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DYNAMIC RESPONSE FOR A SYSTEM MODELED AS A SOLID BODY WITH CONSTRUCTIVE SYMMETRY AND INCLINED DISTURBING FORCE

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Abstract: *This paper presents the dynamic analysis of a solid body with elastic bearings and constructive symmetries (mass and moments of inertia symmetry, sizes/geometric symmetry, the arrangement and the stiffness of the elastic supports symmetry), modeled as a system with six degrees-of-freedom. The dynamic analysis consisted in determining the proper modes for the system with decoupled movements due to the constructive symmetry and establishing operational calculation relations for determining the amplitudes of the harmonic forced vibrations produced by a harmonic disturbing force inclined in a longitudinal vertical plane. For each uncoupled differential equation of motion produced by the harmonic inclined force, there were determined the amplitudes of the steady-state forced vibrations for each displacement (3 translations, 3 rotations).*

Key words: *solid body, 6 degrees-of-freedom, 3D elastic bearing, constructive and structural symmetry, decupled movements, steady-state forced vibration, forced vibration amplitudes.*

1. INTRODUCTION

The analysis of the dynamic parameters of the 6 degrees-of-freedom elastic mechanical systems consist in [10] [12] [14] [16] [19] [20]:

1)modal analysis (determining the eigenpulsations / eigenfrequencies and the eigenvectors / Eigenshapes)

2)determining the amplitudes of harmonical steady-state vibration movements of the solid

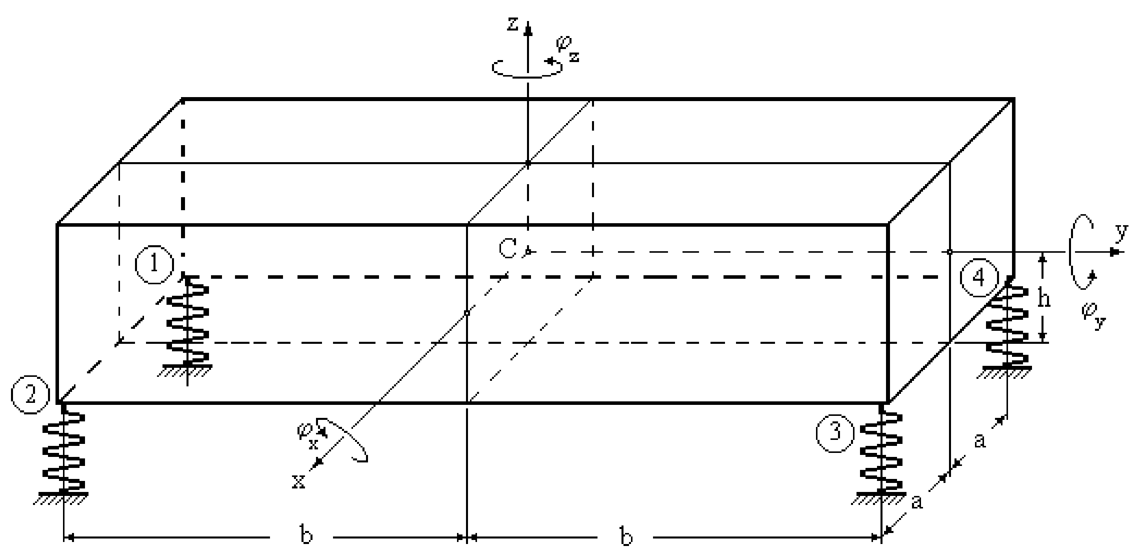

Fig. 1. 6DOF Solid body with constructive symmetries - Central and principal axes of coordinates **Cxyz**

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body (three translation displacements and three rotation displacements).

Figure 1 shows a solid body with constructive and structural symmetries (mass and inertia symmetry, dimensional symmetry, elastic symmetry), supported on four identical elastical bearings with 3D stiffness $k\left(k_x, k_y, k_z\right)$, modeled as a six degrees of dynamic freedom [4] [18].

The dynamic analysis of the system is important in all stages of development and operation of an equipment or machine with vibrating action or subject to harmful vibrations: design, experimental model, prototype, testing, certification, manufacturing, operation, maintenance. [1] [5].

2. DYNAMIC MODEL OF THE 6DOF MECHANICAL SYSTEM

General coordinations used for the dynamic analysis of the six degrees-of-freedom mechanical elastic system from figure 1 are the displacements of the rigid body (3 translations of gravity center **C** and 3 rotations around the central and principal axes **Cx**, **Cy** and **Cz**):

X - lateral sliding displacement of **C**
Y - longitudinal/forward displacement of **C**
Z - vertical displacement of **C**
φ_x - pitching rotation angle
φ_y - rolling rotation angle
φ_z - turning rotation angle

The matrix motion equation of the linear system with six degrees-of-freedom (a system with six differential equations) is [8] [13] [21]

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{Q^F\}, \quad (1)$$

where:

■$\{q\}$, $\{\dot{q}\}$, $\{\ddot{q}\}$ are the column vectors of displacements (general coordinates), (general) velocities and (general) accelerations

$$\{q\} = \{X \quad Y \quad Z \quad \varphi_x \quad \varphi_y \quad \varphi_z\}^T \quad (2)$$
$$\{\dot{q}\} = \{\dot{X} \quad \dot{Y} \quad \dot{Z} \quad \dot{\varphi}_x \quad \dot{\varphi}_y \quad \dot{\varphi}_z\}^T \quad (3)$$
$$\{\ddot{q}\} = \{\ddot{X} \quad \ddot{Y} \quad \ddot{Z} \quad \ddot{\varphi}_x \quad \ddot{\varphi}_y \quad \ddot{\varphi}_z\}^T \quad (4)$$

■$[M]$, $[C]$, $[K]$ are the 6×6 matrices of mass, viscous damping and linear stiffness

■$\{Q^F\}$ is the column vector of general disturbing forces

$$\{Q^F\} = \left\{Q_X^F \quad Q_Y^F \quad Q_Z^F \quad Q_{\varphi_x}^F \quad Q_{\varphi_y}^F \quad Q_{\varphi_z}^F\right\}^T (5)$$

3. DYNAMIC MODEL OF THE SYSTEM WITH CONSTRUCTIVE SYMMETRIES

Considering **Cxyz** the central and principal system of coordinates axes, the mass/inertia matrix is diagonal [9] [11]

$$[M] = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & J_x & 0 & 0 \\ 0 & 0 & 0 & 0 & J_y & 0 \\ 0 & 0 & 0 & 0 & 0 & J_z \end{bmatrix}, \quad (6)$$

where

m is the rigid body mass
J_x, J_y, J_z - the principal moments of inertia

If the bearings of solid body are elastic (or quasi-elastic), the 6×6 damping matrix is null:

$$[C] = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \quad (6)$$

The working part of the technological equipment's and machines modeled as a six degrees-of-freedom rigid body with constructive symmetries on vertical planes (as in figure 1), have four identical elastic supports (stiffness).

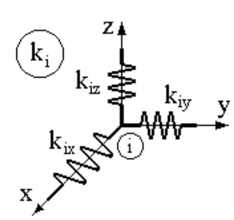

Fig. 2. 3D axis stiffness coefficients of elastic support

Considering

I)$k_i(k_x \quad k_y \quad k_z)\ i=\overline{1,4}$ the elastic parameters of each support (where k_x, k_y and k_z are the "directional" stiffness, figure 2)

II)a convenient positioning of the horizontal plane of the bearing system which contains the center of mass of the solid body (*h=0*),

III)a symmetrical arrangement of supports reported to vertical central axis **Cz** (figure 1) **1***(-a, -b, 0)*, **2***(a, -b, 0)*, **3***(a, b, 0)*, **4***(-a, b, 0)*, the stiffness matrix becomes diagonal:

$$[K]=\begin{bmatrix}[K]_{11} & [0]_{3\times3}\\ [0]_{3\times3} & [K]_{22}\end{bmatrix}\ , \tag{6}$$

where the four sub matrices component are

$$[K]_{11}=\begin{bmatrix}4k_x & 0 & 0\\ 0 & 4k_y & 0\\ 0 & 0 & 4k_z\end{bmatrix}$$

$$[K]_{22}=\begin{bmatrix}4b^2k_z & 0 & 0\\ 0 & 4a^2k_z & 0\\ 0 & 0 & 4\left(a^2k_y+b^2k_x\right)\end{bmatrix}$$

$$[K]_{12}=[K]_{21}=[0]_{3\times3}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

4. DYNAMIC MODEL OF THE SYSTEM WITH CONSTRUCTIVE SYMMETRIES

4.1 Decoupled movements of the system with constructive symmetries. Modal analysis

The constructive symmetries of the system and the central and principal coordinates axis **Cxyz**, the mass/inertia matrix $[M]$ and stiffness matrix $[K]$ are both diagonal. In this case, the system of movement differential equations becomes a system with six decoupled/independent equations. The differential equations of free vibration for 6DOF system with decoupled movements are:

$$m\ddot{X}+4k_xX=0 \tag{8a}$$
$$m\ddot{Y}+4k_yY=0 \tag{8b}$$
$$m\ddot{Z}+4k_zZ=0 \tag{8c}$$
$$J_x\ddot{\varphi}_x+4b^2k_z\varphi_x=0 \tag{8d}$$
$$J_y\ddot{\varphi}_y+4a^2k_z\varphi_y=0 \tag{8e}$$
$$J_z\ddot{\varphi}_z+4\left(a^2k_y+b^2k_x\right)\varphi_z=0 \tag{8f}$$

Dividing each eq. (8a)-(8f) by correspondent inertial coefficient, we obtain the modal pulsations of the decoupled system [6] [7]:

$$p_X=2\sqrt{\frac{k_x}{m}} \tag{9a}$$
$$p_Y=2\sqrt{\frac{k_y}{m}} \tag{9b}$$
$$p_Z=2\sqrt{\frac{k_z}{m}} \tag{9c}$$
$$p_{\varphi_x}=2\frac{b}{i_x}\sqrt{\frac{k_z}{m}} \tag{9d}$$
$$p_{\varphi_y}=2\frac{a}{i_y}\sqrt{\frac{k_z}{m}} \tag{9e}$$
$$p_{\varphi_z}=2\frac{1}{i_z}\sqrt{\frac{a^2k_y+b^2k_x}{m}} \tag{9f}$$

where i_x, i_y and i_z are the radius of inertia.

4.2 Dynamic analysis of the system with constructive symmetries and inclined disturbing force - movement equations

In order to analyze the dynamic response of the mechanic elastic from figure 1 (symmetries on the vertical-longitudinal plane and on the vertical-lateral plane), we consider an inclined force **F** as in figure 3. Taking into consideration the angle β of inclination and the eccentricities of the point where the force is applied, the vector of generalized forces is [15]:

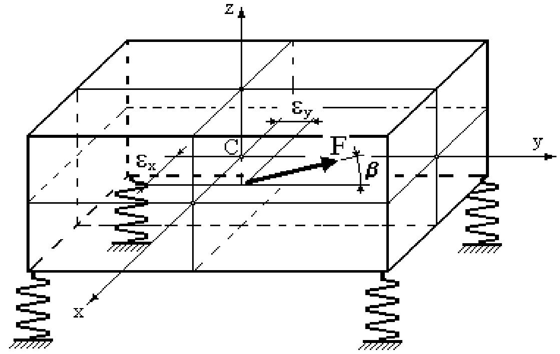

Fig. 3. 6DOF elastic mechanical system with constructive symmetries - inclined disturbed force

$$\{Q^F\} \equiv \begin{Bmatrix} Q_X^F \\ Q_Y^F \\ Q_Z^F \\ Q_{\varphi_x}^F \\ Q_{\varphi_y}^F \\ Q_{\varphi_z}^F \end{Bmatrix} = \begin{Bmatrix} 0 \\ F cos\beta \\ F sin\beta \\ \varepsilon_y F sin\beta \\ -\varepsilon_x F sin\beta \\ \varepsilon_x F cos\beta \end{Bmatrix} \quad (10)$$

Usually, the disturbing one-direction force is produced by the symmetrical rotation of two identical unbalanced masses of vibrating motors (inertial forces). Considering a harmonic disturbing force $F = F_0 sin(\omega t)$ where ω is the pulsation (cyclic frequency), the vector of generalized forces becomes [17]:

$$\{Q^F\} = \begin{Bmatrix} 0 \\ F_0 cos\beta \\ F_0 sin\beta \\ \varepsilon_y F_0 sin\beta \\ -\varepsilon_x F_0 sin\beta \\ \varepsilon_x F_0 cos\beta \end{Bmatrix} sin(\omega t) \quad (11)$$

With the expressions (11) of generalized forces, the system of differential equations of forced vibration for the six degrees-of-freedom system with decoupled movements is [2] [3]:

$$\begin{cases} m\ddot{X} + 4k_x X = 0 \\ m\ddot{Y} + 4k_y Y = F_0 cos\beta \cdot sin(\omega t) \\ m\ddot{Z} + 4k_z Z = F_0 sin\beta \cdot sin(\omega t) \\ J_x \ddot{\varphi}_x + 4b^2 k_z \varphi_x = \varepsilon_y F_0 sin\beta \cdot sin(\omega t) \\ J_y \ddot{\varphi}_y + 4a^2 k_z \varphi_y = -\varepsilon_x F_0 sin\beta \cdot sin(\omega t) \\ J_z \ddot{\varphi}_z + 4(a^2 k_y + b^2 k_x)\varphi_z = \\ = \varepsilon_x F_0 cos\beta \cdot sin(\omega t) \end{cases} \quad (12)$$

The inhomogeneous differential equations (12) have particular solutions which describe the steady-state harmonic forced vibration

$$\{q_f\} = \{a_f\} sin(\omega t) = \begin{Bmatrix} A_X^f \\ A_Y^f \\ A_Z^f \\ A_{\varphi_x}^f \\ A_{\varphi_y}^f \\ A_{\varphi_z}^f \end{Bmatrix} sin(\omega t) \quad (13)$$

where

$$\{a_f\} = \left\{ A_X^f \quad A_Y^f \quad A_Z^f \quad A_{\varphi_x}^f \quad A_{\varphi_y}^f \quad A_{\varphi_z}^f \right\}^T$$

is the amplitudes vector of forced vibration of mechanical elastic system (on the six "direction" of movement: lateral sliding, forward-back, vertical up-down, pitching, rolling, turning).

The vector of general accelerations for the steady-state forced vibration is:

$$\{\ddot{q}_f\} = -\omega^2 \{a_f\} sin(\omega t) \quad (14)$$

The amplitudes of forced vibration are obtained by replacement of (13) and (14) in (1) and identification of vectorial coefficients:

$$A_Y^f = \frac{F_0 cos\beta}{4k_y - m\omega^2} \quad (15a)$$

$$A_Z^f = \frac{F_0 sin\beta}{4k_z - m\omega^2} \quad (15b)$$

$$A_{\varphi_x}^f = \frac{\varepsilon_y F_0 sin\beta}{4b^2 k_z - J_x \omega^2} \quad (15c)$$

$$A_{\varphi_y}^f = \frac{-\varepsilon_x F_0 sin\beta}{4a^2 k_z - J_y \omega^2} \quad (15d)$$

$$A_{\varphi_z}^f = \frac{\varepsilon_x F_0 cos\beta}{4(a^2 k_y + b^2 k_x) - J_z \omega^2} \quad (15e)$$

The expressions of amplitudes of forced vibration function of modal pulsations are as follows:

$$A_Y^f = \frac{F_0 cos\beta}{m(p_Y^2 - \omega^2)} \quad (16a)$$

$$A_Z^f = \frac{F_0 sin\beta}{m(p_Z^2 - \omega^2)} \quad (16b)$$

$$A_{\varphi_x}^f = \frac{\varepsilon_y F_0 sin\beta}{m i_x^2 (p_{\varphi_x}^2 - \omega^2)} \quad (16c)$$

$$A_{\varphi_y}^f = \frac{-\varepsilon_x F_0 sin\beta}{m i_y^2 (p_{\varphi_y}^2 - \omega^2)} \quad (16d)$$

$$A_{\varphi_z}^f = \frac{\varepsilon_x F_0 cos\beta}{m i_z^2 (p_{\varphi_z}^2 - \omega^2)} \quad (16e)$$

5. CONCLUSIONS

a)due to constructive symmetry of the six degrees-of-freedom rigid body with elastic supports, the movements (3 translations and 3 rotations) are decoupled; if the center of mass **C** of the rigid body is in the horizontal plane of the bearings, the mass matrix and the stiffness matrix are diagonal and the differential equations of movements are independent;
b)the amplitudes of forced steady-state vibration, done by rel. (16a)-(16e), can be used to analyze the influence of the structural and dimensional characteristics of the system on the dynamic parameters [21] [22].

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Răspunsul dinamic pentru un sistem modelat ca solid rigid cu simetrie constructivă și forță perturbatoare înclinată

Rezumat: *Articolul prezintă analiza dinamică a unui solid rigid cu reazeme elastice şi simetrii constructive (simetrie a masei şi momentelor de inerţie, simetrie dimensională/geometrică, simetrie a dispunerii şi rigidităţii suporturilor elastice), modelat ca un sistem cu şase grade de -libertate. Analiza dinamică constă în determinarea modurilor proprii sistemului cu mişcări decuplate datorate simetriei constructive şi stabilirea relaţiilor de calcul operaţional pentru determinarea amplitudinilor vibraţiilor forţate armonice produse de o forţă perturbatoare armonică înclinată într-un plan vertical logitudinal. Pentru fiecare ecuaţie diferenţială (decuplată) a mişcărilor produse de forţa armonică înclinată, s-au determinat amplitudinile vibraţiilor forţate în regim staţionare de funcţionare pentru fiecare deplasare (3 translaţii, 3 rotaţii).*

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