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INFLUENCE OF ECCENTRICITIES OF DISTURBING FORCES APPLIED TO AN ELASTIC MECHANICAL SYSTEM MODELED AS A SOLID BODY WITH CONSTRUCTIVE SYMMETRIES

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Abstract: *The paper presents the analysis of the dynamic parameters of a rigid body with elastic bearings and constructive symmetries modeled as a six degrees-of-freedom mechanical system disturbed by a harmonic vertical eccentric force. The constructive symmetries lead to the decoupling of the six movements of the mechanical system (three translations and three rotations) in subsystems with coupled movements (translations and rotations). For each of the decoupled subsystems (with coupled movements), there were determined the calculation relations of the amplitudes of the steady-state forced vibrations of the subsystems disturbed by a harmonic vertical force with eccentricities to the vertical line of the center of gravity of the solid body.*

Key words: *constructive symmetry, eccentric perturbing force, six degrees-of-freedom, solid body, decupled movements, uncoupled subsystems, steady-state vibration, forced vibration amplitudes.*

1. INTRODUCTION

The solid body with elastic and/or viscous-elastic bearings can be modeled as a six degrees-of-freedom system with six general coordinates used for dynamic analysis. In order to create the mathematical model (movement eq.), it uses the next characteristics: sizes/dimensions, masses/inertia moments, stiffness and dampings of the bearings. If the coordinates axis is central and principal **Cxyz**, the inertia matrix is diagonal and the system of differential moving equations of the solid body are coupled by the coefficients of stiffness matrix and damping matrix [1] [2] [3].

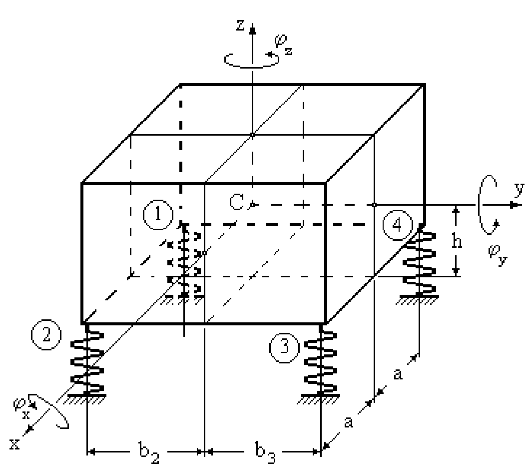

Fig. 1. Six degrees-of-freedom solid body with elastic bearings and vertical-longitudinal plane of symmetry [3]

Figure 1 shows a solid body with constructive vertical-longitudinal plane of symmetry. Constructive symmetry means: dimensional/sizes symmetry, mass/inertia symmetry, elastic symmetry of the bearings. Due to the constructive symmetries, the movements of the mechanical elastic system, modeled as a six degrees-of-freedom system, are decoupled into some subsystems with coupled movements.

2. DYNAMIC MODEL OF 6DOF SYSTEM WITH A VERTICAL-LONGITUDINAL PLANE OF SYMMETRY

2.1 Equations of decoupled free vibration

If the system of coordinates axes is central and principal, **Cxyz**, the 6×6 inertia matrix is diagonal [4] [5]

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$$[M] = DIAG\ (m, m, m, J_x, J_y, J_z\ ) \qquad (1)$$

where

■m is the rigid body mass;

■J_x, J_y, J_z - the principal moments of inertia.

Considering that the general coordinates are the displacements (translations, rotations) as in figure 1, for a convenient positioning of the bearing system in the horizontal plane of the center of mass **C** (*h=0*), the movements of the six degrees-of-freedom system are decupled as follows [6] [7]:

►subsystem (X, φ_z) - lateral sliding coupled with turning rotation;

►subsystem (Z, φ_x) - vertical displacement coupled with pitching rotation;

►"subsystem" (Y) - independent forward displacement;

►"subsystem" (φ_y) - independent rolling rotation.

The differential equations of free decoupled vibration of the elastic mechanical 6DOF system with symmetries are:

►for the subsystem (X, φ_z) with coupled movements of lateral sliding and turning rotation

$$[M_{16}]\begin{Bmatrix}\ddot{X}\\ \ddot{\varphi}_z\end{Bmatrix} + [K_{16}]\begin{Bmatrix}X\\ \varphi_z\end{Bmatrix} = \begin{Bmatrix}0\\0\end{Bmatrix} \qquad (2)$$

where

$$[M_{16}] = \begin{bmatrix}m & 0\\ 0 & J_z\end{bmatrix} \qquad (3)$$

and

$$[K_{16}] = \begin{bmatrix}4k_x & -2k_x(b_3 - b_2)\\ -2k_x(b_3 - b_2) & 4a^2k_y + 2k_x(b_2^2 + b_3^2)\end{bmatrix} \qquad (4)$$

►for the subsystem (Z, φ_x) with coupled movements of vertical displacement and pitching rotation

$$[M_{34}]\begin{Bmatrix}\ddot{Z}\\ \ddot{\varphi}_x\end{Bmatrix} + [K_{34}]\begin{Bmatrix}Z\\ \varphi_x\end{Bmatrix} = \begin{Bmatrix}0\\0\end{Bmatrix} \qquad (5)$$

where

$$[M_{34}] = \begin{bmatrix}m & 0\\ 0 & J_x\end{bmatrix} \qquad (6)$$

and

$$[K_{34}] = \begin{bmatrix}4k_z & 2k_z(b_3 - b_2)\\ 2k_z(b_3 - b_2) & 2k_z(b_2^2 + b_3^2)\end{bmatrix} \qquad (7)$$

►for decoupled forward movement (Y)

$$m\ddot{Y} + 4k_yY = 0 \qquad (8)$$

►for decoupled rolling rotation (φ_y)

$$J_y\ddot{\varphi}_y + 4a^2k_z\varphi_z = 0 \qquad (9)$$

2.2 Eigenpulsation for uncoupled movements

$$p_X = 2\sqrt{\frac{k_x}{m}} \qquad (10a)$$

$$p_Y = 2\sqrt{\frac{k_y}{m}} \qquad (10b)$$

$$p_Z = 2\sqrt{\frac{k_z}{m}} \qquad (10c)$$

$$p_{\varphi_x} = \sqrt{\frac{2k_z(b_2^2+b_3^2)}{J_x}} \qquad (10d)$$

$$p_{\varphi_y} = 2a\sqrt{\frac{k_z}{J_y}} \qquad (10e)$$

$$p_{\varphi_z} = \sqrt{\frac{4a^2k_y+2k_x(b_2^2+b_3^2)}{J_z}} \qquad (10f)$$

2.3 Eigenpulsation for uncoupled subsystems with coupled movements

►subsystem (X, φ_z)

$$p_{1,2} = \sqrt{\frac{1}{2}\left[p_X^2 + p_{\varphi_z}^2 \pm \sqrt{\left(p_X^2 - p_{\varphi_z}^2\right)^2 + 4\gamma_1\gamma_2}\right]} \qquad (11)$$

where:

$$\gamma_1 = \frac{2k_x(b_3 - b_2)}{m}$$

$$\gamma_2 = \frac{2k_x(b_3 - b_2)}{J_z}$$

►subsystem (Z, φ_x)

$$p_{3,4} = \sqrt{\frac{1}{2}\left[p_Z^2 + p_{\varphi_x}^2 \pm \sqrt{\left(p_Z^2 - p_{\varphi_x}^2\right)^2 + 4\delta_1\delta_2}\right]} \qquad (12)$$

where:

$$\delta_1 = \frac{2k_z(b_3-b_2)}{m} \qquad \delta_2 = \frac{2k_z(b_3-b_2)}{J_x}$$

3. AMPLITUDES OF FORCED STEADY-STATE VIBRATION

Figure 2 shows the physical model of the rigid body with elastic bearings and constructive symmetry in a vertical-longitudinal plane, perturbed by a eccentric vertical harmonic force $F_z = F_0 sin(\omega t)$ [8] [9].

3.1 Differential equations of forced vibration

The perturbing eccentric vertical force produces vibrations in a stabilized regime only after the "directions" (Z, φ_x) and (φ_y) [10] [11] [12]. The differential equations of steady-state forced vibration are:

$$[M_{34}]\begin{Bmatrix} \ddot{Z} \\ \ddot{\varphi}_x \end{Bmatrix} + [K_{34}]\begin{Bmatrix} Z \\ \varphi_x \end{Bmatrix} = \begin{Bmatrix} F_0 \\ e_y F_0 \end{Bmatrix} sin(\omega t) \quad (13)$$

$$J_y \ddot{\varphi}_y + 4a^2 k_z \varphi_z = -e_{1x} F_0 sin(\omega t) \quad (14)$$

3.2 Amplitudes of forced vibration

$$A_Z = \frac{F_0[(p^2_{\varphi_x} - \omega^2) - \delta_2 e_y]}{m[(p^2_Z - \omega^2)(p^2_{\varphi_x} - \omega^2) - \delta_1 \delta_2]} \quad (15)$$

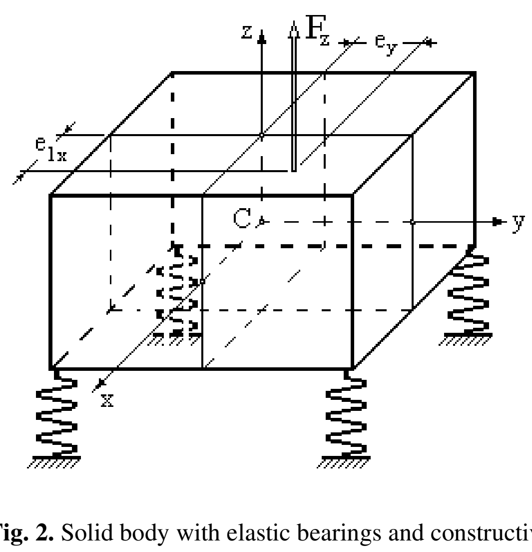

Fig. 2. Solid body with elastic bearings and constructive symmetry (vertical-longitudinal plane of symmetry) disturbed by eccentric vertical harmonic force

$$A_{\varphi_x} = -\frac{F_0[e_y(p^2_Z - \omega^2) - \delta_1]}{J_x[(p^2_Z - \omega^2)(p^2_{\varphi_x} - \omega^2) - \delta_1 \delta_2]} \quad (16)$$

$$A_{\varphi_y} = \frac{-e_{1x} F_0}{4a^2 k_z - J_y \omega^2} = \frac{-e_{1x} F_0}{J_y\left(p^2_{\varphi_y} - \omega^2\right)} \quad (17)$$

4. CONCLUSIONS

►due to the constructive symmetry, the movements of the elastic mechanical system are decoupled in subsystems with coupled movements [13];

►the eccentric vertical force disturbs the system after the following "directions": (Z) vertical translation (jumping), (φ_x) pitching rotation (φ_y) rolling rotation;

►the amplitudes A_Z, A_{φ_x}, A_{φ_y} of the forced steady-state vibrations varies linearly with the amplitude F_0 of the applied force;

►the angular amplitude A_{φ_y} of the roller rotation is linear with the eccentricity e_{1x} of the force relative to the plane of symmetry **Cyz**.

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Influența excentricităților forțelor perturbatoare aplicate unui sistem mecanic elastic modelat ca solid rigid cu simetrii constructive

Rezumat: *Lucrarea prezintă analiza parametrilor dinamici ai unui solid rigid cu reazeme elastice şi simetrii constructive modelat ca un sistem mecanic cu şase grade de libertate, perturbat de o forță armonică verticală şi excentrică. Simetriile constructive conduc la decuplarea celor şase mişcări ale sistemului mecanic (trei translaţii şi trei rotaţii) în subsisteme cu mişcări cuplate (translaţii şi rotaţii). Pentru fiecare dintre subsistemele decuplate (cu mişcări cuplate), sunt determinate relațiile operaționale de calcul ale amplitudinilor vibrațiilor forțate în regim dinamic, în cazul solicitărilor cu o forță verticală armonică cu excentricități față de verticala centrului de greutate al solidului rigid.*

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