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# ANALYTICAL INVESTIGATION ON BUCKLING STABILITY OF GLASS/CARBON HYBRID ANGLE-PLY COMPOSITE LAMINATES

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***Abstract:*** *This research is focused on the buckling stability of a hybrid Glass/Carbon composite laminated plates, we founded the present investigation on a simpler and efficient refined high order analytical model for predicting the critical mechanical buckling loads, the material properties considered in this study are Carbon's and Glass's, furthermore we used a mathematical approach to predict the performance of the mixture of two types of fibers in the same layer of the laminated composite plate. In view of the fact that everyone of these kinds of fibers excel singularly at least in one mechanical property, the mixture is undeniably to provide the best qualities and eliminate any possible deficiencies in these materials.*

***Key words:*** *Critical buckling load, angle-ply laminates, refined theory, shear stress, Carbon, Glass.*

## 1. INTRODUCTION

For the moment composite structures draw more and more attentiveness day after day due to their divers mechanical and physical advantages however it is expensive to carry out various experiments in order to predict their characteristics this obstacle drove scientists to adopt a more complex mathematical modeling to represent the actual behavior of composite structures The most important of these advanced analytical theories are those which take into consideration the effect of shear in terms of inferring stresses and strains like the shear deformation theory of first order originated in the mid-forties by Reissner and Mindlin [1] that includes the effect shearing in strains and stresses but this had a huge shortcoming due to its linear shear stresses distribution in the plate's thickness leading to necessity of inclusion of the coefficients shear stress correction [2],[3],[4]. So as to rectify this preceding deficiency a considerable number of higher order theories has been appeared starting with [5],[6],[7],[8],[9]. The refined theories emerged to facilitate the math formulas by decreasing the number of variables [10],[11],[12].

A numerous number of researchers have been considering the behavior of structures with advanced materials [13].

The most interesting characteristic of the refined theories is that they do not include shear correction coefficients and they are highly similar to the Euler Bernoulli's theory in ease of resolution and simplicity

In the present research the investigation of buckling stability of Glass/Carbon hybrid composite plates submitted to mechanical load is conducted using an efficient refined high order model. This theory allows for a distribution in parabolic shape of transversal shear strains and stresses along the plate's thickness and satisfies the condition of shear stress equal to zero at the lower edge (z=-h/2) and upper edge the plate (z=+h/2) and doesn't include any shear correction coefficients. The equilibrium equations are obtained by using virtual work's principle The plate's buckling loads are deduced thanks to the solution of Navier. In the aim to prove the validity the present model result's accuracy, we compare them against those of other theories from the literature

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## 2. MATERIAL PROPERTIES

The material properties considered in this investigation are those of a hybrid composite laminated plate, where the hybridization is done in each layer of the plate, this means that each layer contains two types of fibers.

In order to calculate the elasticity modulus of the hybrid composite laminates, we exploit the following formula [14]:

$$E_1 = V_f[E_C w_f + E_G(1 - w_f)] + E_E V_E \quad (1)$$

Where $E_1$ is longitudinal Young's modulus. $E_C$, $E_G$and $E_E$ are the Young modulus of the Carbon fiber, the Glass fiber and the matrix respectively. $V_C$, $V_G$, and $V_E$ are the fraction's volume of the Carbon fiber, the Glass fiber and the matrix, respectively, where:

$$V_C + V_G + V_E = 1 \quad (2)$$

And $w_f$ is the Carbon's percentage over the total fiber's volume fraction as follows:

$$w_f = \frac{V_C}{V_f} \quad (3)$$

Using the same approach, the Poisson's coefficient is given by

$$\nu_{12} = V_f[\nu_C w_f + \nu_G(1 - w_f)] + \nu_E V_E \quad (4)$$

## 3. KINEMATICS

The field of displacement in the present investigation is given by:

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w_b}{\partial x} - \frac{4z^3}{3h^2}\frac{\partial w_s}{\partial x} \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w_b}{\partial y} - \frac{4z^3}{3h^2}\frac{\partial w_s}{\partial y} \\ w(x,y,z) &= w_b(x,y) + w_s(x,y) \end{aligned} \quad (5)$$

Where "u" and "v" are the components of displacements in medium plane of the considered plate in the "x" and "y" directions, respectively; "$w_b$" is the bending component and "$w_s$" is the shear component of transversal displacement.

The strain obtained from the displacements in Eq. (5) are:

$$\varepsilon_x = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_b}{\partial x^2} - \frac{4z^3}{3h^2}\frac{\partial^2 w_s}{\partial x^2} \quad \text{(6-a)}$$

$$\varepsilon_y = \frac{\partial v_0}{\partial y} - z\frac{\partial^2 w_b}{\partial y^2} - \frac{4z^3}{3h^2}\frac{\partial^2 w_s}{\partial y^2} \quad \text{(6-b)}$$

$$\gamma_{xy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - z\frac{\partial^2 w_b}{\partial x\,\partial y} - \frac{4z^3}{3h^2}\frac{\partial^2 w_s}{\partial x\,\partial y} \quad \text{(6-c)}$$

$$\gamma_{yz} = \left(1 - \frac{4z^2}{h^2}\right)\frac{\partial w_s}{\partial y} \quad \text{(6-d)}$$

$$\gamma_{xz} = \left(1 - \frac{4z^2}{h^2}\right)\frac{\partial w_s}{\partial x} \quad \text{(6-e)}$$

$$\varepsilon_z = 0 \quad \text{(6-f)}$$

## 4. CONSTITUTIVE EQUATIONS

As the Hybrid composite plates are made of various orthotropic layers with arbitrarily oriented material axes, the equations constitutive of every single layer have to be transformed according to the coordinates of the plate (x, y, z). The strain-stress relations in the plate coordinates of each layer are as fellow

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \quad (7)$$

Where $\bar{Q}_{ij}$are the transformed material constants [10].

## 5. GOVERNING EQUATIONS

Virtual work principle is used in this study in order to obtain the equilibrium equations appropriate to the displacement field and the constitutive equations. It can be given in the following formula

$$\frac{1}{2}\int_V \sigma_{ij}\delta\varepsilon_{ij}dV + \frac{1}{2}\int_A \left[N_x^0\frac{\partial^2(w_b+w_s)}{\partial x^2} + N_y^0\frac{\partial^2(w_b+w_s)}{\partial y^2} + 2N_{xy}^0\frac{\partial^2(w_b+w_s)}{\partial x\,\partial y}\right]dxdy = 0 \quad (8)$$

Where$N_x^0$, $N_y^0$ and $N_{xy}^0$ are in-plane distributed forces.

Substituting Eqs. (7) into Eq. (8) and integrating them by parts, assembling the coefficients of $\delta u_0, \delta v_0$, $\delta w_b$ and $\delta w_s$, the equations of equilibrium for the hybrid laminated plate are given by the formulas:

$$\delta u_0: \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \qquad (9\text{-a})$$

$$\delta v_0: \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \qquad (9\text{-b})$$

$$\delta w_b: \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x\,\partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + N_x^o \frac{\partial^2}{\partial x^2}[w_b + w_s] + N_y^o \frac{\partial^2}{\partial y^2}[w_b + w_s] + 2N_{xy}^o \frac{\partial^2}{\partial x\,\partial y}[w_b + w_s] = 0 \qquad (9\text{-c})$$

$$\delta w_s: \frac{\partial^2 M_x^s}{\partial x^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x\,\partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial Q_{xz}^s}{\partial x} + \frac{\partial Q_{yz}^s}{\partial y} + N_x^o \frac{\partial^2}{\partial x^2}[w_b + w_s] + N_y^o \frac{\partial^2}{\partial y^2}[w_b + w_s] + 2N_{xy}^o \frac{\partial^2}{\partial x\,\partial y}[w_b + w_s] = 0 \qquad (9\text{-d})$$

## 6. ANALYTICAL SOLUTION

The Navier solutions can be developed for an antisymmetric simply supported angle-ply hybrid laminated plates, the boundary conditions are satisfied by:

$$\begin{aligned} u_0 &= \textstyle\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} U_{mn}\, sin(\lambda\, x)\, cos(\mu\, y) \\ v_0 &= \textstyle\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} V_{mn}\, cos(\lambda\, x)\, sin(\mu\, y) \\ w_b &= \textstyle\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{bmn}\, sin(\lambda\, x)\, sin(\mu y) \quad (10) \\ w_s &= \textstyle\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{smn}\, sin(\lambda\, x)\, sin(\mu\, y) \end{aligned}$$

Where $U_{mn}$, $V_{mn}$, $W_{bmn}$ and $W_{smn}$ are unknown variables must be determined, $\lambda = \frac{m\pi}{a}$ and $\mu = \frac{n\pi}{b}$.

Substituting Equation (10) into Equation (9), the solution of Navier of the hybrid laminated antisymmetric angle-ply plate can be deduced from the following equation:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{12} & a_{22} & a_{23} & a_{24} \\ a_{13} & a_{23} & a_{33}+k & a_{34}+k \\ a_{14} & a_{24} & a_{34}+k & a_{44}+k \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{bmn} \\ W_{smn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (11)$$

Where

$$a_{11} = A_{11}\lambda^2 + A_{66}\mu^2 \qquad (12\text{-a})$$

$$a_{22} = A_{66}\lambda^2 + A_{22}\mu^2 \qquad (12\text{-b})$$

$$a_{12} = \lambda\mu\,(A_{12} + A_{66}) \qquad (12\text{-c})$$

$$a_{13} = -(3B_{16}\lambda^2\mu + B_{26}\mu^3) \qquad (12\text{-d})$$

$$a_{14} = -(3B_{16}^s\lambda^2\mu + B_{26}^s\mu^3) \qquad (12\text{-e})$$

$$a_{24} = -(B_{16}^s\lambda^3 + 3B_{26}^s\lambda\mu^2) \qquad (12\text{-f})$$

$$a_{23} = -(B_{16}\lambda^3 + 3B_{26}\lambda\mu^2) \qquad (12\text{-g})$$

$$\begin{aligned} a_{33} &= D_{11}\lambda^4 + 2(D_{12} + 2D_{66})\lambda^2\mu^2 + D_{22}\mu^4 \\ a_{34} &= D_{11}^s\lambda^4 + 2(D_{12}^s + 2D_{66}^s)\lambda^2\mu^2 + D_{22}^s\mu^4 \\ a_{44} &= H_{11}^s\lambda^4 + 2(H_{12}^s + 2H_{66}^s)\lambda^2\mu^2 + H_{22}^s\mu^4 + A_{55}^s\lambda^2 + A_{44}^s\mu^2 \\ k &= N_x^0\lambda^2 + N_y^0\mu^2 \qquad (12\text{-h}) \end{aligned}$$

## 7. VALIDATION AND DISCUSSION OF NUMERICAL RESULTS

An investigation on buckling stability of a Glass/Carbon hybrid laminated composite plate is presented using a refined and efficient shear deformation theory in this study. The critical mechanical buckling loads are obtained by using the solution of Navier. In order to validate the present model, the following mechanical properties of the hybrid composite plate are used [15]: $G_{12} = G_{13} = 0{,}6E_2, G_{23} = 0{,}5E_2$ $E_1 = 40E_2$, $\nu_{12} = 0{,}25$

Where, the results obtained by the present model are compared with those of the Ren [16], the Whitney [17] and the Reddy [18] theories.

The buckling load is obtained in the dimensionless form as fellow:

$$\bar{N} = N_{cr}\left(\frac{a^2}{E_2 h^3}\right) \qquad (13)$$

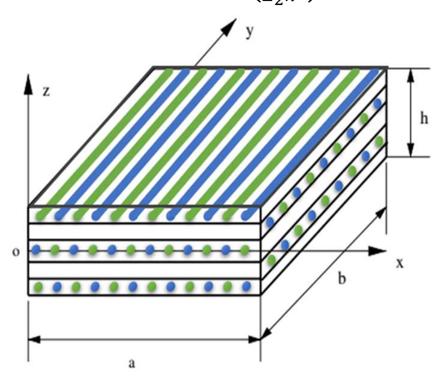


**Fig. 1.** A hybrid composite plate.

The mechanical characteristics of Carbon fibers, Glass fibers and the Epoxy matrix used in this study are given according to Berthelot [19] in Table 1.

*Table 1*

**Mechanical properties**

| Material | Young's modulus (GPa) | Poisson's ratio |
|---|---|---|
| Carbon fibers | 380 | 0.33 |
| Glass fibers | 86 | 0.22 |
| Epoxy matrix | 3.45 | 0.3 |

*Table 2*

**Buckling loads $\bar{N}$ of a square angle-ply $(\theta/-\theta)$ laminated composite plate**

| Orientation | Theory | a/h | | |
|---|---|---|---|---|
| | | 4 | 10 | 100 |
| $(30/-30)$ | Ren [16] | 9.5368 | 15.7517 | 20.4793 |
| | Reddy [18] | 9.3391 | 17.2795 | 20.5040 |
| | Whitney [17] | 7.5450 | 16.6132 | 20.4944 |
| | Present | 9.3518 | 17.2795 | 20.5040 |
| $(45/-45)$ | Ren [16] | 9.8200 | 16.4558 | 21.6384 |
| | Reddy [18] | 8.2377 | 18.1544 | 21.6576 |
| | Whitney [17] | 6.7858 | 17.5522 | 21.6576 |
| | Present | 8.3963 | 18.1544 | 21.6663 |

Table 2 represents an angle-ply oriented (30°/-30°) and (45°/-45°) square composite plate under mechanical load, where, the results of the various theories are putted in comparison. When compared to different high order theories (Ren [16], Reddy [18] and Whitney [17]), we can easily notice the precision and accuracy of our theory in determining axial buckling load for antisymmetric composite laminated plates.

The volume fraction effect on critical buckling load variation of a Glass/Carbon hybrid antisymmetric simply supported composite plates is showed in Fig 2 and table 3 Where the mechanical buckling load is maximum where the fibers are fully Carbon ($w_f = 100\%$ Carbon) and decrease gradually in accordance with the fibers combination $w_f$ until arriving at its minimum value for the fibers are fully Glass ($w_f = 100\%$ Glass) this variation is caused by the influence of the rigidity of the fibers used in the hybrid plate Also the amount of fibers against the plate's volume fraction has a great impact on the variation of Critical buckling load where it increases gradually in accordance with the volume fraction variation until arriving at its maximum for $V_f = 0.45$.

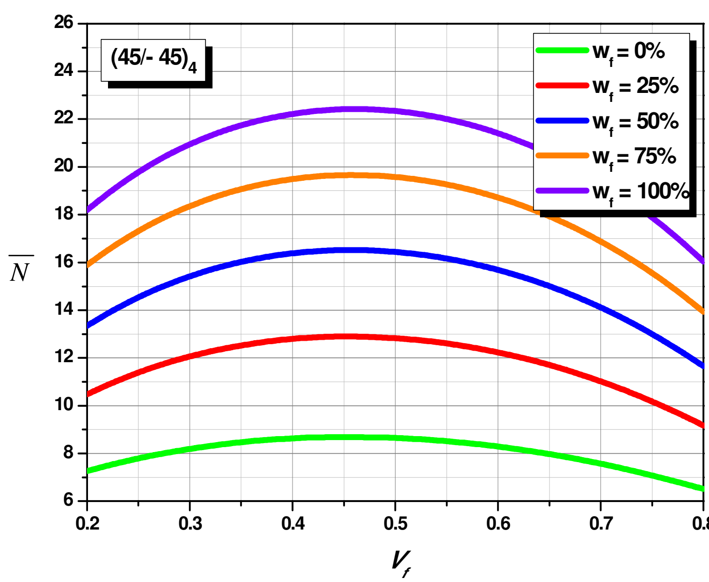


**Fig. 2.** The volume fraction $V_f$ effect on the mechanical buckling load $\bar{N}$ of a hybrid Glass/Carbon angle ply composite plate

*Table 3*

**The volume fraction $V_f$ effect on the mechanical buckling load $\bar{N}$ of a hybrid Glass/Carbon angle ply composite plate (45/-45)**

| Fiber's percentages (%) | | $V_f$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Carbon | Glass | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 |
| 0 | 100 | 6.5407 | 6.6619 | 6.7308 | 6.7520 | 6.7287 | 6.6628 | 6.6556 | 6.4069 | 6.2156 |
| 10 | 90 | 7.2737 | 7.4262 | 7.5133 | 7.5401 | 7.5106 | 7.4276 | 7.2929 | 7.1071 | 6.8695 |
| 20 | 80 | 7.9578 | 8.1424 | 8.2482 | 8.2811 | 8.2459 | 8.1460 | 7.9839 | 7.7612 | 7.4777 |
| 30 | 70 | 8.6131 | 8.8303 | 8.9552 | 8.9943 | 8.9533 | 8.8363 | 8.6466 | 8.3865 | 8.0565 |
| 40 | 60 | 9.2410 | 9.5003 | 9.6445 | 9.6899 | 9.6431 | 9.5088 | 9.2911 | 8.9930 | 8.6159 |
| 50 | 50 | 9.8746 | 10.1583 | 10.3219 | 10.3739 | 10.3210 | 10.1693 | 9.2933 | 9.5868 | 9.1621 |
| 60 | 40 | 10.4905 | 10.8078 | 10.9911 | 11.0495 | 10.9907 | 10.8213 | 10.5468 | 10.1716 | 9.6988 |
| 70 | 30 | 11.0999 | 11.4512 | 11.6542 | 11.7190 | 11.6542 | 11.4671 | 11.1639 | 10.7498 | 10.2285 |
| 80 | 20 | 11.7047 | 12.0899 | 12.3127 | 12.3840 | 12.313 | 12.1082 | 11.7762 | 11.3229 | 10.7529 |
| 90 | 10 | 12.3057 | 12.7245 | 12.9677 | 13.0456 | 12.9686 | 12.7458 | 12.3848 | 11.8922 | 11.2731 |
| 100 | 0 | 12.9037 | 13.3572 | 13.6199 | 13.7043 | 13.6213 | 13.3805 | 12.9905 | 12.4584 | 11.7902 |

*Table 4*

**The thickness ratio *a/h* effect on the critical load $\bar{N}$ of a hybrid Glass/Carbon angle ply composite plate $(45/-45)_4$**

| Fiber's percentages (%) | | a/h | | | | |
|---|---|---|---|---|---|---|
| Carbon | Glass | 5 | 10 | 20 | 50 | 100 |
| 0 | 100 | 4.8304 | 6.1425 | 6.5932 | 6.7318 | 6.7520 |
| 10 | 90 | 5.2423 | 6.7962 | 7.3449 | 7.5151 | 7.5401 |
| 20 | 80 | 5.6154 | 7.4027 | 8.0491 | 8.2514 | 8.2811 |
| 30 | 70 | 5.9608 | 7.9783 | 8.7244 | 8.9597 | 8.9943 |
| 40 | 60 | 6.2849 | 8.5321 | 9.3806 | 9.6502 | 9.6899 |
| 50 | 50 | 6.5917 | 9.0691 | 10.0234 | 10.3287 | 10.3739 |
| 60 | 40 | 6.8838 | 9.5926 | 10.6560 | 10.9987 | 11.0495 |
| 70 | 30 | 7.1630 | 10.1045 | 11.2807 | 11.6623 | 11.7190 |
| 80 | 20 | 7.4307 | 10.6063 | 11.8989 | 12.3212 | 12.3840 |
| 90 | 10 | 7.6880 | 11.0989 | 12.5117 | 12.9763 | 13.0456 |
| 100 | 0 | 7.9357 | 11.5833 | 13.1197 | 13.6283 | 13.7043 |

*Table 5*

**The stacking effect on the mechanical buckling load $\bar{N}$ of a hybrid Glass/Carbon angle ply composite plate $(45/-45)_4$**

| Number of layers | a/h | Fiber's Combination | | | |
|---|---|---|---|---|---|
| | | 100% Glass | 25% Carbon + 75% Glass | 50% Carbon + 50% Glass | 100% Carbon |
| $(45/-45)_1$ | 5 | 4.8304 | 5.7911 | 6.5917 | 7.9357 |
| | 10 | 6.1425 | 7.6937 | 9.0691 | 11.5838 |
| | 20 | 6.5932 | 8.3896 | 10.0234 | 13.1197 |
| | 100 | 6.7520 | 8.6404 | 10.3739 | 13.70423 |
| $(45/-45)_2$ | 5 | 5.8998 | 7.5817 | 8.7707 | 10.3561 |
| | 10 | 8.1776 | 11.8736 | 15.0825 | 20.3913 |

| | | | | | |
|---|---|---|---|---|---|
| | 20 | 9.0574 | 13.8536 | 18.4450 | 27.0834 |
| | 100 | 9.3807 | 14.6366 | 19.8673 | 30.2802 |
| $(45/-45)_4$ | 5 | 6.1890 | 8.0564 | 9.3506 | 11.0414 |
| | 10 | 8.6898 | 12.8980 | 16.5212 | 22.4226 |
| | 20 | 9.6737 | 15.2078 | 20.5126 | 30.4501 |
| | 100 | 10.0381 | 16.1350 | 22.2387 | 30.4170 |
| $(45/-45)_8$ | 5 | 6.26360 | 8.1710 | 9.5034 | 11.2272 |
| | 10 | 8.8187 | 13.1555 | 16.8828 | 22.9341 |
| | 20 | 9.8280 | 15.5465 | 21.0295 | 31.2908 |
| | 100 | 10.2024 | 16.5097 | 22.8315 | 35.4511 |
| $(45/-45)_{16}$ | 5 | 6.2824 | 8.2112 | 9.5422 | 11.2747 |
| | 10 | 8.8510 | 13.2110 | 16.9734 | 23.0624 |
| | 20 | 9.8666 | 15.6313 | 21.1588 | 31.5010 |
| | 100 | 10.2435 | 16.6033 | 22.9797 | 35.7096 |

The tables 4 and 5 also figure 3 represent the effects of geometry side to thickness and piling on the variation of mechanical buckling loads for a Glass/Carbon hybrid composite plate. For each case of fibers combination of Glass and Carbon fibers this critical load accretes with the increase of the layer's number and the decrease of the plate's thickness, this variation is in a good agreement with the fact that the diminution of the thickness for the same amount of layers leads to a concentration of the fibers and consequently an increase of the plate's rigidity.

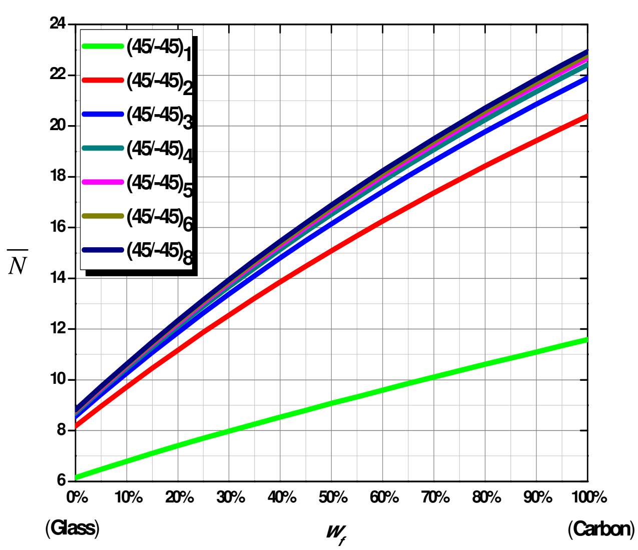


**Fig. 3.** The stacking effect on the mechanical buckling load $\bar{N}$ of a hybrid Glass/Carbon angle ply composite plate

## 8. CONCLUSION

An investigation of the Buckling stability of a hybrid Glass/Carbon composite plates was successfully conducted based on an accurate and precise simple refined high order shear theory.

The present model has been well proved of being accurate and efficient for buckling stability of angle ply antisymmetric hybrid composite plates.

This analytical investigation allows us to describe the behavior of hybrid Glass/Carbon composite plates by predicting the critical mechanical buckling loads in a neat way, which subrogates the costly experimental methods.

The main reason of using the Glass fiber and Carbon fiber in this investigation is that Carbon fibers provide a high rigidity, knowing that these qualities are very costly, in contrast glass fibers are cheaper accompanied with a poor mechanical quality, this is why it is the most used type of fibers in industrial sector.

This association of these kinds of fibers allow industrials to combine in an intelligent manner between their interesting and required qualities, in the first side, we benefit from the high rigidity assured by the Carbon fibers, and in the second side, the glass fibers assures economic materials by designing cheaply, where incorporating a slight quantity of glass fibers to the hybrid composite structures decreases undeniably the composite's production cost only sacrificing a small percentage in terms of rigidity.

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## INVESTIGAȚIE ANALITICA ASUPRA STABILITAȚII LA FLAMBAJ A TICLEI/CARBONULUI PLACI COMPOZITE HIBRIDE CU STRATURI UNGHIULARE

**Rezumat:** Această cercetare se axează pe stabilitatea la flambaj a unei plăci compozite hibride din sticlă/carbon, prezenta investigație se bazează pe un model analitic de deformare prin forfecare rafinat simplu și eficient, proprietățile materialului adoptat în acest studiu sunt ale carbonului și sticlei, de asemenea. am folosit o abordare matematică pentru a prezice performanța combinației a două tipuri de fibre în același strat al plăcii compozite laminate. Având în vedere faptul că fiecare dintre aceste tipuri excelează în mod singular cel puțin într-o proprietate mecanică, amestecul este incontestabil să ofere cele mai bune calități și să elimine eventualele deficiențe ale acestor materiale.

**Cuvinte cheie:** sarcină critică de flambaj, laminate cu straturi unghiulare, teorie rafinată, efort de forfecare, carbon, sticlă.

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