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CONSIDERATIONS REGARDING THE IMPLEMENTATION OF SERIAL ROBOTS IN WORKING PROCESSES

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ABSTRACT: *The implementation of a mechanical robot structure, in a working process, consists in determination of the driving moments of each motor from the kinematic joints. To establish of these driving moments, known as dynamic control equations, can be used a generalized algorithm, based on geometrical, kinematical, and dynamical modelling, by using dedicated algorithms. Generally, a proper estimation of moments of the motors, are revealing the driving motor torques, hence the related breaking systems can be rigorously dimensioned. The paper presents an example of using mathematical algorithms to implement a robot in a working process.*

KEYWORDS*: algorithm, driving moment, dynamic control functions, interpolating functions, robot.*

1. INTRODUCTION

Starting from the idea that a robot is a complex mechanic system, further will be determined the dynamic functions for a mechanical robot structure. The obtained expressions leading to implementation it in a working process. To achieve this goal, it must be considered that the robot is a set of bidirectional elements interconnected by mathematical functions.

Each serial robot is considered having a number of kinematic joint (link). In each kinematic link, is an actuator, which generates the movement of the mechanical element, hence the motion is transmitted to the end effector.

2. GEOMETRICAL MODELING OF THE 2TR SERIAL ROBOT STRUCTURE

Is considered a serial robot structure, having three degrees of freedom, as presented in the Figure 1.

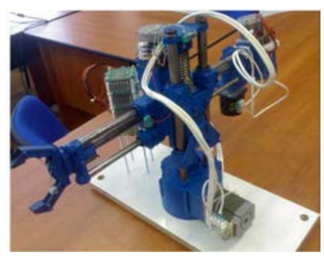

Fig. 1. The 2TR serial robot

According to the figure, the proposed robot is a serial robot, consisting in two translations, and a rotation, the robot denoted 2TR.

The first step in establishing of dynamic control equations, consisting in geometrical modeling, hence resulting a column vector, describing the direct geometric modeling of the type 2TR robot.

In order to obtain the Direct Geometry Equations, according to [1], there is used the Locating Matrix Algorithm, described in [2], considering that q_1, q_2, q_3 are the generalized coordinates from each kinetic link.

As shown in Figure 2, the three d.o.f. of the 2TR serial robot, consisting in a translation along the axis O_{z0}, one along the axis O_{x0}, as well as the rotation of the effector around the O_{x0} axis.

According to mechanical structure of the robot, there is presented the kinematic scheme of the robot, as in Figure 2.

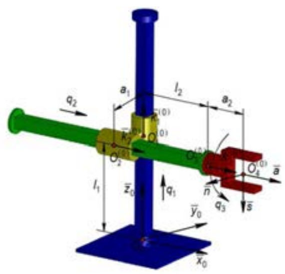

Fig. 2. The Kinematic scheme of serial robot 2TR

For the 2TR serial structure, to apply the Locating Matrix Algorithm as input data is considered the following matrix of nominal geometry, as presented in Table 1.

Table 1

The Matrix of nominal geometry

Joint	*Joint type R,T*	$\bar{k}_i^{(0)T}$			$\bar{p}_i^{(0)T}$		
		$k_{ix}^{(0)T}$	$k_{iy}^{(0)T}$	$k_{iz}^{(0)T}$	$x_i^{(0)T}$	$y_i^{(0)T}$	$z_i^{(0)T}$
1	T	0	0	1	0	0	l_1
2	T	1	0	0	0	- a_1	l_1
3	R	1	0	0	l_2	- a_1	l_1
4	-	-	-	-	$a+l_2$	$-a_1$	l_1

On the basis of matrix of nominal geometry, by applying the algorithm, there is determined the following column vector of generalized coordinates:

$$ {}^0\bar{X}(\bar{\theta}) = \begin{bmatrix} \bar{p} \\ \ldots\ldots \\ \bar{\psi} \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} p_x & p_y & p_z \end{pmatrix}^T \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \begin{pmatrix} \alpha_z & \beta_y & \gamma_z \end{pmatrix}^T \end{bmatrix} = \tag{1} $$

$$ = \left\{ \begin{bmatrix} a_2 + d_2 + q_2 \\ -a_1 \\ l_1 + q_1 \end{bmatrix} \quad \begin{bmatrix} \pi/2 & \pi/2 & \pi + q_3 \end{bmatrix}^T \right\}^T , $$

representing the direct geometry equations.

On the basis of (1), are determined the geometrical control functions, corresponding to the input data regarding the position of the characteristic point in the Cartesian space:

$$ \begin{Bmatrix} q_2 = p_x - a_2 - d_2 \\ q_1 = p_z - l_1 \\ q_3 = \gamma_z - \pi \end{Bmatrix} \tag{2} $$

representing the inverse geometry equations.

3. KINEMATIC CONTROL FUNCTIONS FOR 2TR SERIAL STRUCTURE

In order to obtain the direct kinematics equations, there is used the Algorithm of Matrix Exponential in kinematics, and the time derivative of Jacobian matrix, according to [3], [4]. The direct kinematics equations with respect to fixed reference frame are obtained as [5]:

$$ {}^0\dot{X} \equiv \begin{bmatrix} {}^0\bar{v}_3 \\ --- \\ {}^0\bar{\omega}_3 \end{bmatrix} = {}^0J(\bar{\theta}) \cdot \dot{\bar{\theta}} = \begin{bmatrix} (\dot{q}_2 & 0 & \dot{q}_1)^T \\ (\dot{q}_3 & 0 & 0)^T \end{bmatrix} \tag{3} $$

$$ {}^0\ddot{X} \equiv \begin{bmatrix} {}^0\dot{v}_3 \\ --- \\ {}^0\dot{\omega}_3 \end{bmatrix} = {}^0J(\bar{\theta}) \cdot \ddot{\bar{\theta}} = \begin{bmatrix} (\ddot{q}_2 & 0 & \ddot{q}_1)^T \\ (\ddot{q}_3 & 0 & 0)^T \end{bmatrix} \tag{4} $$

and representing the direct kinematic model. They are characterizing for the gripper`s mechanical structure in Cartesian space, the kinematic motion parameters as velocity and accelerations. In expressions (3) and (4), ${}^0J(\bar{\theta})$ represents the Jacobian matrix, as:

$$ \underset{(6\times 3)}{{}^0J(\bar{\theta})} = \begin{bmatrix} {}^0J_1 & {}^0J_2 & {}^0J_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ \hdashline 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{5} $$

On the basis of the same algorithm, there is determined the direct kinematic model for the proposed 2TR structure as [5]:

$$ \dot{\bar{\theta}}(t) = {}^0J[\bar{\theta}(t)]^{-1} \cdot {}^0\dot{X}(t) = $$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \cdot (\dot{q}_2 \quad 0 \quad \dot{q}_1 \quad \dot{q}_3 \quad 0 \quad 0)^T =$$

$$= (\dot{q}_1 \quad \dot{q}_2 \quad \dot{q}_3)^T \qquad (6)$$

$$\ddot{\bar{\theta}}(t) = {}^0J[\bar{\theta}(t)]^{-1} \cdot {}^0\ddot{X}(t) - {}^0J[\bar{\theta}(t)]^{-1} \cdot {}^0\dot{J}[\bar{\theta}(t)] \cdot \dot{\bar{\theta}} =$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \cdot (\ddot{q}_2 \quad 0 \quad \ddot{q}_1 \quad \ddot{q}_3 \quad 0 \quad 0)^T =$$

$$= (\ddot{q}_1 \quad \ddot{q}_2 \quad \ddot{q}_3)^T \qquad (7)$$

where ${}^0J\left[\bar{\theta}(t)\right]^{-1} \cdot$ is the inverse of the Jacobian matrix.

4. DYNAMIC CONTROL FUNCTIONS FOR 2TR SERIAL ROBOT STRUCTURE

A dynamic model for a mechanical system, consists in the equations of motion of the elements, therefore the establishing of driving moments on the basis of output data from the geometrical and kinematical algorithms. According to the specialized literature, a complete dynamic model comprises the dynamic model of the actuator system belonging to the robot being studied, as well as the dynamic model of the transmission structure of the movement.

According to [3], [6], [7] motion expressions can be determined based on Lagrange-Euler formalism. Due to the fact that the Lagrange-Euler equations are specific to non-conservative mechanical structures the dynamic model of the mechanical structure of the type 2TR serial robot will be determined taking into account the following expressions:

$$Q_{iF}^i + Q_g^i + Q_{SU}^i = Q_m^i\left(\bar{\theta}; \dot{\bar{\theta}}; \ddot{\bar{\theta}}\right), \quad i = 1 \to 3 \qquad (8)$$

In previous expressions there are included $\left\{Q_{i\mathcal{F}}^i; Q_g^i; Q_{SU}^i\right\}$ representing: generalized inertia forces, generalized gravitational forces, and generalized handling forces.[3]

The expressions for generalized inertia forces, is presented as:

$$Q_{iF}^i = \frac{d}{dt}\left[\frac{\partial E_C\left(\bar{\theta};\dot{\bar{\theta}}\right)}{\partial \dot{q}_i}\right] - \frac{\partial E_C\left(\bar{\theta};\dot{\bar{\theta}}\right)}{\partial q_i} \qquad (9)$$

On the basis of König`s theorem [3], the expression for kinetic energy is:

$$E_C^j = (-1)^{\Delta_M} \cdot \frac{1-\Delta_M}{1+3\cdot\Delta_M} \cdot \left\{\frac{1}{2}\cdot M_j \cdot {}^j\bar{v}_{C_j}^T \cdot {}^j\bar{v}_{C_j}\right\} + \\ +\Delta_M^2 \cdot \frac{1}{2} \cdot {}^j\bar{\omega}_j^T \cdot {}^jI_j^* \cdot {}^j\bar{\omega}_j \qquad (10)$$

where:

$\Delta_M = \{\{-1; \mathit{general\ motion}\}; \{0; \mathit{translation}\}; \{1; \text{rotation}\}\}$, ${}^j\bar{v}_{C_j}$ and ${}^j\bar{\omega}_j$ are linear and angular speed of the mass centre of every link; ${}^jI_j^*$ inertial axial-centrifugal tensor of link (j). Based on previous considerations, kinetic energy for the considered structure, is [5]:

$$E_C\left(\bar{\theta}, \dot{\bar{\theta}}\right) = \sum_{j=1}^{3}\left\{\frac{1}{2}\cdot M_j \cdot {}^{\bar{J}}v_{Cj}^T \cdot {}^{\bar{J}}v_{Cj} + \frac{1}{2} \cdot {}^{\bar{J}}\omega_j^T \cdot {}^jI_j^* \cdot {}^{\bar{J}}\omega_j\right\} =$$

$$= \sum_{j=1}^{3}\left\{\frac{1}{2}\cdot M_j \cdot {}^{\bar{J}}v_{Cj}^T \cdot {}^{\bar{J}}v_{Cj}\right\} + \frac{1}{2} \cdot {}^{\bar{3}}\omega_3^T \cdot {}^3I_3^* \cdot {}^{\bar{3}}\omega_3 =$$

$$= 3{,}731 \cdot \dot{q}_1^2 + 1{,}814 \cdot \dot{q}_2^2 + 760{,}45 \cdot 10^{-6} \cdot \dot{q}_3^2 + \\ +\dot{q}_1 \cdot \dot{q}_3 \cdot (5{,}2 \cdot 10^{-3} \cdot \cos q_3 + 1{,}97 \cdot 10^{-3} \cdot \sin q_3); \quad (11)$$

According to the identity (9), the generalized force of inertia is determined by:

$$Q_{iF}^1(\bar{\theta}) = (1{,}972 \cdot 10^{-3} \cdot \cos q_3 - 5{,}209 \cdot 10^{-3} \cdot \sin q_3) \cdot \dot{q}_3^2 + \\ +7{,}463 \cdot \ddot{q}_1 + (5{,}209 \cdot 10^{-3} \cdot \cos q_3 + 1{,}972 \cdot 10^{-3} \cdot \sin q_3) \cdot \ddot{q}_3 \quad (12)$$

$$Q_{iF}^2(\bar{\theta}) = \frac{d}{dt}\left[\frac{\partial E_C}{\partial \dot{q}_2}\right] - \frac{\partial E_C}{\partial q_2} = 3{,}6286 \cdot \ddot{q}_2 \quad (13)$$

$$Q_{iF}^3(\bar{\theta}) = \frac{d}{dt}\left[\frac{\partial E_C}{\partial \dot{q}_3}\right] - \frac{\partial E_C}{\partial q_3} =$$

$$= (5{,}209 \cdot 10^{-3} \cdot \cos q_3 +$$

$+1{,}97 \cdot 10^{-3} \cdot sin\, q_3) \cdot \ddot{q}_1 + 1{,}52 \cdot 10^{-3} \cdot \ddot{q}_3$

(14)

The generalized gravitational forces are:

$$Q_g^i\left(\bar{\theta}\right) = {}^0J_i\left(\bar{\theta}\right)^T \cdot {}^0F_{x_i}\left(\bar{\theta}\right), \quad (15)$$

where ${}^0J_i\left(\bar{\theta}\right)^T$ is the line (i) of the transposed of Jacobian matrix defined in (5), and ${}^0F_{x_i}\left(\bar{\theta}\right)$ is the resultant force-moment vector of gravitational loads in the range $[i \to 3]$, whose expression for the 2TR structure of is presented in [5].

According to (15), the generalized gravitational forces for the considered structure is:

$$Q_g^1\left(\bar{\theta}\right) = 73{,}186; \qquad Q_g^2\left(\bar{\theta}\right) = 0$$
$$Q_g^3\left(\bar{\theta}\right) = 51{,}081 \cdot 10^{-3} \cdot \cos q_3 + 19{,}339 \cdot 10^{-3} \cdot \sin q_3 \quad (16)$$

The generalized handling forces are expressed:

$$Q_{SU}^i(\theta) = {}^0J_i^T(\theta) \cdot {}^0F_X(\theta), \quad i = 1 \to 3. \quad (17)$$

where ${}^0F_X(\theta), i = 1 \to 3$ is, the vector of the resultant force-load handling moment, determined in [5].

For proposed robot, according to (17), there is obtained:

$$Q_{SU}^1\left(\bar{\theta}\right) = 9{,}806 \cdot m_{SU}; \quad Q_{SU}^2\left(\bar{\theta}\right) = Q_{SU}^3\left(\bar{\theta}\right) = 0 \quad (18)$$

According to the same [5], the precise determination of these forces, in keeping with transmission chain, have as starting expressions the above determined

generalized gravitational, manipulation, inertia, forces, which are substituted in the definition expression (8). It results:

$$Q_m^1 = (1{,}97 \cdot 10^{-3} \cdot cos\, q_3 - 5{,}2 \cdot 10^{-3} \cdot sin\, q_3) \cdot \dot{q}_3^2 + $$
$$+(1{,}97 \cdot 10^{-3} \cdot sin\, q_3 + 5{,}2 \cdot 10^{-3} \cdot cos\, q_3) \cdot \ddot{q}_3 + $$
$$+7{,}46 \cdot \ddot{q}_1 + 73{,}18 + 9{,}806 \cdot m_{SU} \quad (19)$$

$$Q_m^2 = 3{,}628 \cdot \ddot{q}_2 \quad (20)$$

$$Q_m^3 = (5{,}2 \cdot 10^{-3} \cdot cos\, q_3 + 1{,}97 \cdot 10^{-3} \cdot sin\, q_3) \cdot \ddot{q}_1 + $$
$$+1{,}52 \cdot 10^{-3} \cdot \ddot{q}_3 + 51{,}08 \cdot 10^{-3} \cdot cos\, q_3 + $$
$$+19{,}33 \cdot 10^{-3} \cdot sin\, q_3 \quad (21)$$

The relationships previously obtained, (19)-(21) are in accordance with the kinematic structure of the 2TR serial robot, without considering the friction.

Taking into account the constructive form of the robot, regarding the system of motion transmission on each axis [9], there are determined the driving moments [8], [10]. Thus, and taking into account the characteristic friction existing in the mechanical systems of transmission of the movement, the driving motor, from the axis according to [5], are:

$$Q_m^{1jk} = 7{,}47 \cdot \ddot{q}_1 + (28{,}7 \cdot 10^{-9} - 11{,}6 \cdot 10^{-7} \cdot cos\, q_3 - 43{,}95 \cdot 10^{-8} \cdot sin\, q_3) \cdot \ddot{q}_2 + $$
$$+(-26{,}55 \cdot 10^{-9} + 52{,}09 \cdot 10^{-4} \cdot cos\, q_3 + 19{,}72 \cdot 10^{-4} \cdot sin\, q_3) \cdot \ddot{q}_3 + $$
$$+(90{,}71 \cdot 10^{-9} + 19{,}72 \cdot 10^{-4} \cdot cos\, q_3 - 52{,}09 \cdot 10^{-4} \cdot sin\, q_3) \cdot \dot{q}_3^2 + $$
$$+(9{,}806 - 20{,}76 \cdot 10^{-5} \cdot cos\, q_3) \cdot m_{SU} + 73{,}186 \quad (22)$$

$$Q_m^{2jk} = 48{,}21 \cdot 10^{-3} \cdot \ddot{q}_2 + 39{,}03 \cdot 10^{-9} \quad (23)$$

$$Q_m^{3jk} = (5{,}2 \cdot 10^{-3} \cdot cos\, q_3 + 1{,}97 \cdot 10^{-3} \cdot sin\, q_3) \cdot \ddot{q}_1 + 1{,}52 \cdot 10^{-3} \cdot \ddot{q}_3 + $$
$$+51{,}08 \cdot 10^{-3} \cdot cos\, q_3 + 19{,}34 \cdot 10^{-3} \cdot sin\, q_3 + $$
$$+824{,}67 \cdot 10^{-6} \cdot \{[\dot{q}_3^2(1{,}97 \cdot 10^{-3} \cdot cos\, q_3 - 5{,}2 \cdot 10^{-3} \cdot sin\, q_3) + 1{,}3 \cdot \ddot{q}_1 + $$
$$+(5{,}2 \cdot 10^{-3} \cdot cos\, q_3 + 1{,}97 \cdot 10^{-3} \cdot sin\, q_3) \cdot \ddot{q}_3 + 9{,}806 \cdot m_{SU} + 12{,}8]^2 + $$
$$+[(5{,}2 \cdot 10^{-3} \cdot cos\, q_3 + 1{,}97 \cdot 10^{-3} \cdot sin\, q_3) \cdot \dot{q}_3^2 + $$
$$+(5{,}2 \cdot 10^{-3} \cdot sin\, q_3 - 1{,}97 \cdot 10^{-3} \cdot cos\, q_3) \cdot \ddot{q}_3]^2\}^{\frac{1}{2}}, \quad (24)$$

where, j,k will be explained in the next paragraph.

5. THE IMPLEMENTATION OF SERIAL 2TR STRUCTURE IN A WORKING PROCESS

Further in the paper is presented, based on determined driving moments from the kinetic link the implementation of the robot in a working sequence, as presented in Figure 3.

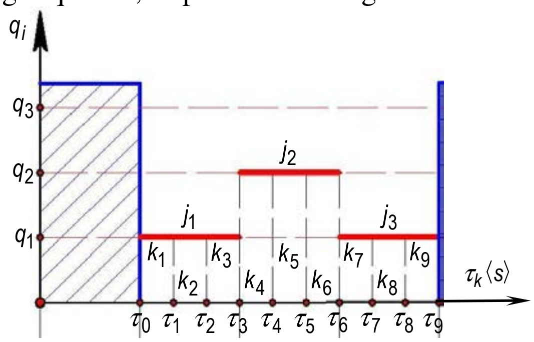

Fig. 3. The robot working sequences

There is considered three kinetic links, on working sequences $j=1\rightarrow 3$. Each sequence is divided in 3 segments, resulting $k=1\rightarrow 9$ configurations.

The cyclogram marks the order as well as the operating time τ_k of each link of the 2TR structure, in time intervals between two adjacent configurations. Further to underscore the oscillation of the kinematic parameters, and the driving moments, according to [3], respectively [5] every sequence of the process, is mathematically modeled by interpolating the trajectory in the configurations space on every $j=1\rightarrow 3$ sequence, using cubic spline functions. The analysis taking account the restrictions imposed by geometry and kinematics of the robot.

Further are used polynomial functions, and interpolation by cubic spline of $(3n)-type$ expressions.

Mathematically the method consists in generation of linear time functions for the generalized accelerations for every kinetic link of the robot. The motion trajectory in space must go through all the points corresponding to the moments $\left[\tau_i\ (i=0\rightarrow n)\right]$.

Based on initial conditions, the trajectory must be controlled in position, velocity and acceleration at τ_0 and τ_n. Also, it must be a continuity in velocity and acceleration at $\left[\tau_k\ (k=1\rightarrow n-1)\right]$. Hence, there is interpolated every $k=1\rightarrow 9$ segment, and is mathematically modeled using cubic spline functions.

In order to establish of generalized accelerations for every kinetic link, a time linear function, is generated [5] as:

$$\ddot{q}_{ji}(\tau)=\frac{\tau_i-\tau}{t_i}\cdot\ddot{q}_{ji}(\tau_{i-1})+\frac{\tau-\tau_{i-1}}{t_i}\cdot\ddot{q}_{ji}(\tau_i) \qquad (25)$$

where $t_i=\tau_i-\tau_{i-1}$ is the time necessary cover the segment $(i=1\rightarrow 3)$. By applying mathematical transformations on (25), results:

$$\dot{q}_{ji}(\tau)=-\frac{(\tau_i-\tau)^2}{2\cdot t_i}\cdot\ddot{q}_{ji-1}+\frac{(\tau-\tau_{i-1})^2}{2\cdot t_i}\cdot\ddot{q}_{ji}+a_{ji1};\qquad (26)$$

$$q_{ji}(\tau)=\frac{(\tau_i-\tau)^3}{6\cdot t_i}\cdot\ddot{q}_{ji-1}+\frac{(\tau-\tau_{i-1})^3}{6\cdot t_i}\cdot\ddot{q}_{ji}+a_{ji1}\cdot\tau+a_{ji2} \qquad (27)$$

For exemplification of the above presented considerations, in Table 2, there are imposed values for coordinates and time of the intervals as presented.

Table 2

Values for coordinates and time

Sequence j=1→3	*Config.* k=0→9	*Time* $\tau_k\langle s\rangle$	*Run time* $t_i\langle s\rangle$	*Values of coord.* $q_{ijk}\langle m,rad\rangle$
1	*0*	*0*	*0*	*0*
	1	*2*	*2*	
	2	*4*	*2*	
	3	*6*	*2*	*2*
2	*3**	*15*	*9*	*2*
	4	*16*	*1*	
	5	*17*	*1*	
	6	*18*	*1*	*1*
3	*6**	*18*	*0*	*1,571*
	7	*18,5*	*0,5*	
	8	*18,75*	*0,25*	
	9	*19,25*	*0,5*	*0*

Based on (25)-(27), the kinematic control functions based on $(3n)-type$ polynomial functions with restrictions, are:

Table 3

Kinematic control functions

Seq. j=1→9	Interval	Coord	Generalized position, speeds and accelerations		
			q_{ljk} $\langle m, rad \rangle$	$\dot{q}_{ljk}$ $\left\langle \frac{m}{s}, \frac{rad}{s} \right\rangle$	$\ddot{q}_{ljk}$ $\left\langle \frac{m}{s^2}, \frac{rad}{s^2} \right\rangle$
1	1	q_1	$0,04 \cdot \tau^3$	$0,125 \cdot \tau^2$	$0,25\tau$
	2		$-0,1\tau^3 + 0,75 \cdot \tau^2 -$ $-1,5 \cdot \tau + 1$	$-0,25\tau^2 +$ $+1,5(\tau - 1)$	$1,5 - 0,5\tau$
	3		$0,04(\tau - 6)^3 + 2$	$0,125 \cdot (\tau - 6)^2$	$0,25\tau - 1,5$
2	1	q_2	$20 - 0,17(\tau - 15$	$-0,5 \cdot (\tau - 15)^2$	$15 - \tau$
	2		$0,34\tau^3 - 16,5\tau^2 +$ $+271,5\tau - 1483,5$	$\tau^2 - 33\tau +$ $+271,5$	$2\tau - 33$
	3		$1 - 0,17(\tau - 18)^3$	$-0,5 \cdot (\tau - 18)^2$	$18 - \tau$
3	1	q_3	$1,6 - 3,4(\tau - 18)$	$-10,05(\tau - 18)^2$	$-10,05(2\tau - 36$
	2		$13,4\tau^3 - 748,9\tau^2 +$ $+13945,7\tau - 86540$	$40,2\tau^2 - 1497,9\tau +$ $+13945,7$	$80,4\tau - 1497,$
	3		$-3,3(\tau - 19,25)$	$-10,0(\tau - 19,25)^2$	$-10,05(2\tau - 38,$

Using as input data, the running time for trajectory, and the coordinate at the beginning and end of the sequence, the expressions of the generalized coordinates, and kinematic parameters, are represented, as in Figures 4-6:

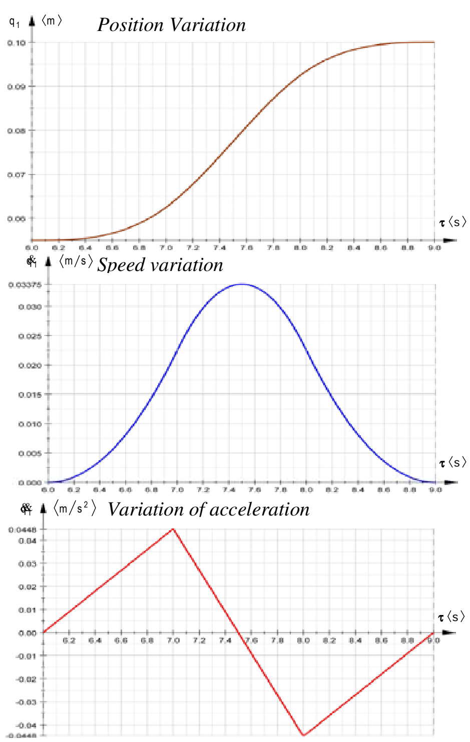

Fig. 4. Representation of kinematical parameters on sequence j=1

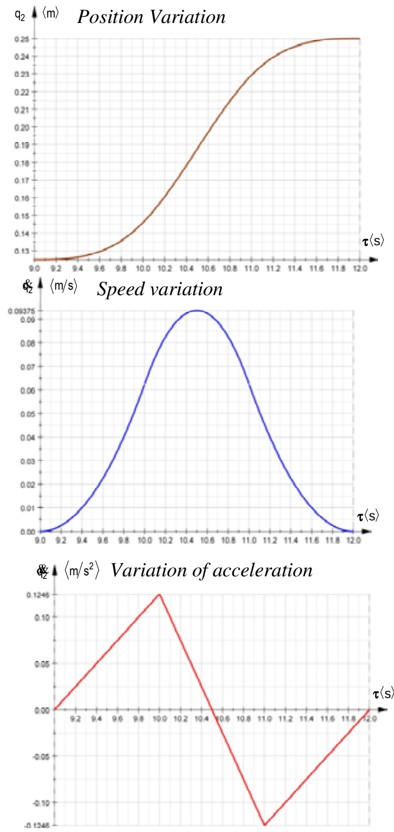

Fig. 5. Representation of kinematical parameters on sequence j=2

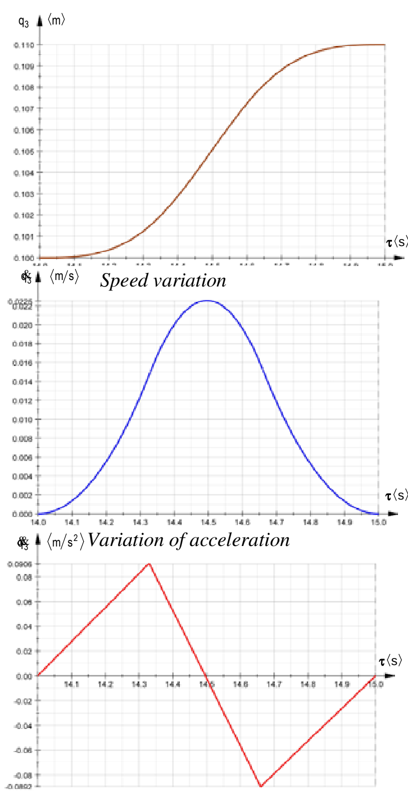

Fig. 6 Representation of kinematical parameters on sequence j=3

On the basis of data contained in Table 3, and functions (22)-(24), the variations of the driving moments are presented in the following (see Figures 7-9).

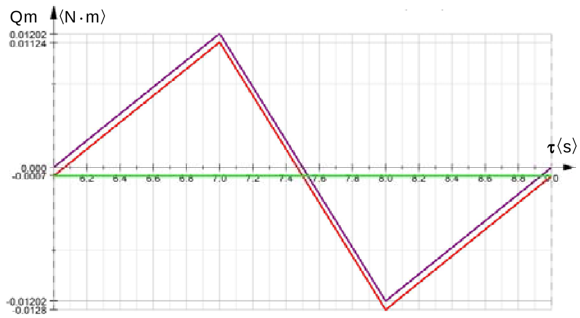

Figure 7 The representation of driving moment on j=1

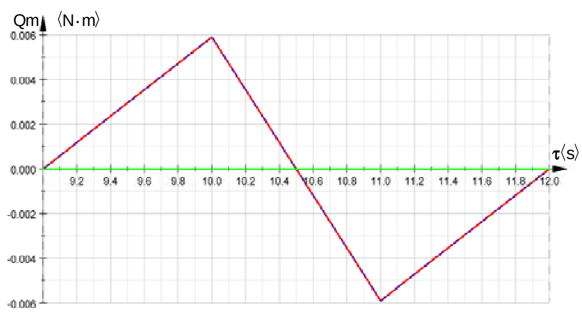

Figure 8 The representation of driving moment on

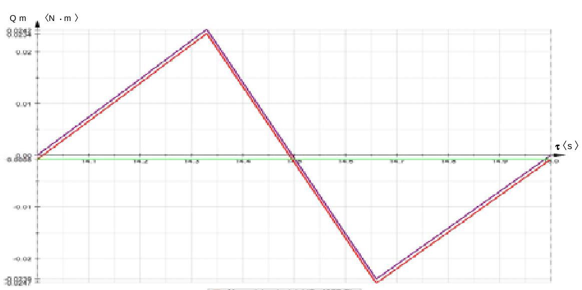

Figure 9 The representation of driving moment on

Analysing the previous graphs, for a correct determination of variation laws for the driving moments, is required a linear variation law for accelerations. Also, on the graphs corresponding to the driving moments, were represented, both static and dynamic components which together constituting the total driving moment.

6. CONCLUSIONS

The paper presents the implementation mode of a serial structure in a working process. In first phase, using the locating matrix algorithm, there are established the direct and inverse geometry equations.

Applying the Matrix Exponential Algorithm in kinematics, has been determined the Jacobian matrix (known as the velocity transfer matrix). In this sense, some of the results obtained by applying the mathematical exponential algorithm in direct geometry as well as the exponential functions in the direct kinematics are used as inputs. As major observation, the algorithmizing mathematical modelling of the robot structures having advantages, as simple visualization of the characteristics of different parameters; the degree of generalization.

The dynamic modelling called dynamic simulation, reveals the differential equations of motion through successive integrations lead to the determination of the motion laws on each kinetic link of mechanical structure.

The variation laws of coordinates, velocities, and accelerations have been established, having as input data the positions of characteristic points of working space. The dynamic control functions have been determined, using as kinetic energy. Using differential principles, specific to the holonomic mechanical systems, and applying the algorithm of the generalized forces in dynamics, the generalized driving forces were directly determined.
The generalized variables for which describing the robot motion can be replaced by polynomial time functions. It have been determined the kinematic control functions for the 2TR robot, and after it were established the polynomial interpolation functions having as real variable the time for description of the process. Substituting expressions into dynamic equations there were determined the variation laws of the driving moments.

As an important remark, by a proper estimation of the motor moments, the driving moments of the motors necessary to start up a kinematic axis, respectively the related braking systems, can be rigorously dimensioned, thus keeping up from damaging the robots.

7. REFERENCES

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Considerații privind implementarea roboților seriali în procese de lucru

Rezumat: Implementarea unei structuri mecanice de robot, într-un proces de lucru, constă în determinarea momentelor de antrenare ale fiecărui motor din cuplele cinematice. Pentru stabilirea acestor momente motoare, cunoscute sub denumirea de ecuații de control dinamic, se poate folosi un algoritm generalizat, bazat pe modelare geometrică, cinematică și dinamică, folosind algoritmi dedicați. În general printr-o determinare corectă și reală a momentelor de antrenare a motoarelor necesare punerii in miscare a unei axe cinematice, sistemele de frânare aferente, pot fi dimensionate riguros, evitându-se astfel situațiile critice, care pot duce la deteriorarea structurii mecanice. Lucrarea prezintă un exemplu de utilizare a algoritmilor matematici pentru a implementa un robot într-un proces de lucru.

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