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SOFTWARE FOR GENERATING A SPUR GEAR'S INVOLUTE PROFILE AND ALTERNATIVE MANUFACTURING METHOD

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Abstract: *The paper analyzes the state of the art in regard to methods of manufacturing spur gears differently than the conventional ones such as hobbing, shaping and skiving. Current research concludes that additive manufacturing cannot be used to properly make gears because their profiles require higher accuracy. The most competitive processes have proven to be different types of milling. Most papers present the milling of gears using expensive 4 or 5 axis machining centers with the end mills oriented radially to the blank. The current paper shows that spur gears can be manufactured efficiently form a time and cost point of view by using a standard 3 axis CNC milling machine with conventional end mills. The end mills are oriented axially to the blank and the only limiting factor is the tool's usable length. A proprietary software was developed to calculate the involute profile of any spur gear using Matlab and a method of generating the CAD model is presented. The machining process was simulated using CAMWorks. A comparison of the costs of the proposed alternative manufacturing method is made with actual spur gear prices and it shows that standard 3 axis CNC machining becomes progressively profitable for gears of large module and/or numbers of teeth.*

Key words: *spur gear milling, gear design software, involute profile calculation, 3 axis machining, gear milling costs.*

1. INTRODUCTION

The current industrial market is mostly centered on series production. But there are small companies which require only a few pieces of a certain component to be manufactured and it's difficult for them to gain access to specialized manufacturing facilities at sustainable costs.

This is also the case when small companies need gears. They might have a hard time finding a local machine shop that has the machines and the know-how to manufacture gears or a local supplier which imports low numbers of parts at appropriate prices and in a short amount of time. However, even the most complex gears can be manufactured using a 5 axis milling machine and the required tools. Spur gears have an even simpler shape and can be machined on a 3 axis CNC milling machine as long as the width of the gear doesn't exceed the usable length of the end mill being used. This opens up a lot of possibilities since many machine shops have 3 axis mills and the CAM software needed to program complex surfaces.

Such machines could mill simple spur gears, but also gears that feature crowning or end relief. An entire array of gears from small to large can be machined. The blanks could be pre-hardened steels or case hardening steels that are semifinished, then hardened and then finished. The end mills would need to be suitable for machining hard steels, but other than that they could have standard sizes which would make them affordable and quick to purchase. This probably can't be said of hobs which are quite complex and expensive.

Paper [1] compares technologies used in the fast prototyping of spur gears. They review both conventional and additive technologies such as milling, wire edm and selective laser melting. Milling offers the best results in terms of cost and quality, however a 5 axis machine is used to mill the profile from a direction perpendicular to the cylindrical blank's main axis. This results in a large number of finishing passes in order to decrease the surface roughness which consequently increases manufacturing time.

Paper [2] reviews new strategies for manufacturing gears using 5-axis CNC milling machines or turn-mill machines. It describes the characteristics of two approaches, InvoMilling by Sandvik and GearMill by DMG. Both are CAD/CAM solutions tailor made for gear machining and can utilize a range of tool types and strategies to develop the machine program for making cylindrical and conical gears. Even though these approaches are very flexible in terms of results and provide accurate gears, they still require specialized and expensive software packages and even more costly 5-axis milling machines.

Paper [3] presents a solution for milling spur gears using end mills that are parallel to the axis of the gear. The paper uses a macro program of the available milling machine, Dyna 2009 Myte, and produces a plastic gear. It proves the manufacturing concept, but doesn't provide a calculation of costs, a comparison with market prices and an analysis of the part accuracy.

The current paper presents a low-cost solution for manufacturing spur gears using a simple 3-axis CNC machine and conventional and readily-available end mills. The gears are machined with end mills that are parallel to the axis of the gear, such as in paper [3]. However, a proprietary piece of software was developed to generate the tooth flank profiles using Matlab. It generates points of the involute profile for a certain spur gear which can then be used to make a CAD model of the respective gear. The CAD model can be used to generate the machine code for the actual machining with the aid of a CAM software, such as CAMWorks. An example of such a calculation and machine program for a spur gear are presented. Manufacturing costs were calculated for a series of gears based on machining time and industry data. Even though the software cannot design and machine helical gears or bevel gears, it is a simple, fast and cost effective solution for making spur gears which have numerous applications.

2.EQUATIONS USED FOR CALCULATING THE INVOLUTE PROFILE OF THE GEAR

The involute profile is calculated as a series of points starting with the tooth thickness measured as an arc and using the equations described by [4]. At first the reference, base, tip and root diameters are calculated using the following equations:

$$d_{ref} = m \cdot z \quad (1)$$

$$d_{base} = d_{ref} \cdot \cos\alpha \quad (2)$$

$$d_{tip} = d_{ref} + 2 \cdot m \quad (3)$$

$$d_{root} = d_{ref} - 2.5 \cdot m \quad (4)$$

where: m - gear modulus; α - pressure angle.

The length of the arc on the reference diameter that corresponds to the tooth thickness is calculated next:

$$S_{tref} = \frac{\pi \cdot d_{ref}}{2 \cdot z} \quad (5)$$

The pressure angle α_y and the arc thickness of the tooth Sty at any given radius r_y are determined with the following equations.

$$\alpha_y = \mathrm{acos}\left(\frac{r_b}{r_y}\right) \quad (6)$$

$$S_{ty} = d_y \cdot \left(\frac{S_{tref}}{d_{ref}} + \mathrm{inv}\,\alpha - \mathrm{inv}\alpha_y\right) \quad (7)$$

Function inv represents the involute of the angle in radians:

$$\mathrm{inv}\,\alpha = \tan\alpha - \alpha \quad (8)$$

The angle formed by the arc S_{ty} and the center of the gear is calculated as:

$$\theta_1 = \frac{S_{ty}}{r_y} \quad (9)$$

The coordinates of the points on the involute profile of the tooth are obtained using the following equations.

$$x = r_y \cdot \sin\frac{\theta_1}{2} \quad (10)$$

$$y = r_y \cdot \cos\frac{\theta_1}{2} \quad (11)$$

3. THE MATLAB PROGRAM

CAD software packages, such as Solidworks, don't typically include functions for designing spur gears with an involute profile or have a series of toolbox components which approximate this profile. This is a functionality rarely required, so it was never implemented by the CAD developers.

A piece of software was developed using Matlab that determines the involute profile of a flank of one tooth of the gear. The program takes as input the number of teeth of the gear z, the module m and the pressure angle α. It then calculates coordinates x and y of points on the tooth flank taking r_y in the interval $d_{base}/2$ to $d_{tip}/2$ with an

increment i that is defined by the user. The smaller the increment, the more precise the involute profile becomes. It is important to remember that the precision needed is finite, since milling machines with a good setup can hold tolerances of around 0.01 mm at best.
The program outputs the calculated point coordinates x and y and adds a third coordinate of 0 corresponding to the z axis. This is necessary because Solidworks requires three coordinates for the points through which it constructs a curve.

4. MAKING THE CAD MODEL OF THE GEAR USING THE MATLAB PROGRAM

The following gear was chosen as an example to test and showcase the gear calculation and modelling method: a gear with 20 teeth, a module of 4 and a pressure angle of 20°.
Solidworks was used to build the CAD model of the gear starting from the involute flank of the tooth calculated with Matlab and the base diameter.

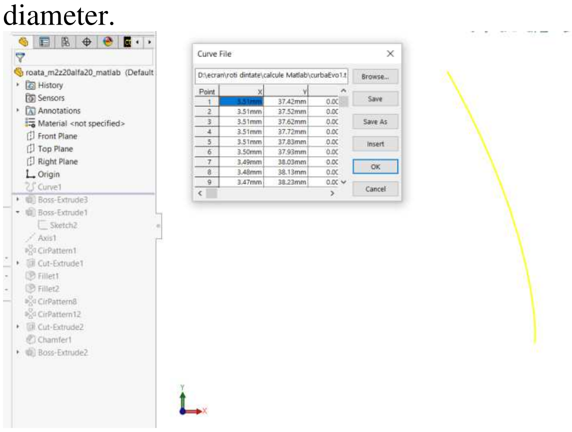

Fig. 1. Inserting the flank spline curve

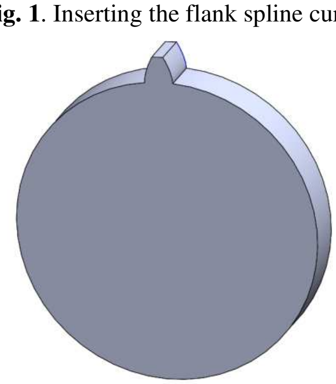

Fig. 2. Extruding one tooth

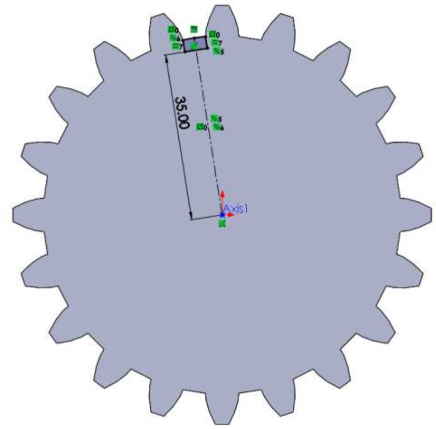

Fig. 3. Cutting the root relief

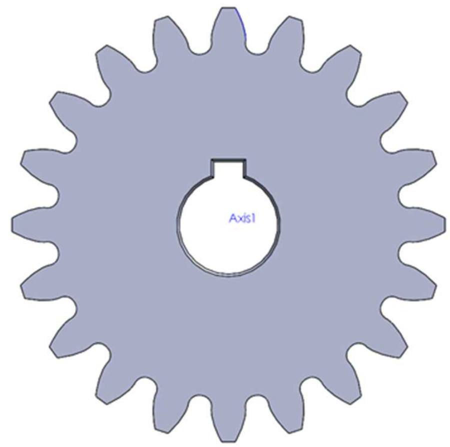

Fig. 4. Finished gear

The following steps are applied to build the CAD model:

- a spline curve is inserted using the calculated set of points with coordinates x, y and 0 (fig. 1).
- the base circle is drawn and extruded next.
- one tooth is extruded based on the involute curve (fig. 2).
- a circular pattern of the complete number of teeth is made.
- the root relief between two adjacent teeth is cut and rounded (3).
- a circular pattern of the reliefs is made.
- the central part of the gear is modified to correspond to the shaft assembly method (e.g.: parallel key).

The finished gear can be seen in figure 4.

5. CAD MODEL OF THE GEAR USING THE GRAPHICAL METHOD

Solidworks is once again used to build the same gear, only this time based on the graphical method. The graphical method simulates unwinding a chord from the base circle in order to draw the profile of the one tooth flank. This is actually done by choosing a starting point for one tooth's flank and choosing increasingly longer chords of the base circle. These are then measured and redrawn as tangents to the circle. The end points of the tangents that don't touch the base circle represent point of the involute profile of the tooth. This profile is next mirrored in order to create the entire tooth and patterned to finish the entire gear.

Figure 5 shows the graphical construction of the tooth's flank.

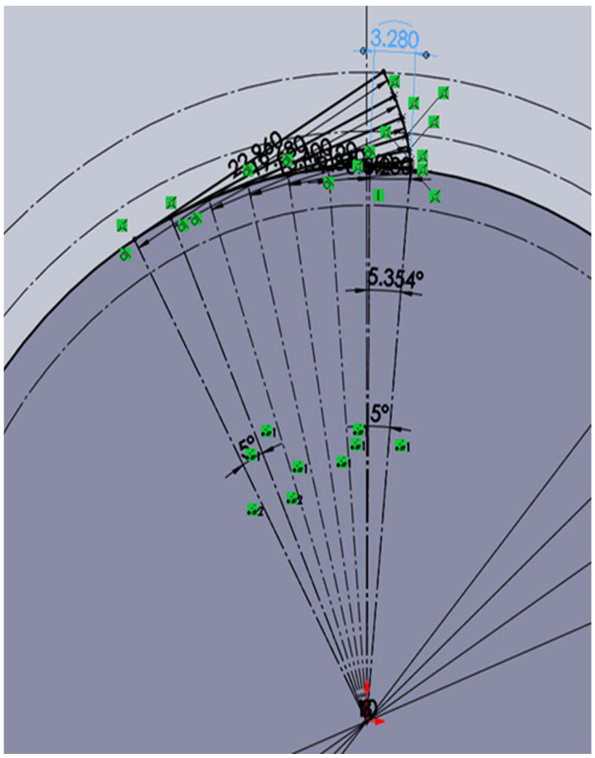

Fig. 5. Graphical construction of the involute profile

6. COMPARISON OF THE MATLAB AND GRAPHICALLY DESIGNED INVOLUTE PROFILE

The flanks of the gear designed using Matlab calculations and the one based on the graphical method were compared, both visually and numerically. Figure 6 shows both profiles superimposed, and no difference can be seen. There are numerical differences, which are presented in table 1, but they are small, reaching a maximum value of 1.2 µm, which is in the range of very precise grinding. This fact supports the validity of the Matlab calculated tooth flank.

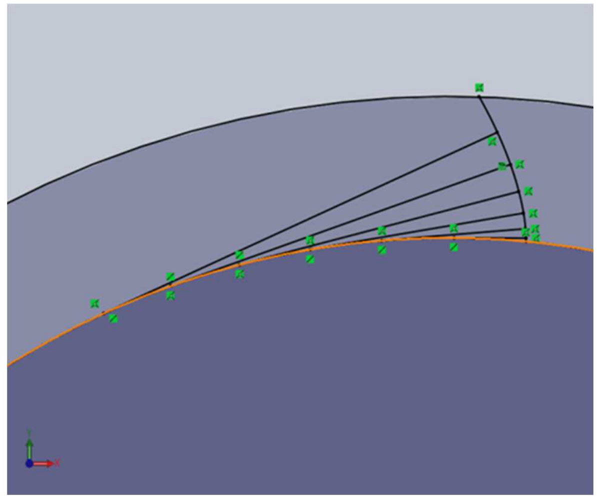

Fig. 6. The superimposed Matlab and graphical method involute tooth flank

Table 1

Comparison of profile points obtained by the two methods

No.	Matlab		Graphical method		Distance
	x	y	x	y	
1	3.5072	37.4237	3.5073	37.4235	0.0002
2	3.5124	37.5665	3.5122	37.5665	0.0002
3	3.4942	37.9954	3.4939	37.9953	0.0003
4	3.4031	38.7049	3.4027	38.7049	0.0004
5	3.1904	39.6836	3.1898	39.6835	0.0006
6	2.8088	40.9135	2.8081	40.9133	0.0007
7	2.2136	42.3705	2.2128	42.3701	0.0008
8	1.3895	43.9781	1.3883	43.9781	0.0012
				max:	**0.0012**

7. CHECKING THE INTERFERENCE BETWEEN THE TWO MODELS USING A CAD ASSEMBLY

An assembly was made in Solidworks using two pieces of the gear designed with the proprietary software. Figures 7 and 8 show how the gears mesh.

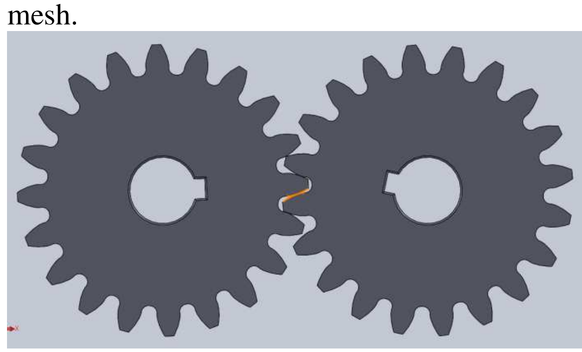

Fig. 7. Front view of the gears meshing

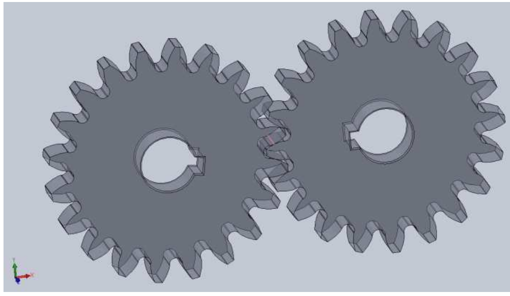

Fig. 8. Perspective view of the gears meshing

Solidworks detects interference when a tooth engages and disengages with the other gear, but this has a very small value 3.3e-6 mm^3 and is not encountered when moving away from the first or last point of contact. This could be due to actual interference of the calculated profiles or it could be just an approximation error resulting from the tangency condition being applied to the ends of the involute profile of the flank.

8. CAM SOFTWARE MANUFACTURING SIMULATION

Camworks was used to calculate the tool paths for milling two spur gears:

- 20 teeth, module 4 and pressure angle 20°;
- 20 teeth, module 8 and pressure angle 20°.

The machining process was simulated. The tools used for the simulation were extra-long 4 flute end mills made by Guehring. The machining data recommended in the company's catalog [5] was used.

The strategy consisted of choosing a larger end mill, 6 mm in diameter, for roughing to spaces between the teeth first. Then a 4 mm diameter end mill was chosen to rough the remaining material at the root of the tooth. Finally, the same 4 mm end mill was used for finishing the profile.

Figures 9 to 14 show the two roughing operations and the finish machining of the gear's profile simulated using CAMWorks for the module 4 gear with 20 teeth. The pictures show the end mills' paths and the rest material after each operation.

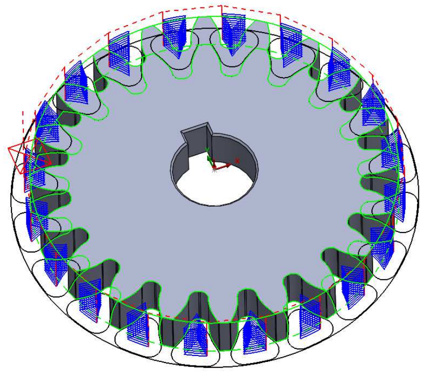

Fig. 9. Roughing using the larger diameter end mill

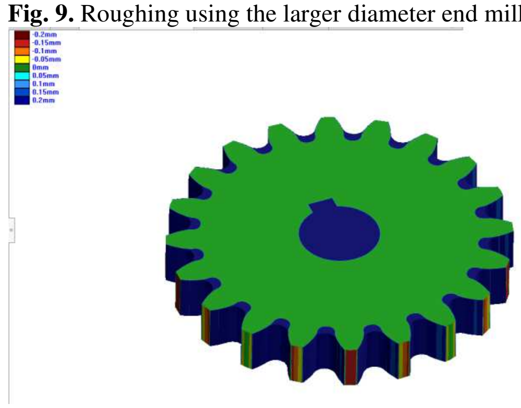

Fig. 10. Rest material after roughing using the larger diameter end mill

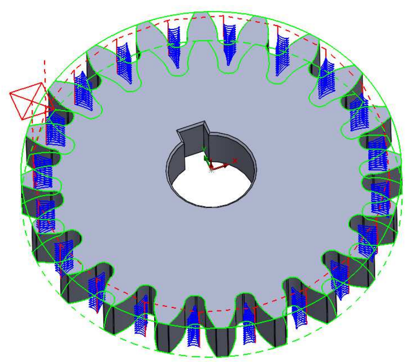

Fig. 11. Roughing the rest using the smaller diameter end mill

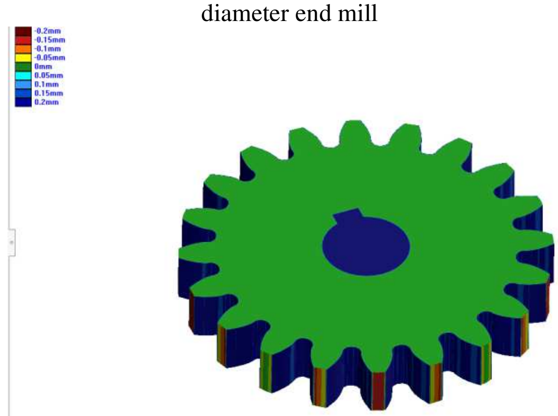

Fig. 12. Rest material after roughing using the smaller diameter end mill

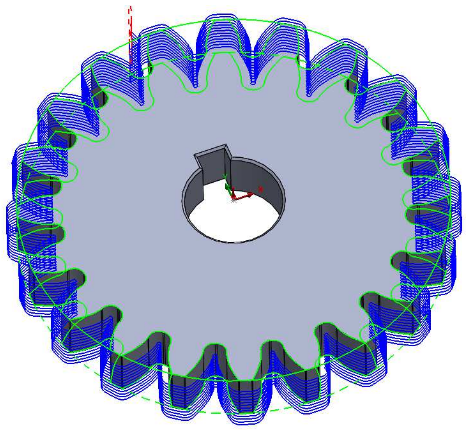

Fig. 13. Finishing the gear using the smaller end mill

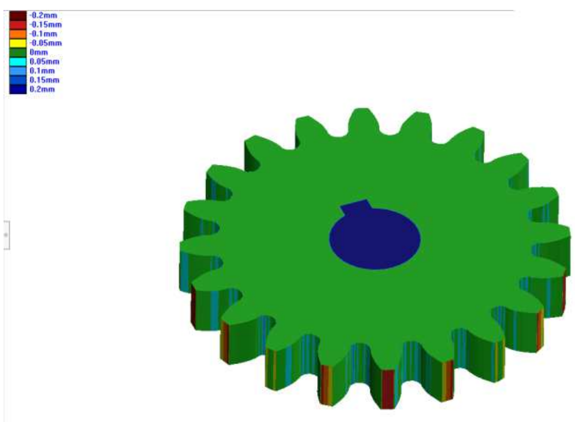

Fig. 14. Rest material after finishing the gear using the smaller end mill

9. SPUR GEAR MILLING COSTS

The authors have calculated the cost of milling spur gears on a 3-axes milling machine. The initial assumption of this paper is that the shop that wants to manufacture a small number of gears already owns the milling machine, so the machine's cost wasn't added to the calculation.
The following costs were considered:
the workpiece material.
an average cost of 50 euro/hour for the milling which includes tooling, programming and operator supervision.
The considered material is medium carbon C45 steel. This is a common steel used for the manufacturing of gears since it is relatively low cost, readily available and can be heat treated by quenching and tempering for increased hardness. This is supplied in hot rolled bar stock and has a price of 1420 euro/tonne [6]. Before milling, cutting of the workpiece from the bar is carried out. This operation wasn't taken into consideration when calculating the costs.
The milling consists of three operations:
roughing of the tooth spaces using a larger diameter end mill;
roughing of the tooth spaces using a smaller diameter end mill;
finishing of the tooth faces using the same smaller diameter end mill.
The authors have simulated the machining of 2 gear models, of modules 4 and 8, both of 20 teeth and 20° pressure angle using CAMWorks in order to calculate the machining time. They have extrapolated these machining times to numbers of teeth 50 and 100 for both modules. A total of 6 gears was obtained represented by combinations of modules 4 and 8 and numbers of teeth 20, 50 and 100 . Costs were calculated based on blank size and machining time. Actual industry prices for these spur gears were collected from the sites of companies such as HPC Europe (all module 4 gears) and KHK Gears USA (20 and 50 teeth module 8 gears). The data are presented in tables 2 and 3. The price for the 100 teeth module 8 gear was extrapolated based on the other two module 8 gears.

Table 2

Comparison between the manufacturing cost and the price of spur gears of module 4

Module 4									
No of teeth	Time for op. 1	Time for op. 2	Time for op. 3	Total time [min]	Blank mass [kg]	Blank cost [EUR]	Milling cost [EUR]	Total cost [EUR]	HPC Europe cost [EUR]
20	11.3	12.3	14.8	38.4	1.897	2.2	32.000	34.23	26.93
50	28.3	30.8	37.0	96.0	10.596	12.5	80.000	92.46	120.74
100	56.5	61.5	74.0	192.0	40.770	47.9	160.000	207.95	417.40

Table 3

Comparison between the manufacturing cost and the price of spur gears of module 8

Module 8									
No of teeth	Time for op. 1	Time for op. 2	Time for op. 3	Total time [min]	Blank mass [kg]	Blank cost [EUR]	Milling cost [EUR]	Total cost [EUR]	KHK-USA [EUR]
20	15.7	22.3	18.0	56.0	15.173	17.8	46.67	64.510	340.754
50	39.3	55.8	45.0	140.0	84.770	99.7	116.67	216.356	1596.553
100	78.5	111.5	90.0	280.0	326.163	383.6	233.33	616.901	3193.107

Figures 15 and 16 show a comparison between prices from an online seller and the costs approximated by the authors. They show that while there isn't a significant difference between the price of small gears calculated in the present paper and the online company, the prices increase disproportionately when assessing gears of increasing module and/or number of teeth.

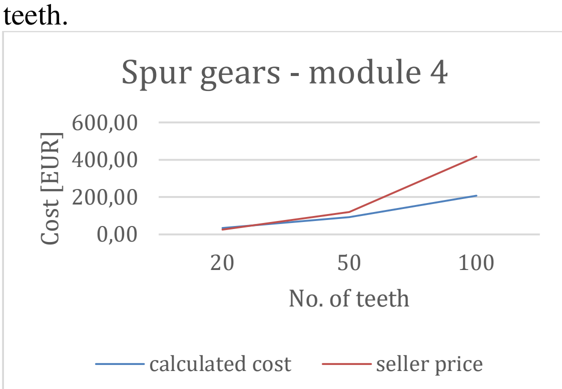

Fig. 15. Calculated cost vs. price for module 4 spur gears

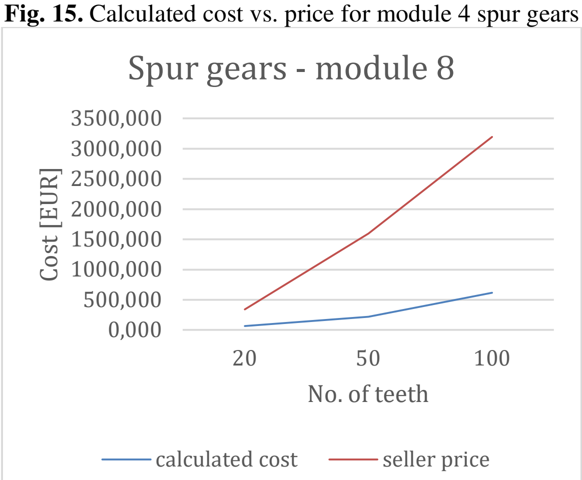

Fig. 16. Calculated cost vs. price for module 8 spur gears

10. CONCLUSIONS

The paper presents a proprietary piece of software designed using Matlab that takes as input the module, the number of teeth and the pressure angle of a spur gear and generates the tooth profile through a series of points. The authors then present a method of constructing a CAD model based on it and compare the proprietary software generated profile with a graphically constructed one. The results prove that the generated profile is accurate and can be used for designing spur gears.

The authors show an example of designing and machining such a spur gear using a 3-axis CNC milling machine and conventional end mills. This data is extrapolated to a total of 6 gears and the machining time and blank sizes are used to calculate manufacturing costs. The costs are compared to prices offered by companies such as HPC Europe and KHK Gears USA. The comparison shows that the method of manufacturing gears described in the present paper is economically viable for spur gears of large modules and/or numbers of teeth. Even though more expensive, smaller gears can be produced in a more timely fashion than would otherwise be possible by buying from a retailer.

It is estimated that the precision of the gears and the surface quality manufactured using a 3-axis CNC machine would be at least as good as the ones obtained by hobbing based on the fact that the end mill is parallel to the axis of the gear and the conclusions presented in Hyatt et al (2014).

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Software pentru generarea profilului evolventic al unei roti dințate cilindrice cu dinți drepți și metodă alternativă de fabricație

Rezumat: *Lucrarea analizează stadiul tehnicii în căutarea de metode alternative de fabricare a roților dințate cilindrice cu dinți drepți față de cele convenționale, cum ar fi frezarea, mortezarea și procedeul skiving. Cercetările actuale concluzionează că fabricarea aditivă nu poate fi folosită pentru a face roti dințate funcționale, deoarece profilele acestora necesită o precizie mai mare. Cele mai competitive procese s-au dovedit a fi diferite tipuri de frezare. Majoritatea lucrărilor prezintă frezarea angrenajelor folosind centre de prelucrare scumpe cu 4 sau 5 axe, cu frezele orientate radial față de semifabricat. Lucrarea actuală arată că roțile dințate drepte pot fi fabricate eficient din punct de vedere al timpului și al costului, folosind o mașină de frezat CNC standard cu 3 axe cu freze convenționale. Frezele sunt orientate axial față de semifabricat și singurul factor limitativ este lungimea utila a sculei. S-a dezvoltat un software propriu pentru a calcula profilul evolventic al oricărei roti cilindrice folosind Matlab și s-a prezentat o metodă de generare a modelului CAD. Procesul de prelucrare a fost simulat folosind CAMWorks. Se face o comparație a costurilor metodei alternative de fabricație propuse cu prețurile reale ale angrenajelor cilindrice și se arată că prelucrarea CNC standard cu 3 axe devine progresiv profitabilă pentru roți dințate cu module mari și/sau număr mare de dinți.*

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