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CONSIDERATIONS ABOUT THE MOTION TRAJECTORY PLANNING OF A 5R ROBOT, IMPLEMENTED IN A WELDING PROCESS

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Abstract: *the article describes the process of planning the motion trajectory of the 5R articulated industrial robot used in welding processes. The work cycle is described and divided into phases. The (4-3-4) method of motion planning is introduced and applied for a phase of the technological process. The phase is then divided into path segments and the variation table of the generalized coordinates is established. The restrictions in coordinates, velocities and accelerations are imposed, used as input data in the path modeling algorithm. As an example, the polynomial interpolation functions for the first phase of the technological process are determined, as well as their corresponding graphs, in normalized time.*
Key words: *motion planning, articulated robot, 5R, robotic welding.*

1. INTRODUCTION

In order to analyze the geometric, kinematic and dynamic behavior of articulated industrial robots, the 5R articulated robot [1] is considered, implemented in a welding process [2] and shown in fig. 1.

The planning of the trajectories of the 5R robot will be performed using the method of polynomial interpolation functions of type (4-3-4) [3], [4], [5], [6], [9]. After establishing the working cycle of the 5R robot, the variation of the generalized coordinates with respect to time follows. The technological process will be divided into phases, and each phase will be divided, in turn, into trajectory segments.

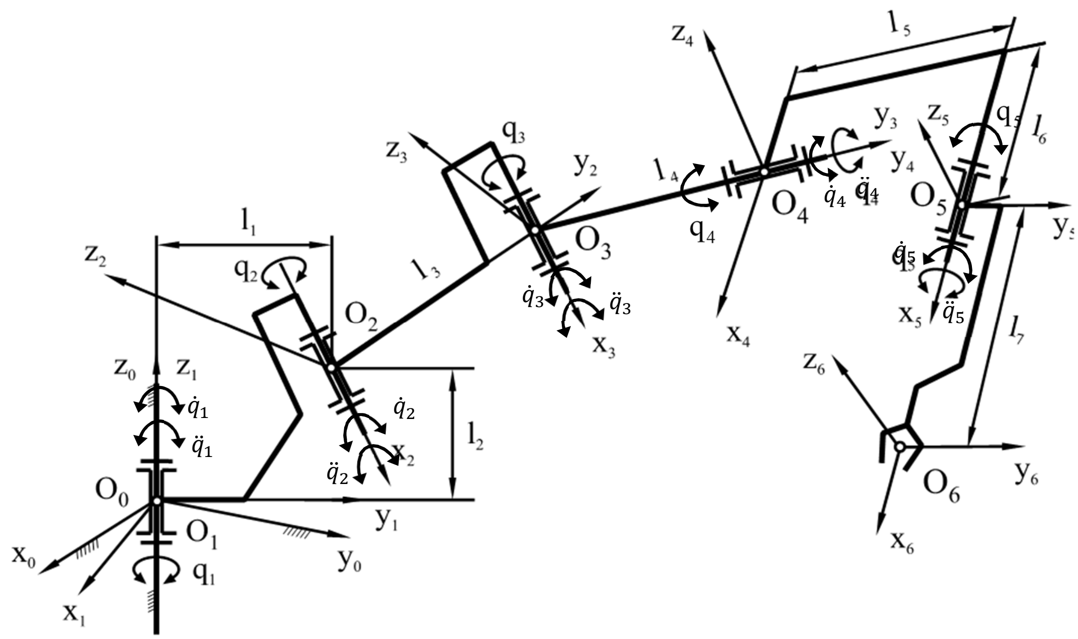

Fig. 1. 5R articulated industrial robot

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In order to determine the polynomial coefficients of the interpolation functions of type (4-3-4), the initial, final and continuity conditions in coordinates, velocities and accelerations for the analyzed phase will be imposed.

The numerical data thus obtained will be substituted in the equations of the geometric model [1], resulting in graphs of variation of the coordinates of the characteristic point of the welding tool with respect to time, as well as the 3D trajectory of the characteristic point in the analyzed phase.

The replacement of the numerical data in the equations of the kinematic model [2] leads to the drawing of the graphs of variation of the operational, linear, and angular velocities and accelerations, corresponding to the characteristic point, with respect to time.

Of a great utility are the graphs of the generalized driving forces with respect to time, determined on the basis of the equations of the dynamic model. Based on their extreme values, the corresponding actuators of the five robot joints will be chosen.

2. PRESENTATION OF THE WORK CYCLE AND ITS DIVISION INTO PHASES

The 5R articulated industrial robot will be implemented in a welding process. It will be programmed to start from rest, from the zero configuration, characterized by the values of the vector of generalized coordinates:

$$\theta_0 = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 & q_5 \end{bmatrix}_0^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \quad (1)$$

at the time $\tau = 0$. The first phase of the technological process is the positioning of the welding tool at the starting point of the welding line. Considering the continuous welding line, the second phase of the technological process will be dedicated to tracing the welding line, from the initial point to the final point, passing through certain intermediate points previously established, respecting the continuity in position, velocity and acceleration.

The third phase of the technological process will be represented by the return movement of the robot in the zero configuration, after the end of the welding operation.

As the movement of the robot during the welding operation does not require high speeds and accelerations, the phase I, corresponding to the positioning operation at the initial welding point, will be further analyzed from a geometric, kinematic and dynamic point of view.

Phase I is decomposed into three segments, the robot being forced to pass through two intermediate points.

The end of the first segment is marked by the column vector, at the time moment $\tau_1 = 3{,}33$:

$$\theta_1 = \begin{bmatrix} -7{,}6 & 56{,}61 & 7{,}08 & -7{,}03 & -0{,}103 \end{bmatrix}^T \quad (2)$$

The second segment corresponds to the following column vector, at the time moment $\tau_2 = 6{,}67$:

$$\theta_2 = \begin{bmatrix} -15{,}2 & 113{,}22 & 14{,}16 & -14{,}16 & -0{,}206 \end{bmatrix}^T \quad (3)$$

The end of phase I, respectively of the third segment, is represented by the following column vector, at the moment $\tau_3 = 10$:

$$\theta_3 = \begin{bmatrix} -15{,}2 & 116{,}82 & -1{,}3 & -1{,}4 & -0{,}206 \end{bmatrix}^T \quad (4)$$

Note: all the above generalized coordinates are expressed in [deg] and the time is expressed in [s].

3. PRESENTATION OF THE (4-3-4) METHOD OF PATH PLANNING

According to [4], [5] and [10], the trajectory of the 5R robot is determined by the initial and final point of the movement, to which, in order to avoid the collision with the external obstacles, a finite set of intermediate points is added. In the case of trajectories of type (4-3-4), the end segments are interpolated with degree 4 polynomials, and the intermediate segments with degree 3 polynomials.

The first trajectory segment is interpolated by polynomial time functions of degree 4 for position, degree 3 for velocities and degree 2 for accelerations.

In the following relations, j = 1÷5 represents the joint index, i = 1÷5 represents the path segment index, and n is the number of path segments into which the phase is divided. According to [3], the interpolation functions for coordinates, velocities and accelerations corresponding to the first segment are:

$$h_{j1}(t) = q_{j1}(t) = a_{j14}t^4 + a_{j13}t^3 + a_{j12}t^2 + a_{j11}t + a_{j10}$$

$$v_{j1}(t) = \frac{\dot{q}_{j1}(t)}{t_1} = \frac{1}{t_1}(4a_{j14}t^3 + 3a_{j13}t^2 + 2a_{j12}t + a_{j11})$$

$$a_{j1}(t) = \frac{\ddot{q}_{j1}(t)}{t_1^2} = \frac{1}{t_1^2}(12a_{j14}t^2 + 6a_{j13}t + 2a_{j12}). \quad (5)$$

The variable:

$$t = \frac{\tau - \tau_{i-1}}{t_i} \qquad (6)$$

is called the normalized time and it can take values in the range [0, 1]. The length of the time interval corresponding to the segment is denoted by:

$$t_i = \tau_i - \tau_{i-1} \qquad (7)$$

$h_{ji}(t)$ represents the normalized polynomial function of interpolation of the motion trajectory for the joint j, segment i;

$v_{ji}(t)$ is the normalized polynomial function for velocities, corresponding to the joint j and the segment i;

$a_{ji}(t)$ designates the normalized polynomial function for accelerations, corresponding to the joint j and the segment i;

a_{jik} represent the polynomial coefficients corresponding to the joint j, the segment i, respectively the order term k from the normalized polynomial function of the generalized coordinate (k = 0÷4 for degree 4 polynomials, respectively k = 0÷3 for degree 3 polynomials).

The last path segment is also interpolated by polynomial functions of degree 4, but for the easier determination of the polynomial coefficients, the following substitution is made:

$$\bar{t} = t - 1; \quad \bar{t} \in [-1, 0]. \qquad (8)$$

Therefore, the polynomial functions are of the form:

$$h_{jn}(\bar{t}) = q_{jn}(\bar{t}) = a_{jn4}\bar{t}^4 + a_{jn3}\bar{t}^3 + a_{jn2}\bar{t}^2 + a_{jn1}\bar{t} + a_{jn0}$$

$$v_{jn}(\bar{t}) = \frac{\dot{q}_{jn}(\bar{t})}{t_n} = \frac{1}{t_n}(4a_{jn4}\bar{t}^3 + 3a_{jn3}\bar{t}^2 + 2a_{jn2}\bar{t} + a_{jn1})$$

$$a_{jn}(\bar{t}) = \frac{\ddot{q}_{jn}(\bar{t})}{t_n^2} = \frac{1}{t_n^2}(12a_{jn4}\bar{t}^2 + 6a_{jn3}\bar{t} + 2a_{jn2}), \quad (9)$$

Expressed in the variable $t = \bar{t} + 1$, they can be described as follows:

$$\begin{aligned} h_{jn}(t) = q_{jn}(t) &= a_{jn4}t^4 + (-4a_{jn4} + a_{jn3})t^3 + (6a_{jn4} - 3a_{jn3} + a_{jn2})t^2 + \\ &+ (-4a_{jn4} + 3a_{jn3} - 2a_{jn2} + a_{jn1})t + (a_{jn4} - a_{jn3} + a_{jn2} - a_{jn1} + a_{jn0}) \\ v_{jn}(t) = \frac{\dot{q}_{jn}(t)}{t_n} &= \frac{1}{t_n}[(4a_{jn4}t^3 + (-12a_{jn4} + 3a_{jn3})t^2 + (12a_{jn4} - 6a_{jn3} + 2a_{jn2})t + \\ &+ (-4a_{jn4} + 3a_{jn3} - 2a_{jn2} + a_{jn1})] \\ a_{jn}(t) = \frac{\ddot{q}_{jn}(t)}{t_n^2} &= \frac{1}{t_n^2}[12a_{jn4}t^2 + (-24a_{jn4} + 6a_{jn3})t + (12a_{jn4} - 6a_{jn3} + 2a_{jn2})] \end{aligned} \qquad (10)$$

The intermediate segments, $i = 2 \div n-1$, have the following form of interpolation polynomial functions:

$$h_{ji}(t) = q_{ji}(t) = a_{ji3}t^3 + a_{ji2}t^2 + a_{ji1}t + a_{ji0}$$

$$v_{ji}(t) = \frac{\dot{q}_{ji}(t)}{t_i} = \frac{1}{t_i}(3a_{ji3}t^2 + 2a_{ji2}t + a_{ji1})$$

$$a_{ji}(t) = \frac{\ddot{q}_{ji}(t)}{t_i^2} = \frac{1}{t_i^2}(6a_{ji3}t + 2a_{ji2}). \quad (11)$$

By imposing the restrictive, initial, final and continuity conditions, results a number of (4 + n) coefficients determined by direct calculation. The remaining 4 + 3 (n - 2) coefficients are determined by solving a system of 4 + 3 (n - 2) equations.

In the simplest case, when there is only one intermediate segment, the total number of polynomial coefficients is 14, of which 7 result directly, and 7 will be determined by solving a system of 7 equations with 7 unknowns.

4. DIVIDING INTO PATH SEGMENTS

To make it possible to apply the path planning method of type (4-3-4), phase I will be divided into a number of three path segments. Two of them, namely the end segments, will be modeled by degree 4 polynomials, and the intermediate segment by degree 3 polynomial. Table 1 shows the variation of the generalized coordinates q_i, i = 1 ÷ 5, with respect to the real time, τ.

Table 1

Variation of generalized coordinates during phase I of the technological process

τ_i, [s]	q_1, [°]	q_2, [°]	q_3, [°]	q_4, [°]	q_5, [°]
0,00	0,00	0,00	0,00	0,00	0,00
3,33	-7,60	56,61	7,08	-7,03	-0,103
6,67	-15,20	113,22	14,16	-14,06	-0,206
10,00	-15,20	116,82	-1,30	1,40	-0,206

5. IMPOSING THE RESTRICTIVE CONDITIONS IN COORDINATES, VELOCITIES AND ACCELERATIONS

The start and end points of each phase are characterized by zero speed and acceleration. Therefore, denoting with τ_0 the initial moment of the first phase, τ_1 and τ_2 the intermediate moments, corresponding to the passing through the intermediate points, respectively τ_3 the final moment, the following clarifications can be made.

For the 1st joint, at the initial moment, $\tau_0 = 0$ (t = 0), the initial conditions are established:

$$h_{11}(0) = 0; \qquad v_{11}(0) = 0; \qquad a_{11}(0) = 0. \quad (12)$$

At the intermediate moment $\tau_1 = 3{,}33\,\mathrm{s}$ (t = 1), the value of the generalized coordinate is imposed:

$$h_{11}(1) = h_{12}(0) = -7{,}60°, \quad (13)$$

and the conditions of continuity in velocities and accelerations are established:

$$v_{11}(1) = v_{12}(0); \qquad a_{11}(1) = a_{12}(0). \quad (14)$$

At the intermediate moment $\tau_2 = 6{,}67$ s ($\bar{t} = -1$), the value of the generalized coordinate is imposed:

$$h_{12}(1) = h_{13}(-1) = -15{,}20°, \quad (15)$$

and the conditions of continuity in velocities and accelerations are established:

$$v_{12}(1) = v_{13}(-1); \qquad a_{12}(1) = a_{13}(-1). \quad (16)$$

At the final moment $\tau_3 = 10\,\mathrm{s}$ ($\bar{t} = 0$), the final conditions are established:

$$h_{14}(0) = -15{,}20°; \quad v_{14}(0) = 0; \quad a_{14}(0) = 0 \quad . \quad (17)$$

By imposing these restrictive conditions, a part of the polynomial coefficients results by direct calculation.

6. DETERMINATION OF POLYNOMIAL INTERPOLATION FUNCTIONS

In order to easily determine all the polynomial coefficients, the inter434() function is used, which was defined in the [3]. This function is called for each phase, each segment, and each joint, respectively.

The input data of the function are the following:
n - the number of trajectory segments;
c_id - joint identifier;
qmin, qmax - limits in generalized coordinates;
qzero - zero position;
deltaq - the vector of relative displacements;
tau - the absolute time vector;
qdot, qddot - restrictions in velocities and accelerations at the ends of the phase;
inc_timp – a time increment.

The output data of the function are:
gh1, gv1, ga1, gh2, gv2, ga2, ghn, gvn, gan - vectors with values of generalized coordinates, velocities and accelerations, in order to graphically represent the three segments;
tt - the associated real time vector.

An example of using this function, for the restrictive conditions from §5 is provided in the following code sequence:

```
phase=1;
%% joint 1
c_id=1;
% joint limitations [degrees]
qmin=-160*deg;
qmax=160*deg;
% zero position
qzero=0;
% no. of configurations
m=3;
% defining the configurations
% by relative displacements [degrees]
deltaq=deg*[-15.2/2 -15.2/2 0];
% absolute time vector [s]
tau=1/2*[0 6.67 13.33 20];
% restrictions at the phase ends
qdot(0+1)=0;
qddot(0+1)=0;
qdot(m+1)=0;
qddot(m+1)=0;
% calculation of generalized
% coordinates, velocities, accelerations
[gh1, gv1, ga1, gh2, gv2, ga2, ghn, ... gvn,
gan, tt] = inter434(m, c_id, ... qmin, qmax,
qzero, deltaq, tau, ...
qdot, qddot, inc_timp);
```

The polynomial coefficients are determined in the inter434() function and saved in a log file, being contained in the matrix a_{3x5}.

The three lines of the matrix a represent the trajectory segments, and the columns represent the order of the coefficients in the polynomial functions, in reverse order: column 1 represents the free term, column 2 the coefficient of t, ... column 5 the coefficient of t^4.

For example, the steps for determining the polynomial functions for joint 1, phase I, segments 1-3, in normal time are presented in detail.

The first segment is characterized by the following polynomial functions, expressed in relation to the normalized time:

$$h_{11}(t) = q_{j1}(t) = 0{,}1659t^4 - 0{,}2985t^3$$
$$v_{11}(t) = \frac{\dot{q}_{j1}(t)}{t_1} = \frac{1}{3{,}33}(4 \cdot 0{,}1659t^3 - 3 \cdot 0{,}2985t^2)$$
$$a_{11}(t) = \frac{\ddot{q}_{j1}(t)}{t_1^2} = \frac{1}{3{,}33^2}(12 \cdot 0{,}1659t^2 + 6 \cdot 0{,}2985t). \quad (18)$$

The second segment is described by the following polynomial functions:

$$h_{12}(t) = -0{,}0002 \cdot t^3 + 0{,}1 \cdot t^2 - 0{,}2324 \cdot t - 0{,}1326$$
$$v_{12}(t) = \frac{1}{3{,}33}(-3 \cdot 0{,}0002 \cdot t^2 + 2 \cdot 0{,}1 \cdot t - 0{,}2324) \quad (19)$$
$$a_{12}(t) = \frac{1}{3{,}33^2}(-6 \cdot 0{,}0002 \cdot t + 2 \cdot 0{,}1).$$

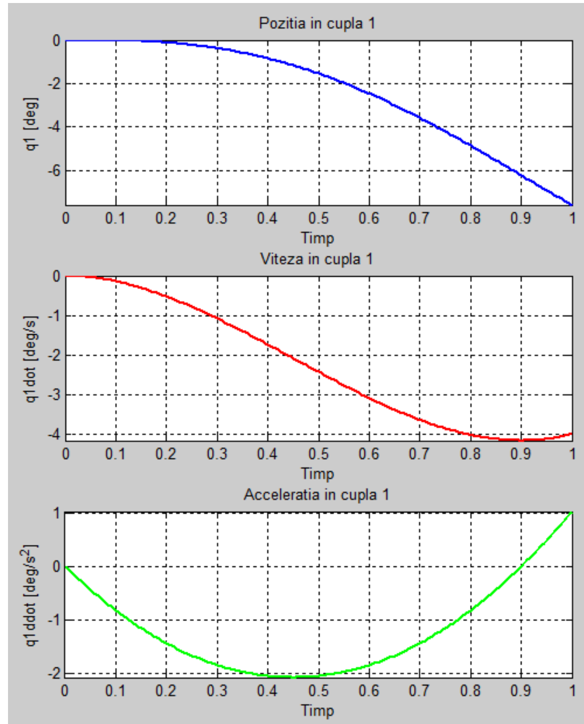

Fig. 2. Polynomial interpolation functions for generalized coordinates, velocities and accelerations, corresponding to phase I, joint 1, segment 1, in normalized time

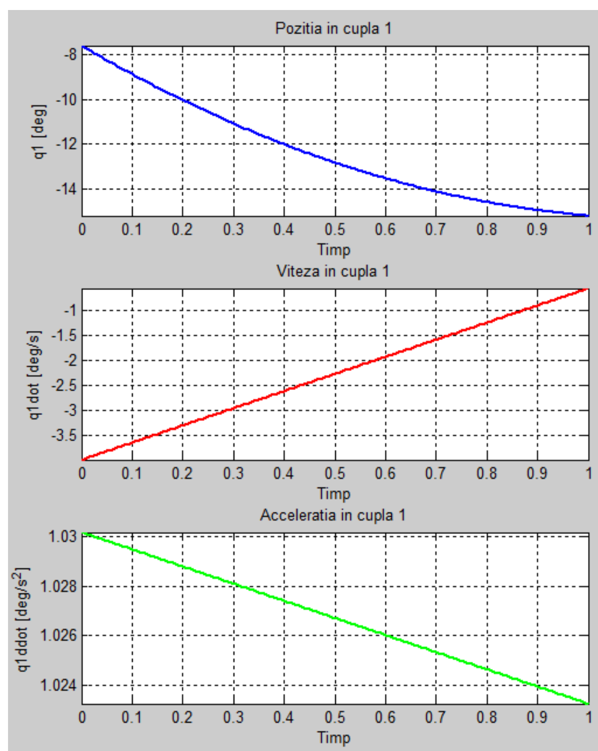

Fig. 3. Polynomial interpolation functions for generalized coordinates, velocities and accelerations, corresponding to phase I, joint 1, segment 2, in normalized time

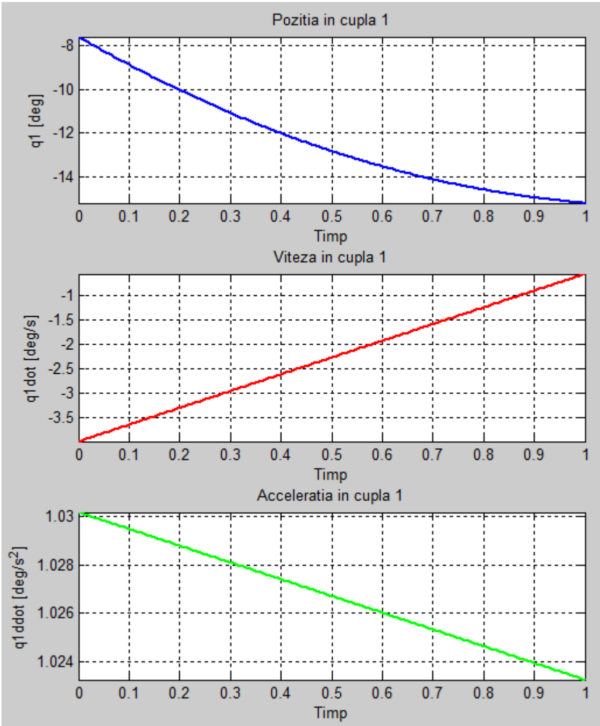

Fig. 4. Polynomial interpolation functions for generalized coordinates, velocities and accelerations, corresponding to phase I, joint 1, segment 3, in normalized time

The third segment is characterized by the following polynomial functions:

$$
\begin{aligned}
h_{13}(\bar{t}) &= 0{,}0331 \cdot \bar{t}^4 + 0{,}0331 \cdot \bar{t}^3 - 0{,}2653 \\
v_{13}(\bar{t}) &= \frac{1}{3{,}33}(4 \cdot 0{,}0331 \cdot \bar{t}^3 + 3 \cdot 0{,}0331 \cdot \bar{t}^2) \\
a_{13}(\bar{t}) &= \frac{1}{3{,}33^2}(12 \cdot 0{,}0331 \cdot \bar{t}^2 + 6 \cdot 0{,}0331 \cdot \bar{t}).
\end{aligned}
\qquad (20)
$$

Their graphical representation is made in figures 2-4.

Figure 5 represents the polynomial interpolation functions for generalized coordinates, velocities and accelerations, corresponding to phase I, joint 1, in real time.

Next, in figures 6-8, the graphs of the interpolation functions for the coordinates, velocities and generalized accelerations, in real time, for the joints 2-5, phase I are presented.

These functions will be used in a further development of the work, to determine the trajectory, operational velocities and accelerations, respectively the generalized driving forces, corresponding to phase I of the technological process.

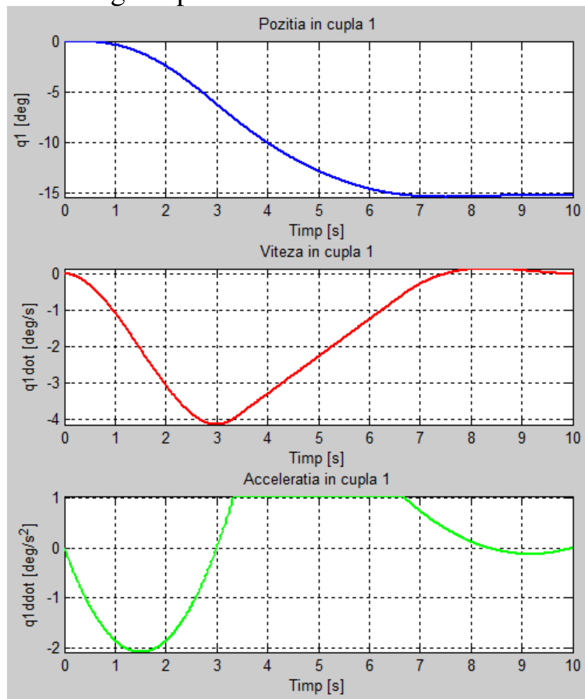

Fig. 5. Polynomial interpolation functions for generalized coordinates, velocities and accelerations, corresponding to phase I, joint 1, in real time

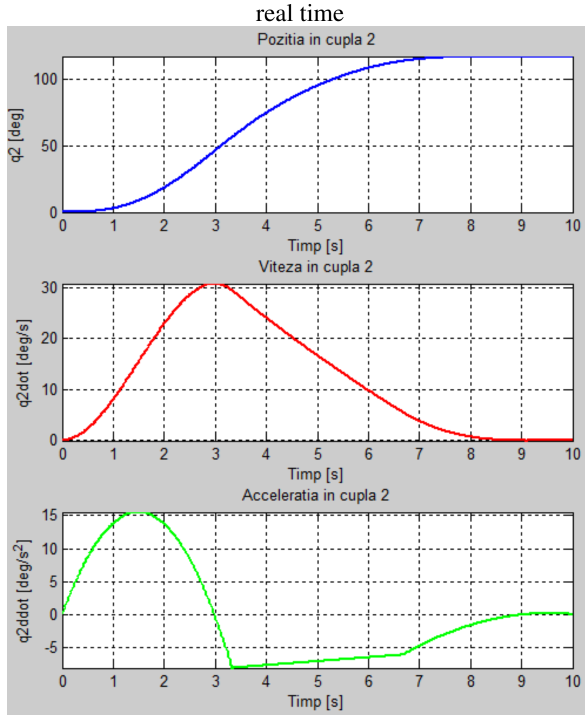

Fig. 6. Polynomial interpolation functions for generalized coordinates, velocities and accelerations, corresponding to phase I, joint 2, in real time

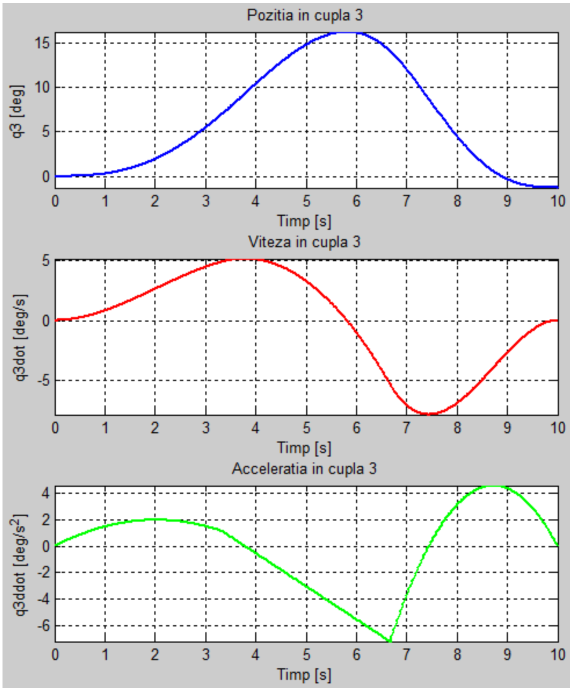

Fig. 7. Polynomial interpolation functions for generalized coordinates, velocities and accelerations, corresponding to phase I, joint 3, in real time

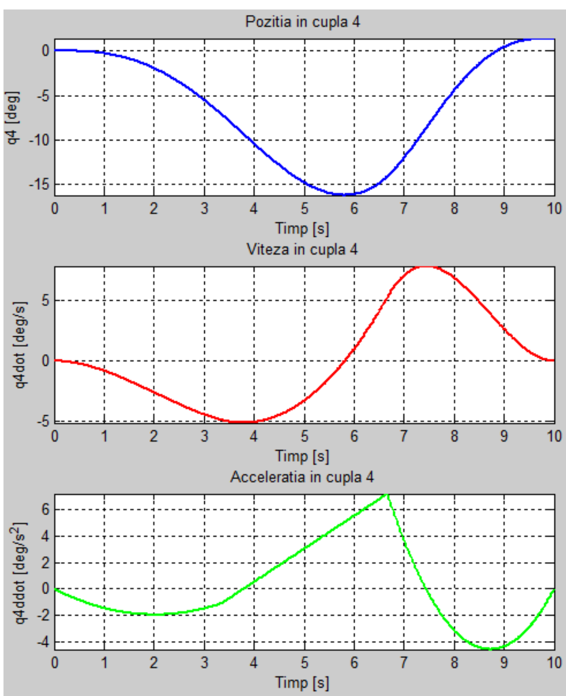

Fig. 8. Polynomial interpolation functions for generalized coordinates, velocities and accelerations, corresponding to phase I, joint 4, in real time

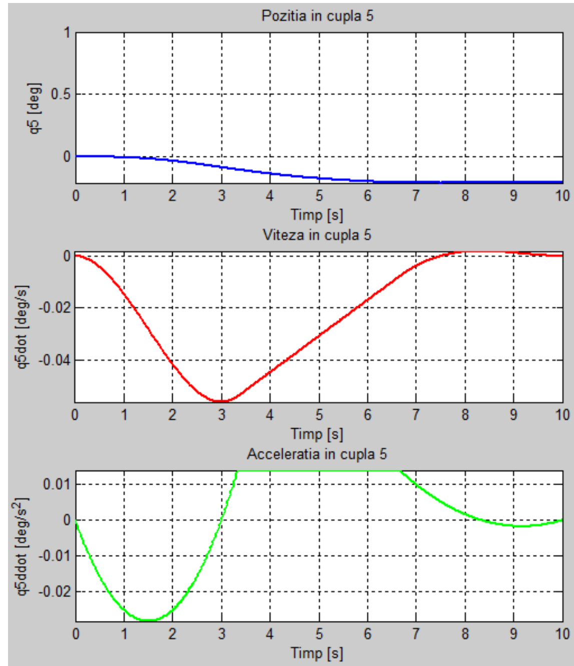

Fig. 9. Polynomial interpolation functions for generalized coordinates, velocities and accelerations, corresponding to phase I, joint 5, in real time

7. CONCLUSION

The steps presented in this paper are an important part of the 5R articulated robot trajectory planning algorithm. Based on these steps, the polynomial interpolation functions, necessary for the robot control, will be determined. The originality of this work consists in applying the MATLAB environment of the modeling method on this particular articulated robot model, implemented in a welding process of large cylindrical parts, whose axis of symmetry coincides with the first axis of rotation of the robot. To carry out the welding process, the robot can be mounted both on the floor and upside-down.

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Considerații despre planificarea traiectoriei de mişcare a unui robot 5R, implementat într-un proces de sudare

Articolul descrie procesul de planificare a traiectoriei de mișcare a robotului industrial articulat 5R utilizat în procesele de sudare. Ciclul de lucru este prezentat și împărțit în faze. Metoda (4-3-4) de planificare a mişcării este descrisă şi aplicată pentru o fază a procesului tehnologic. Faza este apoi împărțită în segmente de traiectorie și se stabileşte tabelul de variație a coordonatelor generalizate. Se impun restricţiile în coordonate, viteze și accelerații, utilizate ca date de intrare în algoritmul de modelare a traiectoriei. Ca exemplu, sunt determinate funcţiile polinomiale de interpolare pentru prima fază a procesului tehnologic, precum și graficele corespunzătoare acestora, în timp normalizat.

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