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THEORETICAL MODEL FOR CALCULATING THE ELASTIC AND CONTACT DEFORMATIONS OF THE POSITIONING ELEMENTS OF A 3-2-1 FIXTURE SYSTEM

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Abstract: *Regardless the machining, assembly and inspection domain, the fixture design is crucial its main purpose being to locate accurately and hold the workpiece, preserve these during manufacturing process, without displacements, rotations or deformation of the workpiece. During various operations the clamping forces and cutting forces act on workpiece. The paper presents a theoretical model for calculate the elastic and contact deformations of the positioning elements and the movement of the part under their action in the case of a 3-2-1 fixture system having elastic locators around a rigid rectangular workpiece. The analysis and calculations are performed by means of a software program made in the Python programming language. They help to estimate the various errors that may occur during the manufacturing process, to estimate the reactions on the positioning elements and to estimate the fixing and processing errors.*
Key words: *Fixture design, Location error, Clamping forces, Machining forces, Reaction forces.*

1. INTRODUCTION

The main purpose of a workholding device is to hold the workpiece in a precise location while the manufacturing operation is being performed and within the required tolerance. Fixtures are not met only in case of machining operations but also in welding, assembly, inspection etc.

The most important criteria for fixture are workpiece position accuracy and deformation. A good fixture design minimizes workpiece geometric, machining accuracy errors and must limit deformation of the workpiece, workpiece stability [1], non-interference with the cutting tool [2].

Machining accuracy is directly dependent on workpiece fixture and tooling fixture. The deflections due to clamping and cutting forces during manufacturing operations should be minimum in order to maintain the workpiece displacements within the tolerance-required boundaries. The magnitude and direction of the cutting forces will influence both the location and clamping of the workpiece in the workholder.

Accuracy, deformations and distortions of workpiece can be minimized by optimizing locating scheme (the number, type and disposition of locating elements) and clamping scheme (the number, type and disposition of clamping elements and clamping force magnitude) [3].

The quality of final products is dependent on accuracy, stability and reliability of manufacturing processes. The fixtures performance and costs influences the result of the whole manufacturing process of a product and this justifies the interest in properly design fixtures and think new methods and ways of designing fixtures. There is a vast amount of research done on fixture layout design, the setup planning, fixture planning, unit design, verification, CAFD technique for verifying and improving existing fixture designs is presented in [4], [5], [6], synthesis and optimization of fixture layout using various methods like FEA and GA [1]. A genetic algorithm is proposed, which uses a fitness function that evaluates the positioning error of the workpiece under external forces and torque [7], multi-objective optimization considering locator displacement and force–deformation [8], analyze modular fixture tool contact area deformation and optimize support locations, using finite element analysis (FEA) [9], clamping forces optimization to reduce the location error due to machining forces [10], the problem of compliance of interface between clamping/locating fixture elements and workpiece, under dynamic loads during machining studied by Tadic et al. [3]. Novel methods for optimum workpiece positioning and a geometry-based method is proposed to find the optimal location of the workpiece relative to the fixture system [11].

Workpiece displacement relative to fixturing element was analyzed using analytical solution of the Lagrange differential equations of motion [12], improving workpiece location accuracy in fixture was studied by [10] where reduction in workpiece locating error due to rigid body displacements is achieved through optimal placement of locators and clamps around the workpiece. The development of a novel multi-purpose solution (addressing deformations and displacements) for a modular fixture design with higher efficiency, higher accessibility and flexibility is proposed in [13].

Joel M. et al [14] investigated the influence of friction and contact stiffness between workpiece and fixture parts and the displacement and rotation of the workpiece are investigated as a function of the clamping force. Bo Li and N. Melkote [15] presents a model for improving workpiece location accuracy in fixture device. The reduction in workpiece locating error due to rigid body displacements is achieved through optimal placement of locators and clamps around the workpiece.

The subject of fixture design is intensively debated in specialized literature and efforts in the area of integrating different methodologies established in fixture design area are still of interest [16] and presents an integrated methodology for cohesive fixture design.

2. CALCULATION OF THE REACTIONS AND THE ELASTIC AND CONTACT DEFORMATIONS ON LOCATING PINS

The work presented in this paper is related to previous work on calculation reactions on locating pins based on force and moment separation method using a software program [17]. The software program in written in Python 3.4.0 language and allows a quickly computing of reactions of locating pins using the method described in [17]. The software program was improved and completed with the determination of reactions on the positioning elements in the conditions of the semi-finished product processing and allows also the calculation of their elastic and contact deformations which in turn will generate translations and rotations of the semi-finished product.

The positioning mechanisms of the fixing devices ensure the localization and orientation of the semi-finished products. This positioning, locating of the workpiece and preserve this position, must be maintained under the action of cutting forces and is most often achieved through clamping forces developed by the fixing devices mechanisms.

The precision of the workpiece processing installed in the devices and the deviations of form, orientation, position and dimension of the workpiece are generated by a complex of existing factors and at the same time due to the technological process itself.

The processing errors are due to the workpiece installation in the clamping device, the variation of the forces acting on workpiece during processing, errors obtained after adjusting the technological system, control errors, temperature, tool wear, machine tool inaccuracy [18], [19].

The errors of installing workpiece in the device depend on the precision of their orientation and fixing in the device. The complete elimination of these causes in order to completely eliminate any kind and completely all of the deviations is practically impossible. What can be done, however, is to research these causes in detail and try to reduce or counteract situations that generate deviations.

Reaction forces on the positioning elements

During the installation operation of workpiece in the device, a clamping force *S* will act on workpiece. Due to the limited stiffness of the positioning elements, the forces applied on the workpiece will generate elastic and contact deformations, which will change the initial position of workpiece. The application of the clamping force *S* [*Sx, Sy, Sz*] will therefore generate the fixing errors.

During the processing of the workpiece, the processing force *F* [*Fx, Fy, Fz*] will additionally act on it. In turn this will generate deformations which are called processing errors. These will overlap with the fixing errors contributing to the global errors, called processing errors.

The deformations caused by these forces will change the position of the workpiece, which undergoes translations (*Tx*, *Ty*, *Tz*) and rotations (*Rx*, *Ry*, *Rz*) because the adjustment of the processing operation is made after the installation phase of the semi-finished product in the device. If there is a mathematical model that can estimate these errors, adjustments can be made, but only for workpiece translations. Errors due to its rotations during processing are difficult to compensate. However, their estimation is welcome in order to be able to estimate the processing errors of the part. Workpiece is considered a perfectly rigid body, locators are elastic and they deform under clamping and machining causing part displacement.

A 3-2-1 fixture system having elastic locators around a rigid rectangular workpiece is used. The positioning mechanism of the device is composed of three bases: (BPA), formed by the positioning elements (EP) EP1, EP2 and EP3, (BPD), consisting of EP4 and EP5 and (BPR), consisting of EP6.

It should be noted that the clamping mechanisms of the devices are of two types [20]:

1. Clamping mechanisms with self-braking properties;
2. Clamping mechanisms without self-braking properties.

In the case presented in this paper it was considered the case of the type 2 mechanism, without self-braking and considering an insufficient transversal stiffness of the final clamping element, in which case the frictional force resulting on this fastening element cannot be counted on.

The clamping force *S* and the processing force *F* will generate on EP the reactions *N1,…,N6*, and due to the relative displacement tendencies on EP, the frictional forces $\mu N_1,\ldots,\mu N_6$ (Figure 1) will also appear.

In order to be able to estimate the deformations on EP, it is necessary to calculate first the reactions $N_1 \ldots N_6$. A variant for calculating the reactions on EP [21] is the one that uses the force reduction torsor in the origin of the reference system O. We have a torsor of the active τ_a forces and one of the reactive forces τ_r. In this sense, the forces shown in figure 1 must be reduced in the origin of the reference system O where in order for the workpiece to be in equilibrium, the resultant of the forces *R* and the momentum *Mo* resulting from the origin O of the reference system have to be equal to 0.

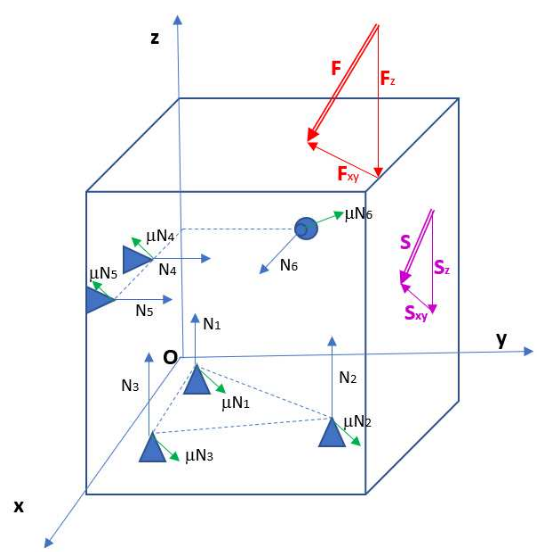

Fig. 1. Forces acting on workpiece

The active forces *S* and *F* will have in origin O the torsor τ_a presented in equation (1):

$$\tau_a: \begin{cases} R_a = (S_x + F_x) \cdot \bar{i} + (S_y + R_y) \cdot \bar{j} + (S_z + R_z) \cdot \bar{k} \\ M_a = (Ms_x + Mf_x) \cdot \bar{i} + (Ms_y + Mf_y) \cdot \bar{j} + (Ms_z + (Ms_z + Mf_z) \cdot \bar{k} \end{cases} \quad (1)$$

EP positions are given by vectors presented in equation (2):

$$\vec{r} = [\vec{r_1}, \vec{r_2}, \vec{r_3}, \vec{r_4}. \vec{r_5}, \vec{r_6}] \quad (2)$$

Each vector has components on the axes of the reference system, so the matrix of the position vectors of the EP is presented in equation (3):

$$Mat_{EP} = \begin{bmatrix} r_{1x} & r_{1y} & r_{1z} \\ r_{2x} & r_{2y} & r_{2z} \\ r_{3x} & r_{3y} & r_{3z} \\ r_{4x} & r_{4y} & r_{4z} \\ r_{5x} & r_{5y} & r_{5z} \\ r_{6x} & r_{6y} & r_{6z} \end{bmatrix} \quad (3)$$

On each EP will act a normal reaction N_i and a friction force μN_i, and they have the following components presented in equation (4):

$$N_i, \mu N_i = \begin{bmatrix} \mu_1 N_1 cos\gamma_{xy} & \mu_1 N_1 sin\gamma_{xy} & N_1 \\ \mu_2 N_2 cos\gamma_{xy} & \mu_2 N_2 sin\gamma_{xy} & N_2 \\ \mu_3 N_{13} cos\gamma_{xy} & \mu_3 N_3 sin\gamma_{xy} & N_3 \\ \mu_4 N_4 cos\gamma_{xz} & N_4 & \mu_4 N_4 sin\gamma_{xz} \\ \mu_5 N_5 cos\gamma_{xz} & N_5 & \mu_5 N_5 sin\gamma_{xz} \\ N_6 & \mu_6 N_6 cos\gamma_{yz} & \mu_6 N_6 sin\gamma_{yz} \end{bmatrix} \quad (4)$$

The resulting friction forces on the EP will be inverse to the displacement tendency on these EPs. The displacement trends will be according to the directions of the projection of the resultant of the forces *S* and *F* (*R*): on the Oxy plane (*Rxy*) for the BPA frictions, on the Oxz plane projection (*Rxz*) for the BPD frictions and respectively on the Oyz plane (*Ryz*) for the friction from BPR. The angles that these projections make with the Ox, Oy, and Oz axes are: γ_{xy}, γ_{yz} and γ_{xz}.

These reaction forces N_i and the friction forces generated by them can be reduced [2] in the origin of the reference system O, forming the reactive torsor τ_r. The equilibrium condition imposes the equality of the two torsos, from where the values of the N_i reactions can be calculated.

The disadvantage of the method is that the mathematical solution of the system can lead to one or more reactions with negative values, or the reactions can only be null or acting on the semi-finished product. There is a variant that the negative reactions are cancel and the other reactions are solved, but we consider that the method becomes cumbersome and that is why we will apply the method of calculating the reactions, presented in [17].

3. WORKPIECE DISPLACEMENT DURING THE MANUFACTURING PROCESS

The N_i reactions on positioning elements allowed the calculation of their elastic and contact deformations. For this, the elastic and contact yields (CECi) of each EP must be known. Experimentally, these CECi values can be obtained, as well as the values of the exponent of the force *n*. EP deformations in this case are expressed as in equation (5):

$$def_i = -CEC_i * N_i^n \quad (5)$$

These deformations will generate translations and rotations of the workpiece. The software allows the graphical representation (Figure 2) of the position of a prismatic workpiece in the phases:

- Placing the workpiece in the device (lack of strength), represented with green color;
- Fixing it by applying the clamping force *S* [*Sx, Sy, Sz*], represented with blue color;
- workpiece processing (acts the clamping force *S* [*Sx, Sy, Sz*] and the processing force *F* [*Fx, Fy, Fz*], represented with red color.

For a proper visualization of workpiece position changing in the phases presented above, the deformations suffered by the positioning elements (EP) were increased by the factor *fmd* (e.g. *fmd* = 300 in the case of the figure 2).

The software also visualizes the clamping force *S* (the blue vector), the processing force *F* (the red vector), the N_i reactions in the processing phase (the orange vectors) with the application points in the red dots (EPi).

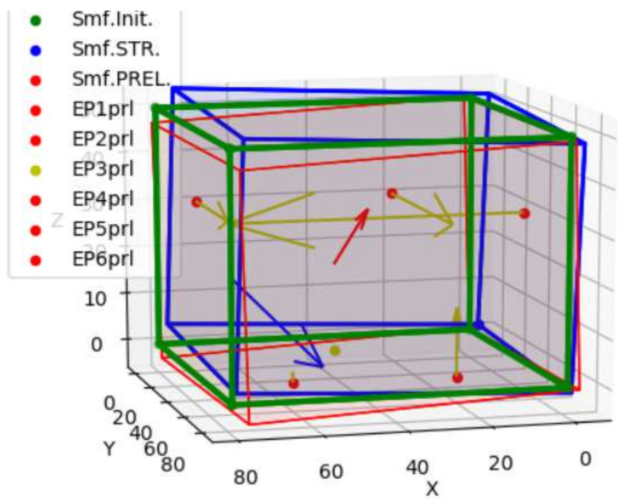

Fig. 2. Workpiece position: initial (green), clamping (blue), processing (red)

If a reaction on the EP becomes zero (does not exist) the point changes colour to orange (in figure 2, one point of BPA is orange, the other two are red). In the orange points there is a possibility that the workpiece will lose contact with EP and the program warns us in writing.

During machining, the position of the force *F* changes due to the trajectory of the machining tool. It was considered a linear trajectory in space *L* [*Lx, Ly, Lz*], which was segmented into *nI* parts (iterations), in the case of work *nI* = 10, thus obtaining for each iteration the values of EP deformations.

The reactions that appear on the EP positioning elements, when processing the workpiece, are shown in figure 3. There is a tendency to increase the reaction on EP2 and EP4. On EP3 the reaction values are low, there is the possibility to loose contact. The reaction on EP1 decreases towards the end of processing, where the risk of losing contact may even occur. An increase of the *Sz* component, of the clamping force is required, in order to eliminate the possible loss of the contact with the BPA elements. Contact with BPD and BPR is maintained in the case of this processing.

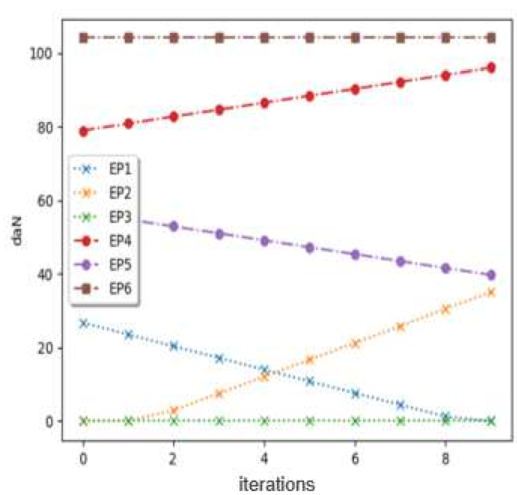

Fig. 3. Reactions on locating pins (EP)

These reactions led to deformations and implicitly to changes in the position of the workpiece during processing, respectively to its rotations *R* [*Rx, Ry, Rz*] around the axes Ox, Oy and Oz, as well as to translations of *T* [*Tx, Ty, Tz*].

The software has the possibility to calculate these *R* and *T* in the cases:

- Clamping position from Initial position;
- Processing position compared to Initial position;
- Processing Position versus Tightening Position;

Since the adjustment of the tool is made relative to the position of the WP in the device, it becomes important to know the change of workpiece position in case of processing compared to the position when it was tightened.

The rotations *Rx*, *Ry* and *Rz* of the semi-finished product around the axes Ox, Oy and Oz, during processing, compared to the position held at tightening, are presented in figure 4.

The translations of the semi-finished product from its position in the tightening phase, during processing L, are presented in figure 5.

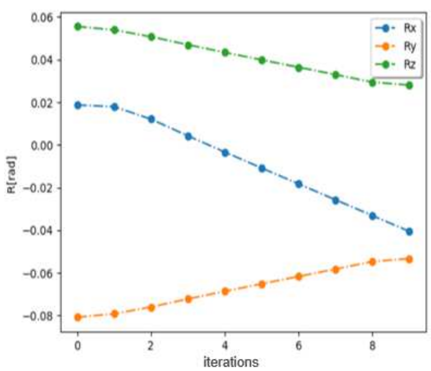

Fig. 4. Rotations of workpiece in the Reference system

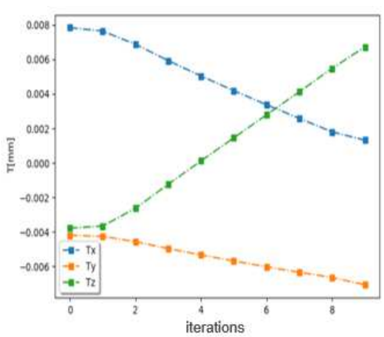

Fig. 5. Workpiece translations regarding its position in clamping position

Counteracting the effect of workpiece rotations during processing is more difficult to achieve. If the machine tool is equipped with an adaptive feed control system, which can control the size of the machining force *F* and its components *Fx*, *Fy*, *Fz*, then a rotations compensation for workpiece could be tried.

4. CONCLUSIONS

During manufacturing it is important to minimize product manufacturing errors and this becomes possible if engineers have software to help them estimate the various errors that occur during the manufacturing process.

The paper presented a software designed to estimate the reactions on the EP and to estimate the fixing and processing errors. Knowing the reactions can lead to a fixing of the workpiece in devices, without their surfaces leaving the contact with EP (which are positioning bases (BP), respectively during processing to also avoid contact loss, in which case the workpiece can be removed from the device with damage or even work accidents.

Estimation of processing errors allows the improvement of processing accuracy and therefore implicitly the quality of products.

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Model teoretic pentru calculul deformaților elastice și de contact ale elementelor de poziționare într-un sistem de fixare tip 3-2-1

Rezumat: *Indiferent de domeniul de prelucrare, asamblare și inspecție, proiectarea dispozitivului de fixare este crucială, scopul său principal fiind de a localiza cu precizie și fixa piesa de prelucrat, păstrând aceeași stare în timpul procesului de fabricație, fără deplasări, rotații sau deformări ale piesei de prelucrat. În timpul diferitelor operații, forțele de strângere și forțele de așchiere acționează asupra piesei de prelucrat. Lucrarea prezintă un model teoretic pentru calcularea deformațiilor elastice și de contact ale elementelor de poziționare și a mișcării piesei sub acțiunea acestor forțe în cazul unui sistem de fixare tip 3-2-1, având elemente de poziționare elastice în jurul unei piese dreptunghiulare rigide. Analiza și calculele sunt efectuate cu ajutorul unui program software realizat în limbajul de programare Python, care ajută la estimarea diferitelor erori ce pot apărea în timpul procesului de fabricație, la estimarea reacțiunilor asupra elementelor de poziționare și a erorilor de fixare și de prelucrare*.

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