o T}(0)=N_{X}^{L}(0)+N_{X}^{P}(0)+N_{X}^{T}(0)=\frac{x^{2}}{2}$.
$\gamma_{L^{\prime}}-\frac{\gamma_{L} L^{2}}{8}-\frac{L^{2}}{4 r} \cdot \gamma_{p} \cdot h\left(1-\frac{4 x^{2}}{L^{2}}\right)-\frac{\mathrm{L}^{2}}{4 \mathrm{r}} \cdot q_{T}\left(1-\frac{4 \mathrm{x}^{2}}{\mathrm{~L}^{2}}\right)(26)$
- on account of $\theta=\frac{\pi}{2}, N_{\theta}^{\text {ToT }}\left(\frac{\pi}{2}\right)=N_{\theta}^{L}\left(\frac{\pi}{2}\right)+$
$N_{\theta}^{P}\left(\frac{\pi}{2}\right)+N_{\theta}^{T}\left(\frac{\pi}{2}\right)=\gamma_{L} \cdot r^{2}{ }_{\text {med }}, N_{X \theta}^{T O T}\left(\frac{\pi}{2}\right)=N_{X \theta}^{L}\left(\frac{\pi}{2}\right)+$
$N_{X \theta}^{P}\left(\frac{\pi}{2}\right)+N_{\theta X}^{T}\left(\frac{\pi}{2}\right)=-2 x \cdot q_{T}-2 x \cdot \gamma_{P} \cdot h-x \cdot \gamma_{L}$.
$r_{\text {med }}, N_{X}^{T o T}\left(\frac{\pi}{2}\right)=N_{X}^{L}\left(\frac{\pi}{2}\right)+N_{X}^{P}\left(\frac{\pi}{2}\right)+N_{X}^{T}\left(\frac{\pi}{2}\right)=0$
- on account of $\theta=\pi, N_{\theta}^{T O T}(\pi)=N_{\theta}^{L}(\pi)+$
$N_{\theta}^{P}(\pi)+N_{\theta}^{T}(\pi)=q_{T} \cdot r_{\text {med }}+\gamma_{P} \cdot h \cdot r_{\text {med }}+2 \gamma_{L}$.
$r_{\text {med }}^{2}, N_{X \theta}^{T O T}(\pi)=N_{X \theta}^{L}(\pi)+N_{X \theta}^{P}(\pi)+N_{\theta X}^{T}(\pi)=0$
$N_{x}^{T O T}(\pi)=N_{X}^{L}(\pi)+N_{X}^{P}(\pi)+N_{X}^{T}(\pi)=-\frac{x^{2}}{2} \cdot \gamma_{L}+$
$\frac{\gamma_{L} \cdot L^{2}}{8}+\frac{L^{2}}{4 r} \cdot \gamma_{P} \cdot h\left(1-\frac{4 x^{2}}{L^{2}}\right)+\frac{\mathrm{L}^{2}}{4 \mathrm{r}} \cdot q_{T}\left(1-\frac{4 \mathrm{x}^{2}}{\mathrm{~L}^{2}}\right)$
- on account of $\theta=\frac{3 \pi}{2}, N_{\theta}^{T O T}\left(\frac{3 \pi}{2}\right)=N_{\theta}^{L}\left(\frac{3 \pi}{2}\right)+$
$N_{\theta}^{P}\left(\frac{3 \pi}{2}\right)+N_{\theta}^{T}\left(\frac{3 \pi}{2}\right)=\gamma_{L} \cdot r^{2}{ }_{\text {med }}, N_{\theta x}^{T O T}\left(\frac{3 \pi}{2}\right)=$
$N_{x \theta}^{L}\left(\frac{3 \pi}{2}\right)+N_{x \theta}^{P}\left(\frac{3 \pi}{2}\right)+=N_{x \theta}^{T}\left(\frac{3 \pi}{2}\right)=2 \mathrm{x} \cdot q_{T}+2 x$.
$\gamma_{P} \cdot h++x \cdot \gamma_{L} \cdot r_{m e d} \cdot N_{X}^{T O T}\left(\frac{3 \pi}{2}\right)=N_{X}^{L}\left(\frac{3 \pi}{2}\right)+$
$N_{X}^{P}\left(\frac{3 \pi}{2}\right)+N_{X}^{T}\left(\frac{3 \pi}{2}\right)=0$


## 3. RESULTS

The determined results are constituted as own contributions of the authors. The $x=0$ and $y=0$ coordinates coincide with the middle section of the buried pipe presented in figure 1.


Fig. 1 Longitudinal view section of buried pipe
For the evaluation of the sectional efforts along the pipe for $x=\mp \frac{L}{2}$, in different points $\theta=[0$, $\left.\frac{p i}{2}, p i, 3 * \frac{p i}{2}\right]$, whose expressions were determined in the previous subchapters, the MATLAB software was used and the calculation Program 1 was realized.
\% Program 1
\% Initial data of the variables in the expressions of the stresses on the circumference of the pipe ring;
\% GL, specific weight of liquid $\left[\mathrm{KN} / \mathrm{m}^{\wedge} 3\right]$;
GL=11
$\% \mathrm{gC}$, the thickness of the pipe wall [m];
$\mathrm{gC}=0.013$
\% L, length of the pipe between supports;
L=5
\% GP, specific weight of the soil [KN/m^3];
$\mathrm{GP}=19$
$\% \mathrm{~h}$, pipe depth to upper generator [m];
$\mathrm{h}=0.5$
\% r, average radius of the pipe [m]
r=0.60395
\% qT, load intensity from road traffic [KN/m^2]
$\mathrm{qT}=1.367$
$\mathrm{x}=(-2.5: 1: 2.5)$
Nteta0 $=-\mathrm{GP}^{*} \mathrm{~h}^{*} \mathrm{r}-\mathrm{qT} * \mathrm{r}$

NXteta0=0
NX0 $=((x . \wedge 2 * \mathrm{GL}) /(2))-\left(\left(\mathrm{GL} * \mathrm{~L}^{\wedge} 2\right) /(8)\right)-$
$\left(\left(\mathrm{L}^{\wedge} 2 * \mathrm{GP} * \mathrm{~h}\right) /(4 * \mathrm{r})\right) . *\left(1-(4 * \mathrm{x} . \wedge 2) /\left(\mathrm{L}^{\wedge} 2\right)\right)-$
$-\left(\mathrm{L}^{\wedge} 2^{*} \mathrm{qT} / 4^{*} \mathrm{r}\right) . *\left(\left(1-\left(4 * \mathrm{x} .{ }^{\wedge} 2\right) / \mathrm{L}^{\wedge} 2\right)\right)$
\% FOR teta=pi/2
Ntetapi=GL*r^2
NXteta=-2*x.*qT-2*x.*GP*h-x.*GL*r
NX=0
\% FOR teta=pi
Ntetapi $=\mathrm{qT} * \mathrm{r}+\mathrm{GP} * \mathrm{~h} * \mathrm{r}+2 * \mathrm{GL} * \mathrm{r} \wedge 2$
NXtetapi=0
NXpi $\left.=\left(\left(x . \wedge^{\wedge}\right)^{*} \mathrm{GL}\right) /(2)\right)+\left(\left(\mathrm{GL} * \mathrm{~L}^{\wedge} 2\right) /(8)\right)+\left(\left(\mathrm{L}^{\wedge} 2^{*}\right.\right.$
$\left.\left.\mathrm{GP}^{*} \mathrm{~h}\right) /(4 * \mathrm{r})\right) . *\left(1\left(4^{*} \mathrm{x} \cdot{ }^{\wedge} 2\right) /\left(\mathrm{L}^{\wedge} 2\right)\right)+\left(\mathrm{L}^{\wedge} 2^{*}\right.$
*qT/4*r).*((1-(4*x.^2)/L^2))
\% FOR teta=3*pi/2
Nteta3pi=GL* ${ }^{\wedge} \wedge 2$
NXteta3pi=2*x.*qT+2*x.*GP*h+x.*GL*r
NX3pi=0
In figure 2 is presented the print screen with the calculation program made and run by the authors


Fig. 2 Print screen of the calculation Program 1
The results obtained by running the developed program are presented below.

$$
\begin{array}{llll}
\gg x & =-2.5000 & -1.5000 & -0.5000
\end{array} 0.5000
$$

$$
\begin{array}{ll}
1.5000 \quad 2.5000
\end{array}
$$

$$
\text { Nteta0 }=-6.5631
$$

NXteta0 $=0$
NX0 $=0 \quad-88.2215-132.3323-132.3323-88.2215$
0
Ntetapi $=4.0123$
NXteta $=70.9436 \quad 42.5662 \quad 14.1887-14.1887$ -
42.5662-70.9436

NX =0
Ntetapi $=14.5877$
NXtetapi $=0$
NXpi $=0 \quad 88.2215132 .3323132 .3323 \quad 88.2215$
0
Nteta3pi $=4.0123$
NXteta3pi $=-70.9436-42.5662-14.1887 \quad 14.1887$
42.566270 .9436

NX 3 pi $=0$

The results obtained by running the developed program
> $\mathrm{GL}=11$
$\mathrm{gC}=0.0130$
$\mathrm{L}=5$
GP $=19$
$\mathrm{h}=0.5000$
$\mathrm{r}=0.6039$
$\mathrm{qT}=1.3670$
Sigmaech $=118\left[\mathrm{~N} / \mathrm{mm}^{\wedge} 2\right]$

## 4. CONCLUSIONS

Following and analyzing the data resulting from the calculations performed in the work, the following conclusions can be drawn regarding the good mechanical functioning of the analyzed buried pipe.

1. it is found that in the case of applying the beam theory to the determination of efforts and stresses, their expressions are the same as those obtained in the theory of thin cylindrical coatings;
2. the major disadvantage of the application of the beam theory compared to the theory of thin cylindrical coatings is given by the fact that the expression of the effort cannot be determined $\mathrm{N}_{\theta}$;
3. obtaining rigorous results is related to the values of the ratio between the length of the pipes between two successive supports and their average radius. The condition is that the ratio:

$$
\frac{L}{r_{\text {med }}} \geq 8
$$

4. it is found that depending on the size of the cross section of the pipe ring, the bearing length plays a major role in the successful application of the theory of thin cylindrical coatings;
5. different expressions of these efforts and tensions were determined by applying the two theories, when loading the pipe, respectively its ring with the forces due to the liquid in the pipe;
6. the data obtained for the sectional efforts were centralized in table 1 being expressed in $\left[\frac{N}{m m}\right]$.

Table 1
Sectional stresses according to the theory of thin

| cylindrical coatings |  |  |  |  |  |  |  |
| :---: | :---: | :--- | :--- | :--- | :--- | :--- | :---: |
| Efforts $\mathrm{x}=-2.5$ $\mathrm{x}=-1.5$ $\mathrm{x}=-5$ $\mathrm{x}=.5$ $\mathrm{x}=1.5$ <br> $N_{\theta}$ <br> 0 $\quad(\theta=2.5$ -6.56 -6.56 -6.56 -6.56 <br> -6.56 -6.56     |  |  |  |  |  |  |  |


| $N_{\theta} \quad(\theta=$ <br> $p i / 2)$ | 4.012 | 4.012 | 4.012 | 4.012 | 4.012 | 4.012 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $N_{\theta}$ <br> $p i)$$\quad(\theta=$ | 14.58 | 14.58 | 14.58 | 14.58 | 14.58 | 14.58 |
| $N_{\theta} \quad(\theta=$ <br> $3 * p i / 2)$ | 4.012 | 4.012 | 4.012 | 4.012 | 4.012 | 4.012 |
| $N_{X \theta} \quad(\theta=$ <br> $0)$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $N_{X \theta} \quad(\theta=$ <br> $p i / 2)$ | 70.94 | 42.56 | 14.18 | -14.18 | -42.56 | -70.94 |
| $N_{X \theta} \quad(\theta=$ <br> $p i)$$\quad 0$ | 0 | 0 | 0 | 0 | 0 |  |
| $N_{X \theta} \quad(\theta=$ <br> $3 * p i / 2)$ | -70.94 | -42.56 | -14.18 | 14.18 | 42.56 | 70.94 |
| $N_{X} \quad(\theta=$ <br> $0)$ | 0 | -88.22 | -132.33 | -132.33 | -88.22 | 0 |
| $N_{X} \quad(\theta=$ <br> $p i / 2)$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $N_{X} \quad(\theta=$ <br> $p i)$$\quad 0$ | -88.22 | -132.33 | -132.33 | -88.22 | 0 |  |
| $N_{X} \quad(\theta=$ <br> $3 * p i / 2)$ | 0 | 0 | 0 | 0 | 0 | 0 |

7. analyzing table 1 it is found that the most requested section is the section $x=0$ for which the maximum sectional effort is for $(\theta=0),(\theta=p i)$.

$$
\left|N_{X}\right|=132.3323\left(\frac{N}{\mathrm{~mm}}\right)
$$

8. in figures 3, 4, 5 and 6 according to [4], the variations of the efforts are represented $N_{X}, N_{X \theta}$, for $\theta=0, \theta=\frac{\pi}{2}, \theta=\pi, \theta=3 \pi / 2$, which have a parabolic variation for NX efforts and linear for NXteta efforts.


Fig. 3 Variation for NX on $\theta=0$


Fig. 4 Variation for NXteta for $\theta=\frac{\pi}{2}$


Fig. 5 Variation for NX on $\theta=\pi$


Fig. 6 Variation for NXteta on $\theta=3 \pi / 2$
The paper provides a clear picture for the applicability of the theory of thin cylindrical coatings and the theory of beams in certain dimensional cases of buried pipes to obtain rigorous solutions as close as possible to the real values. By simply modifying the initial data, the designed programs allow the determination of the maximum stresses and of the sectional efforts for any case of loading of the buried pipes.

## 5. REFERENCES

[1] Bors I, MECANICA, Compendiu teoretic si aplicatii de cinematica, dinamica si mecanica analitica, UTpres, Cluj-Napoca 2003;
[2] Marius FETEA, Ancuța ABRUDAN., MECHANICAL STUDY REGARDING THE LIMITS OF USE OF POLYPROPYLENE PIPES, ACTA TECHNICA NAPOCENSIS-Series: APPLIED MATHEMATICS, MECHANICS, and ENGINEERING, Volumul 63, Numărul 4, 2020/12/24;
[3] Ille V, Bia C, Soare M., Rezistenta Materialelor si Teoria Elasticitatii, Editura Didactica si Pedagogica, Bucuresti, 1983;
[4] Ghinea M., Fireteanu V., Matlab, Calcul numeric, grafica, aplicatii. Editura Teora. Bucuresti 2004;
[5] Furis D., Teodorescu M, Calculul structurilor pentru transportul apei, Editura Conspress, Bucuresti 2012;
[6] Posea N, Anghel Al, Manea C, Hotea Gh., Rezistenta Materialelor probleme, Editura Stiintifica si Enciclopedica, Bucuresti, 1986.
[7] Grandpipe, Sistemul de conducte din GRP/PAFSIN.
https://www.grandpipe.ro/wp-content/uploads/2015/12/GRANDPIPE-CATALOG-ROMANA.pdf

## STUDIUL ANALITIC SI NUMERIC AL STARII DE EFORTURI LA CONDUCTELE INGROPATE IN TEORIA DE MEMBRANA

Rezumat: Lucrarea prezinta un studiu numeric al eforturilor ce apar într-o conductă ingropată în teoria de membrană, punând accentul pe analiza statică a eforturilor ce apar în pereții conductei pentru mai multe tipuri de încărcări, respectiv încărcările generate de lichidul transportat, încarcările generate de greutatea solului și încărcările generate de traficul rutier.

Marius Serban FETEA, PhD. Eng. Lecturer, Technical University of Cluj Napoca, Faculty of Building Services Engineering, Building Services Department, marius.fetea@insta.utcluj.ro, +40765211237, Cluj-Napoca, 39/6 Unirii Street, Cluj-Napoca, ROMÂNIA.
Ancuța Coca ABRUDAN, Assoc. Prof. PhD. Eng., Technical University of Cluj-Napoca, Faculty of Building Services Engineering, Building Services Department, ancuta.abrudan@insta.utcluj.ro, +40264202556, 128-130 21 Decembrie 1989 Blv, Cluj-Napoca, 400604, ROMÂNIA
Constantin CILIBIU, PhD. Eng. Assistant Lecturer, Technical University of Cluj Napoca, Faculty of Building Services Engineering, Building Services Department, constantin.cilibiu@insta.utcluj.ro, +40724311138 , Cluj-Napoca, ROMÂNIA.

