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COMPARATIVE ANALYSYS OF THE POSSIBILITIES TO EVALUATE THE DYNAMIC PERFORMANCES FOR INTERNAL COMBUSTION, ELECTRIC AND HYBRID VEHICLES BY MATHEMATICAL MODELING

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Abstract: *The paper evaluates mathematically the performance of the vehicles by identifying the possibilities of evaluating their driving performances in different considered operating situations. The forces at the wheels, forward resistance forces and the forces and powers required to overcome them, for different driving conditions, are achieved by developing a numerical calculation model in the MathCad program, which provides comparative results, with graphical interpretation, on the characteristics of traction and power of the vehicles under study, taking into account the propulsion system, the construction parameters, the used gears, different tilt angles of the road, etc. Thus, it is possible to identify the influence of the vehicle's construction parameters and the driving conditions, on the dynamic performances, with reference to the traction and acceleration possibilities of the considered vehicles, but also to identify the gears that can be used in different operating conditions.*

Key words*: vehicle, propulsion, forward resistance, traction, dynamic performance*

1. INTRODUCTION

Given the statistical data [16] from recent years, it has been shown that with the increase in the number of vehicles, fuel consumption has also increased, with significant decreases in natural resources, thus increasing the pollution of the urban environment. The automotive industry has created new alternative fuel systems, but also alternative propulsion systems [2, 4, 11, 14], trying to reduce the resource consumption and urban environmental pollution.

The continuous increase in the number of vehicles each year brings with it an increase in fuel consumption and environmental pollution due to pollutant emissions from vehicles powered by internal combustion engines. Thus, according to studies [1, 2, 4, 5, 6, 11, 15] in the field of the car industry, numerous comparisons have been made between vehiches equipped with internal combustion propulsion systems and those with electric and hybrid propulsion systems, mainly focused on fuel consumption and pollutant emissions in different operating conditions. The obtained results are mostly favorable for hybrid and electric vehicles [11].

Studies to date [1, 3, 4, 5, 6, 11, 14, 15] show that the strategy of dynamic energy management of electric and hybrid vehicles is much more economical for these vehicles' users, being beneficial as well as in terms of pollution reduction. The transmission systems of the alternative propulsion vehicles offer better control of the power transmitted to the drive wheels, through faster acceleration and braking with energy recovery.

In this paper, a study was performed on three vehicles equipped with: internal combustion, electric and hybrid propulsion systems, comparable in terms of performance, obtaining the dynamic parameters of each vehicle under study, by numerical modeling in the MathCad program, taking into account the technical data provided by the manufacturers. Following the numerical modeling, results were obtained in graphical form of the dynamic performance of the vehicles considered, in terms of their traction and power characteristics, taking into account different driving conditions.

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2. NUMERICAL EVALUATION METHOD

2.1. Notations used in the numerical calculation model

In order to develop the numerical calculation algorithm in the MathCAD program, the construction parameters of the vehicles and the different driving conditions are taken into account - longitudinal inclination, nature and condition of the road, etc.

Table 1 [8] shows the meanings of the units used in the numerical model.

Table 1

The main notations used in the calculation algorithm

Unit	Notation	Measurement unit
❑ maximum engine power	P_{max}	kW
❑ power at the wheel	$P_{R_{j,s}}$	kW
❑ maximum engine torque	M_{max}	daN·m
❑ torque corresponding to the maximum engine power	M_P	daN·m
❑ torque at the wheel	$M_{R_{i,j}}$	daN·m
❑ wheel force	$F_{Rv_{j,s}}$	daN
❑ the rolling resistance force and the power necessary to overcome it	R_{r_p} $P_{r_{vD,p}}$	daN, kW
❑ the resistance force due to the longitudinal inclination of the road and the power necessary to overcome it	R_{p_p} $P_{p_{vD,p}}$	daN, kW
❑ the total resistance force of the road and the power necessary to overcome it	R_{ψ_p} $P_{\psi_{vD,p}}$	daN, kW
❑ the air resistance force and the power necessary to overcome it	$R_{a_{vD}}$ $P_{a_{vD,p}}$	daN, kW
❑ the sum of external resistances and powers, which do not depend on the character of the movement	$R_{ext_{vD,p}}$	daN, kW
❑ vehicle weight	G_a	daN
❑ wheel's rolling radius	r_d	m
❑ wheel's free radius	r_0	m
❑ wheel's nominal radius	r_n	m
❑ gravitational acceleration	g	m/s^2
❑ cross-section area of the vehicle	S_f	m^2
❑ vehicle travel speed	v	km/h
❑ aerodynamic coefficient	k_a	$daN{\cdot}s^2/m^4$
❑ number of forward gears in the gearbox	k	-
❑ a certain gear (j = 1÷k)	j	-
❑ minimum vehicle speed for the first gear	v_{min_1}	km/h
❑ minimum vehicle speed for gear j	v_{min_j}	km/h
❑ maximum vehicle speed for gear k	v_{max_k}	km/h
❑ maximum possible vehicle speed	v_{max_c}	km/h
❑ engine idle speed	n_{min}	rpm
❑ speed corresponding to the maximum effective torque (M_{max}) of the engine	n_M	rpm
❑ speed corresponding to the maximum power (P_{max}) of the engine	n_P	rpm
❑ angular speed and rotational speed of the electric motor, specific to the start-up mode of the electric motor operation at its maximum power and the final operating mode at the maximum torque	n_{MP} ω_{MP}	rpm rad/s
❑ maximum engine speed	n_{max}	rpm
❑ rolling resistance coefficient	f	-
❑ the transmission ratio from the gearbox, which is engaged in gear j	i_{cv_j}	-
❑ the transmission ratio of the main transmission	i_0	-
❑ the total transmission ratio of the transmission, the gearbox being engaged in gear j	i_{t_j}	-
❑ the coefficient of total road resistance or the total specific resistance of the road	ψ	-
❑ total mechanical transmission efficiency	η_t	-
❑ tire deformation coefficient	λ	-

2.2. Numerical modeling

In order to develop the numerical calculation model, the following steps were carried out:

- *Calculation of the dynamic rolling radius of the vehicle wheels.* The marking of the tires is identified, and their main dimensions are determined for each vehicle under study, and based on the numerical calculation algorithm the free radius of the wheel is obtained $r_0 \cong r_n$, in m, at an air pressure inside the tire of $p_i = 0.22$ MPa. Thus, the dynamic radius r_d of the wheels can be determined according to the relation [7, 10, 12, 13]:

$$r_d = \lambda \cdot r_0, \text{ in m}, \quad (1)$$

in which, λ = 0.945÷0.950 [7, 10, 12, 13].

The technical specifications [17, 18, 19] of the vehicles under study (Vehicle 1 - internal combustion: Mitsubishi Lancer; Vehicle 2 - Electric: Volkswagen e-Golf; Vehicle 3 - Hybrid: Hyundai Ioniq) are listed in Table 2. It is mentioned that all the vehicles considered have the same year of manufacture, this being 2017.

Table 2

Technical specifications of the vehicles under study

	Parameter	Notation	Vehicle 1 - internal combustion	Vehicle 2 - electric	Vehicle 3 - hybrid	Measurement unit
Engine	❑ maximum power	$P_{1,2,3\ max}$	109	101	105	kW
	❑ speed at maximum power	$n_{1,2,3\ P}$	6000	3000	5700	rpm
	❑ maximum torque	$M_{1,2,3\ max}$	19.7	29.0	14.7	daN·m
	❑ speed at the maximum torque	$n_{1,2,3\ M}$	4200	3000	4000	rpm
	❑ maximum speed	$v1,2,3_{max}$	194	150	185	km/h
Transmission	❑ gearbox type		Manual	CVT	Automatic	-
	❑ number of gears	k1,2,3	5	1	6	-
	❑ marking of the tires	-	205/60 R16	205/55 R16	195/65 R16	-
	❑ the transmission ratio of the main transmission	$i1,2,3_0$	3.94	3.61	3.61	-
Mass	❑ own vehicle mass	$m1,2,3_0$	1310	1540	1370	kg
	❑ maximum permissible mass	$m1,2,3_{at}$	1800	2020	1870	kg

- *Calculation of the transmission ratios in the gearbox of each vehicle studied.* The technical specifications of the vehicles under consideration shall be identified, with reference to: engine characteristics - maximum power P_{max} at the speed n_P corresponding to it; maximum torque M_{max} at speed n_M corresponding to it; transmission characteristics - number of gears, transmission ratio of the main transmission i_0, transmission ratios from the gearbox i_{cv_j}; specific masses - maximum permissible mass m_{at}. The transmission ratio i_{cv_1} in the first gear of the gearbox shall be determined according to the relation [7, 10, 12, 13]:

$$i_{cv_1} = \frac{\pi \cdot r_d \cdot n_M}{30 \cdot i_0 \cdot v_{cr_1}}, \quad (2)$$

in which, v_{cr_1} is the critical speed in the first gear of the vehicle's gearbox, and is calculated with the relation [7, 10, 12, 13]:

$$v_{cr_1} = \frac{\eta_t \cdot P_M}{G_a \cdot \psi_{max}}, \text{ in m/s,} \quad (3)$$

where: η_t - is the total transmission efficiency; ψ_{max} - coefficient of total road resistance:

$$\psi_{max} = f \cdot \cos\alpha_{max} + \sin\alpha_{max}, \quad (4)$$

f - rolling resistance coefficient, which is determined with the relation [7, 10, 12, 13]:

$$f = 0.0125 + 0.0085 \cdot \left(\frac{v_{max}}{100}\right)^{2.5}, \quad (5)$$

α_{max} - the angle of longitudinal road inclination (α_{max}= 12÷15° - for single-axle motor vehicles) [7, 10, 13];

P_M - the power corresponding to the maximum torque, is obtained from the relation [7, 10, 12, 13]:

$$P_M = \frac{M_{max} \cdot \pi \cdot n_M}{30}, \text{ in W.} \quad (6)$$

Transmission ratio i_{cv_j}, corresponding to the gears, for each vehicle, it is determined with the relation [7, 10, 12, 13]:

$$i_{cv_j} = \sqrt[k-2]{i_{cv_1}^{(k-1-j)}}, \quad (7)$$

where: k - is the number of forward gears in the gearbox, j - the considered gear.

- *Calculation of extreme speeds corresponding to gears.* In order to determine the total transmission ratio i_{t_j}, corresponding to each gear, the following relation was used [4, 7, 10, 12, 13, 14]:

$$i_{t_j} = i_0 \cdot i_{cv_j}. \tag{8}$$

Minimum vehicle travel velocities v_{min_j} and maximum v_{max_j}, in km/h, are determined for each gear in a situation where gear change is optimal, thus [7, 8, 9, 10, 12, 13]:

$$v_{min_j} = \begin{cases} 0.377 \cdot \frac{n_{min} \cdot r_d}{i_{t_j}}, \text{if } j = 1 \\ 0.377 \cdot \frac{n_M \cdot r_d}{i_{t_j}}, \text{if } j \geq 2 \end{cases}, \tag{9}$$

$$v_{max_j} = \begin{cases} 0.377 \cdot \frac{n_P \cdot r_d}{i_{t_j}}, \text{if } j \leq (k-2) \\ 0.377 \cdot \frac{n_{max} \cdot r_d}{i_{t_j}}, \text{if } j = k-1 \\ 0.377 \cdot \frac{n_{lim} \cdot r_d}{i_{t_j}}, \text{if } j = k \end{cases}, \tag{10}$$

in which: the speed limit n_{lim}, is calculated with the relation [7, 8, 9, 10]:

$$n_{lim} = \frac{v_{maxc} \cdot i_{t_k}}{0.377 \cdot r_d}, \text{ in rpm}, \tag{11}$$

where i_{t_k} is the transmission ratio corresponding to the last gear.

- *Determination of the power transmitted to the driving wheels*. After identifying the necessary data from the technical specifications of the motor vehicles, namely: maximum power P_{max} of the engine, in kW, at speed n_P, in rpm, respectively the maximum torque M_{max} of the engine, in daN·m, at speed n_M, in rpm, the moment developed by the engine can be determined at its maximum power M_P, in daN·m, according to the relation [7, 10, 12, 13]:

$$M_P = 955.4 \cdot \frac{P_{max}}{n_P}. \tag{12}$$

Engine power at different operating modes of the engine, i.e. at different engine speeds, can be determined according to the relation [7, 10, 12, 13]:

$$P_{n_i} = \begin{cases} P_{max} \cdot \left[\alpha \cdot \left(\frac{n_i}{n_P}\right) + \beta \cdot \left(\frac{n_i}{n_P}\right)^2 - \gamma \cdot \left(\frac{n_i}{n_P}\right)^3\right], \text{if } n_i \leq n_{med} \\ P_{max} \cdot \left[\alpha' \cdot \left(\frac{n_i}{n_P}\right) + \beta' \cdot \left(\frac{n_i}{n_P}\right)^2 - \gamma' \cdot \left(\frac{n_i}{n_P}\right)^3\right], \text{if } n_i > n_{med} \end{cases}, \text{ in kW}, \tag{13}$$

in which: n_i are the engine speeds in the range of $[n_{min} \div n_{max}]$, which can be determined at its different operating modes, thus [7, 8, 9, 10]:

$$n_i = n_{min} + \frac{n_{max} - n_{min}}{99} \cdot i, \tag{14}$$

with $n_{min} \cong 0.2 \cdot n_P$, $n_{max} \cong 1.1 \cdot n_P$, $i = 0 \div 99$; n_{med} - average engine speed $\left(n_{med} = \frac{n_M + n_P}{2}\right)$;

α, β, γ, α', β', γ'- unitless form coefficients that depend on the elasticity of the engine and the width of its stability zone, the condition of which exists between them [7, 8, 9, 10, 12, 13]:

$$\begin{cases} \alpha + \beta - \gamma = 1 \\ \alpha' + \beta' - \gamma' = 1 \end{cases}. \tag{15}$$

The power transmitted to the wheel, at different speeds of the crankshaft of the engine, is given by the relation [4, 7, 8, 9, 10, 12, 13]:

$$P_{Rn_i} = \eta_t \cdot P_{n_i}, \text{ in kW}. \tag{16}$$

For the determination of the engine power and the power transmitted to the wheels according to the vehicle travel velocity, the following are established: a variable s, (s = 0 ÷ 77), and intermediate velocities $v_{j,s}$ of each gear, in the range of $\left[v_{min_j} \div v_{max_j}\right]$, for each type of transmission [7, 8, 9, 10]:

$$v_{j,s} = v_{min_j} + \frac{v_{max_j} - v_{min_j}}{77} \cdot s. \tag{17}$$

The engine power, depending on the speed gears corresponding to the gears, is determined from the relation (13), in which the engine speed is expressed function of the vehicle velocity, in the form of: $n = \frac{v \cdot i_t}{0.377 \cdot r_d}$, in rpm, obtaining [7, 8, 9, 10]:

$$P_{j,s} = \begin{cases} P_{max} \cdot \left[\alpha \cdot \frac{\frac{v_{j,s} \cdot i_{t_j}}{0.377 \cdot r_d}}{n_P} + \beta \cdot \left(\frac{\frac{v_{j,s} \cdot i_{t_j}}{0.377 \cdot r_d}}{n_P}\right)^2 - \gamma \cdot \left(\frac{\frac{v_{j,s} \cdot i_{t_j}}{0.377 \cdot r_d}}{n_P}\right)^3\right], \text{ if } v_{j,s} \leq v_{med_j} \\ P_{max} \cdot \left[\alpha' \cdot \frac{\frac{v_{j,s} \cdot i_{t_j}}{0.377 \cdot r_d}}{n_P} + \beta' \cdot \left(\frac{\frac{v_{j,s} \cdot i_{t_j}}{0.377 \cdot r_d}}{n_P}\right)^2 - \gamma' \cdot \left(\frac{\frac{v_{j,s} \cdot i_{t_j}}{0.377 \cdot r_d}}{n_P}\right)^3\right], \text{ if } v_{j,s} > v_{med_j} \end{cases}, \text{ in kW}, \tag{18}$$

for the situation where intermediate vehicle speeds $v_{j,s}$, in km/h, of each gear, varies between limits v_{min_j} and v_{max_j}, calculated with the relations (9) and (10), using a relation of the form (17), and the travel speed v_{med_j} can be determined according to the relation [7, 8, 9, 10]:

$$v_{med_j} = 0.377 \cdot \frac{r_d \cdot n_{med}}{i_{t_j}}, \text{ in km/h}, \quad (19)$$

and the power transmitted to the wheel, with the relation [4, 7, 8, 9, 10, 12, 13]:

$$P_{R_{j,s}} = \eta_t \cdot P_{j,s}, \text{ in kW}. \quad (20)$$

• *Determination of the torque transmitted to the driving wheels.* Engine torque, depending on the crankshaft speeds of the engine, can be determined according to the relation [7, 10, 12, 13]:

$$M_i = \begin{cases} M_P \cdot \left[\alpha + \beta \cdot \left(\frac{n_i}{n_P}\right) - \gamma \cdot \left(\frac{n_i}{n_P}\right)^2\right], \text{if } n_i \le n_{med} \\ M_P \cdot \left[\alpha' + \beta' \cdot \left(\frac{n_i}{n_P}\right) - \gamma' \cdot \left(\frac{n_i}{n_P}\right)^2\right], \text{if } n_i > n_{med} \end{cases}, \text{ in daN·m}, \quad (21)$$

and the torque transmitted to the wheel, for each gear, depending on the engine speed, it is determined according to the relation [7, 8, 9, 10, 12, 13, 14]:

$$M_{R_{i,j}} = M_i \cdot \eta_t \cdot i_{t_j}, \text{ in daN·m}. \quad (22)$$

The engine torque, depending on the gear speeds corresponding to the gears, for each type of vehicle under study can be determined from the relation (21), in which the engine speed is expressed according to the speed of the vehicle, in the form of: $n = \frac{v \cdot i_t}{0.377 \cdot r_d}$, in rpm, or according to the relation [7, 8, 9, 10]:

$$M_{j,s} = 955.4 \cdot \frac{P_{j,s}}{\left(\frac{v_{j,s} \cdot i_{t_j}}{0.377 \cdot r_d}\right)}, \text{ in daN·m}, \quad (23)$$

and for the determination of the torque transmitted to the wheel, for each gear, depending on the speed of travel, can be used the relation [7, 8, 9, 10, 12, 13, 14]:

$$M_{Rv_{j,s}} = M_{j,s} \cdot \eta_t \cdot i_{t_j}, \text{ in daN·m}. \quad (24)$$

In the case of an electric motor vehicle, for determination of the torque and power at the wheel, the speed required is n_{MP} specific to the start-up mode of operation of the electric motor at its full power $P_{max_{el(n_{MP})}}$ and the final operating mode at the maximum torque $M_{max_{el(n_{MP})}}$. The angular speed ω_{MP} of the electric motor can be defined according to the relation [10]:

$$\omega_{MP} = \frac{P_{max_{el(n_{MP})}}}{M_{max_{el(n_{MP})}}}, \text{ in rad/s}, \quad (25)$$

but various other angular speed can be determined by the following relations:

$$\omega_{elA_{i_{el}}} = \omega_{elA_{min}} + \frac{\omega_{elA_{max}} - \omega_{elA_{min}}}{87} \cdot (i_{el} - 1), \quad (26)$$

$$\omega_{elB_{i_{el}}} = \omega_{elB_{min}} + \frac{\omega_{elB_{max}} - \omega_{elB_{min}}}{87} \cdot (i_{el} - 1), \quad (27)$$

where: $\omega_{elA_{min}} = 0$; $\omega_{elA_{max}} = \omega_{MP}$; $\omega_{elB_{min}} = \omega_{MP}$; $\omega_{elB_{max}} = \omega_{max_{el}}$; $i_{el} = 1 \div 88$; $\omega_{max_{el}}$ - the maximum possible angular rotation speed of the electric motor.

Thus, the torque and power of the electric motor can be expressed according to the different angular speeds of the motor, using relations of the form:

$$M_{elA_{i_{el}}} = M_{max_{el(n_{MP})}}, \quad (28)$$

$$P_{elA_{i_{el}}} = M_{elA_{i_{el}}} \cdot \omega_{elA_{i_{el}}}, \quad (29)$$

$$P_{elB_{i_{el}}} = P_{max_{el(n_{MP})}}, \quad (30)$$

$$M_{elB_{i_{el}}} = \frac{P_{elB_{i_{el}}}}{\omega_{elB_{i_{el}}}}. \quad (31)$$

The relations (28)÷(31) can be adapted so that they are written according to the speed of the electric vehicle, by expressing the angular speed of the electric motor according to the vehicle speed, as follows:

$$\omega_{el} = \frac{v_D \cdot i_{t_{el}}}{3.6 \cdot r_d}, \text{ in rad/s}, \quad (32)$$

with $v_D = 0 \div v_{max}$, in km/h, and r_d, in m.

The torque and the power at the wheel shall be determined similarly to those mentioned above, also in the case of an electric-powered vehicle, the computing equations being adapted to the quantities which characterise such propulsion.

• *Calculation of the force transmitted to the driving wheels.* The force transmitted to the driving wheels, for each gear, depending on the travel speed, can be determined by means of the relation [7, 8, 9, 10, 12, 13]:

$$F_{Rv_{j,s}} = \frac{M_{Rv_{j,s}}}{r_d}, \text{ in daN.} \quad (33)$$

2.3. Evaluation of the vehicle performance at driving on roads with different longitudinal inclinations

The steps for determining the performance of the studied vehicles are as follows:

- *The realization of the dynamic model for each type of studied vehicle* (Fig. 1) [10]. The notations used in the dynamic model captured in Figure 1 are as follows: A - wheelbase; a, b - horizontal distance along the longitudinal axis of the vehicle from the centre of gravity (C_G) on the front and rear axles, respectively; r_1, r_2 - rolling radius (dynamic radius - r_d) of the front and rear axle wheels, respectively; F_R - traction force at the wheel; R_r - rolling resistance force; R_p - resistance force due to ramp/slope; R_a - air resistance force; R_d - resistance force to acceleration or start-up; G_a - the total weight of the vehicle; h_g - height of the gravity centre; h_a - the height of the pressure centre in which the air force is considered to be applied, in the front centre of pressure C_p; M_{r1}, M_{r2} - rolling-resistant moments, at the front and rear axles; Z_1, Z_2 - normal road reactions to wheels/axles (front and rear); α_p - the inclination angle of the road in the longitudinal plane.

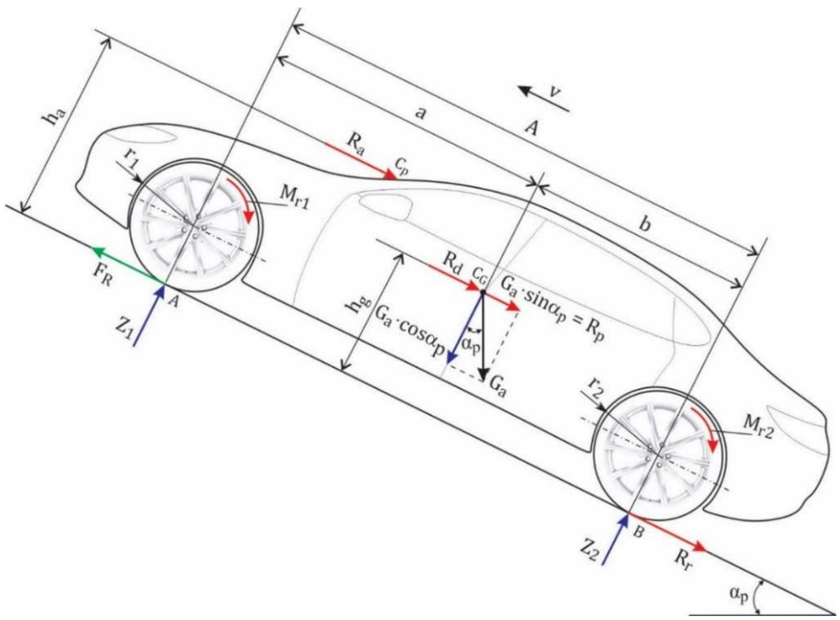

Fig. 1. Forces acting on a vehicle (organized 4x2, with front-wheel drive), in the situation of ascent on a road with longitudinal inclination.

- *Determination of vehicle forward resistance* [7, 8, 9, 10, 12, 13]. The characteristics of the road are established, so an average value of the coefficient of rolling resistance is chosen (f = 0.018), specific to an asphalt or concrete road in good condition, and various situations of longitudinal inclination of the road are considered α_p, captured by using the variable p=1÷3 ($\alpha_{p1}=0^o$, $\alpha_{p2}=5^o$, $\alpha_{p3}=10^o$).

The rolling resistance force R_{r_p}, is determined by the relation [7, 8, 9, 10, 12, 13]:

$$R_{r_p} = f \cdot G_a \cdot \cos(\alpha_p), \text{ in daN.} \quad (34)$$

The resistance force of the longitudinal inclination of the road R_{p_p}, is determined by the relation [7, 8, 9, 10, 12, 13]:

$$R_{p_p} = G_a \cdot \sin(\alpha_p), \text{ in daN.} \quad (35)$$

The determination of the total resistance force of the road, for the ascent situation, is made by the relation [7, 8, 9, 10, 12, 13]:

$$R_{\psi_p} = R_{r_p} + R_{p_p}, \text{ in daN.} \quad (36)$$

Resistance force due to air $R_{a_{vD}}$, is determined according to the relation [7, 8, 9, 10, 12, 13]:

$$R_{a_{vD}} = k_a \cdot S_f \cdot \left(\frac{vD}{3.6}\right)^2, \text{ in daN,} \quad (37)$$

where: k_a - aerodynamic coefficient, in $daN \cdot s^2/m^4$; S_f - cross-sectional area of the passenger cars, in m^2; vD - speed range $[0 \div v_{max}]$, in km/h.

The sum of all external resistance forces R_{ext}, is determined by the relation [7, 8, 9, 10, 12, 13]:

$$R_{ext_{vD,p}} = R_{\psi_p} + R_{a_{vD}}, \text{ in daN.} \quad (38)$$

The variable vD, which characterises the variation of the speed of the vehicle $[0 \div v_{max}]$, is also used to evaluate the powers necessary to overcome the forces of resistance to the vehicles forward.

The power necessary to overcome the rolling resistance force P_r, is calculated as follows [7, 8, 9, 10, 12, 13]:

$$P_{r_{vD,p}} = \frac{R_{r_p} \cdot vD}{360}, \text{ in kW.} \quad (39)$$

The power necessary to overcome the resistance force of the longitudinal inclination of the road, P_p, is calculated as follows [7, 8, 9, 10, 12, 13]:

$$P_{p_{vD,p}} = \frac{R_{p_p} \cdot vD}{360}, \text{ in kW.} \quad (40)$$

The power needed to overcome the total resistance force of the road P_ψ, is calculated with the relation [7, 8, 9, 10, 12, 13]:

$$P_{\psi_{vD,p}} = \frac{R_{\psi_p} \cdot vD}{360}, \text{ in kW.} \quad (41)$$

The power required to overcome the air resistance force P_a, is calculated with the relation [7, 8, 9, 10, 12, 13]:

$$P_{a_{vD,p}} = \frac{R_{a_{vD}} \cdot vD}{360}, \text{ in kW.} \quad (42)$$

The power needed to overcome the sum of all external resistance forces P_{ext}, is calculated with the relation [7, 8, 9, 10, 12, 13]:

$$P_{ext_{vD,p}} = \frac{R_{ext_{vD,p}} \cdot vD}{360}, \text{ in kW.} \quad (43)$$

3. OBTAINED RESULTS

The developed working algorithm allows to obtain the results in graphical form. Thus, the performance of the studied vehicles can be identified by means of the traction characteristics, captured in figures 2, 3 and 4, or by means of power characteristics, captured in figures 5, 6 and 7.

Wheel force curves (Fig. 2) $F1_{Rv1_{j,s}}$, which are above the curve $R1_{ext_{vD,p}}$, characterises the fact that these gears can be used in those operating conditions of the internal combustion -powered vehicle and the wheel force curves below the curve $R_{ext_{vD,p}}$, may not be used under the same conditions.

Thus, from the intersections of the force curves to the wheel $F_{Rv_{j,s}}$ corresponding to each gear, and to all external resistance forces $R1_{ext_{vD,p}}$, results in the maximum speeds, specific to the considered situations, marked with: $v1_{max0}$, $v1_{max5}$ and $v1_{max10}$, which the vehicle can reach at each of the three longitudinal inclinations of the road, and the availability of gears in each case (Fig. 2).

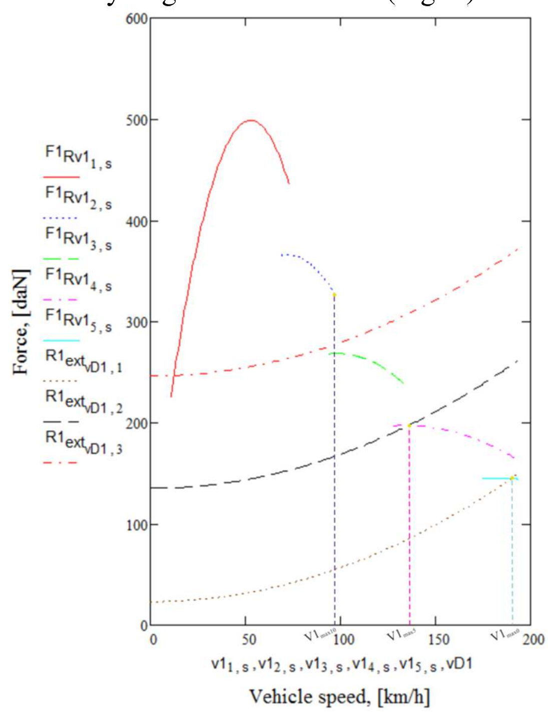

Fig. 2. The traction characteristic for the situation of various longitudinal inclinations of the road and the maximum speed of the vehicle with internal combustion engine, corresponding to each inclination.

Force available for acceleration, capable of overcoming vehicle starting resistance, whatever the engine operating mode, is given by the area between the curves of the forces at the wheel and those of the external resistances $(R_{d_j} = F_{R_j} - R_{ext})$, thus being able to identify the vehicle acceleration availability at each gear.

Figure 3 shows the curves of the forces at the wheel $F2_{Rv2_{j,s}}$ of the electric-powered vehicle and the curves of the external resistance forces $R_{ext_{vD,p}}$, determined by the three situations of longitudinal inclination of the road.

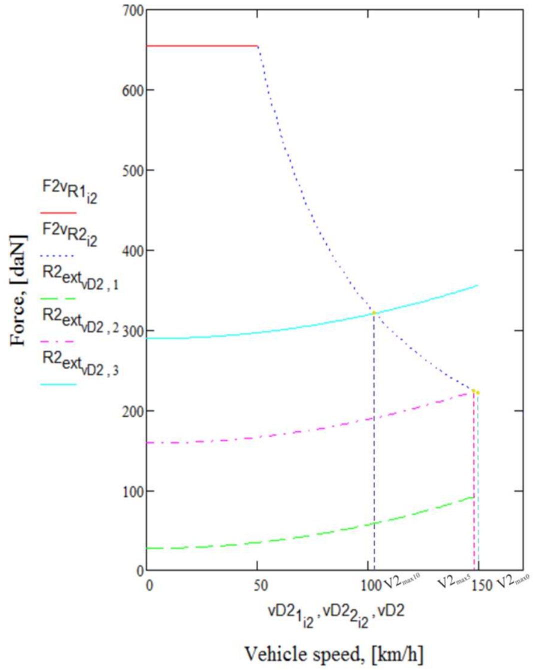

Fig. 3. Traction characteristic for the situation of the different longitudinal inclinations of the road and the maximum speed of the electric-powered vehicle at each considered road tilt.

For this vehicle, having a transmission system with a variable automatic gearbox (CVT) with a single gear and a single gear ratio, Figure 3 shows the availability of the gear.

Thus, as the longitudinal inclination of the road increases, the availability of the gear decreases, and the vehicle speed is inversely proportional to the increase of the ramp. Projections of the intersection points of the wheel force curves $F2_{Rv2_{j,s}}$ with those of the outer resistance forces $R2_{ext_{vD,p}}$, on the speed axis, determines the maximum travel speeds obtained by the electric car, at each longitudinal inclination of the road, these being identified in Figure 3, by the notations: $v2_{max0}$, $v2_{max5}$ and $v2_{max10}$. The portion between the wheel force curves and those of the external resistances characterises the vehicle's accelerating availability ($R_d = F_R - R_{ext}$).

From the results captured in Figure 4, the curves of the forces at the wheel can be identified $F3_{Rv3_{j,s}}$ of the hybrid-powered vehicle, and the three curves of the external resistance forces $R3_{ext_{vD,p}}$, corresponding to each longitudinal inclination of the road.

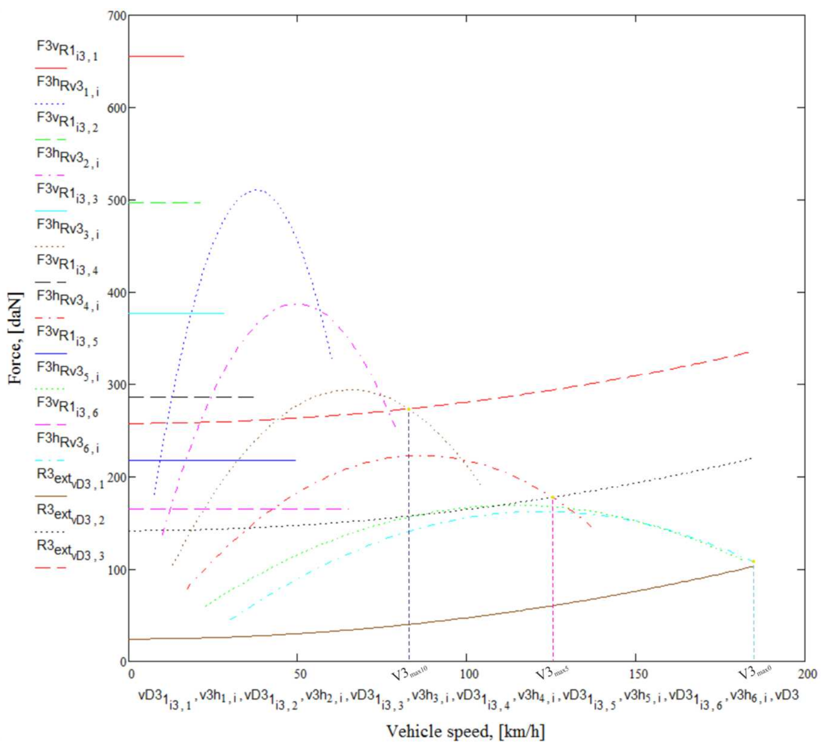

Fig. 4. The traction characteristic of the hybrid propulsion vehicle, for the situation of different longitudinal inclinations of the road and the maximum travel speed corresponding to each road inclination.

Above each curve of the external resistance forces $R3_{ext_{vD,p}}$, of the three cases of longitudinal inclination of the road, the availability of the gears can be noticed in each situation. By intersecting the forces at the wheels with the external resistance forces due to the three longitudinal inclinations of the road, the maximum travel speeds of the hybrid vehicle are obtained, denoted as follows: $v3_{max0}$, $v3_{max5}$ and $v3_{max10}$ (Fig. 4), for every tilt angle of the road, and the difference between the wheel forces and the external resistances indicates the accelerating possibilities of the vehicle, corresponding to each gear.

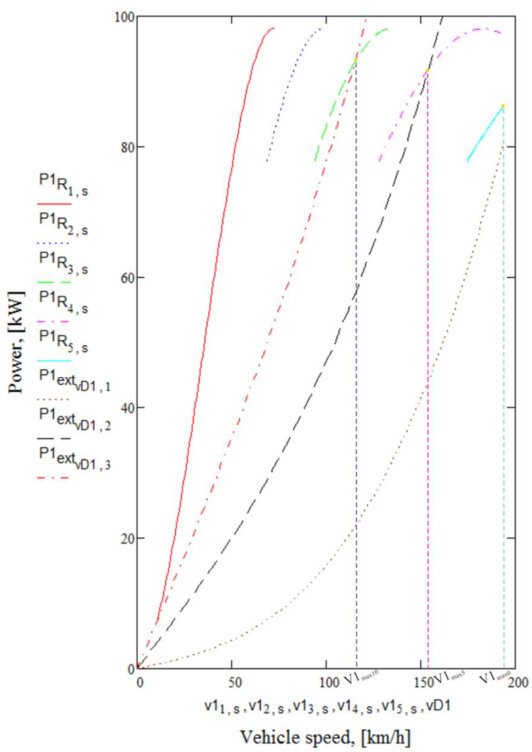

Fig. 5. The power characteristic of the vehicle with internal combustion, for the situation of different longitudinal inclinations of the road and its maximum travel speed at each inclination.

Wheel power curves (Fig. 5) $P1_{R_{j,s}}$, which are above the curve $P1_{ext_{vD,p}}$, characterise the fact that these gears can be used in the operating conditions of the internal combustion vehicle, and the wheel power curves below the curve $P_{ext_{vD,p}}$, may not be used under the same conditions. From the intersection of power curves to the wheel $P1_{R_{j,s}}$, corresponding to each gear, and the curves of powers necessary to overcome the external resistances $P1_{ext_{vD,p}}$, the maximum possible travel speeds are shown, under the conditions considered, marked with: $v1_{max0}$, $v1_{max5}$ and $v1_{max10}$ (Fig. 5), which the vehicle can reach at each of the three longitudinal inclinations of the road, and the availability of gears in each driving situation.

Figure 6 shows the wheel power curves $P2_{R_{j,s}}$ of the electric-powered vehicle and the external resistance power curves $P2_{ext_{vD,p}}$, determined by the three cases of longitudinal inclination of the road.

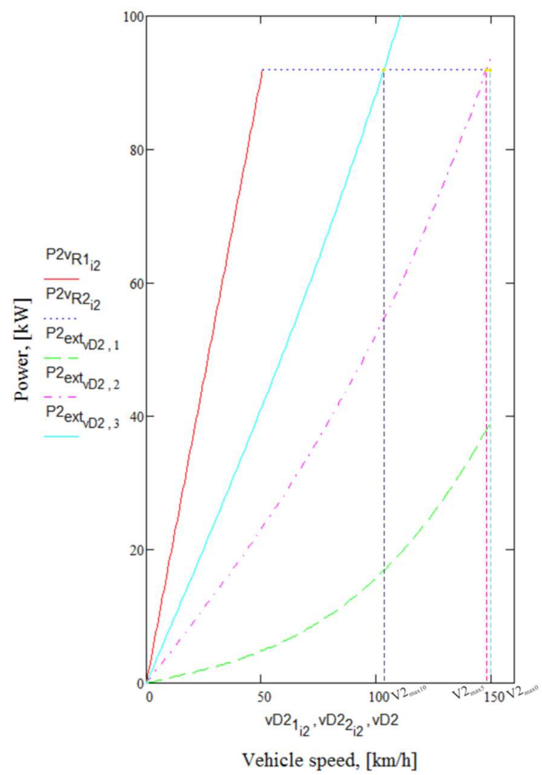

Fig. 6. The power characteristic of the electric-powered vehicle for the situation of the different longitudinal inclinations of the road and the maximum travel speed corresponding to each inclination.

For the electric-powered vehicle with a transmission system with a variable automatic gearbox (CVT) and a single gear, the availability of the gear can be identified from the results obtained in graphic form in Figure 6. Thus, with the increase of the longitudinal inclination of the road, the availability of the gear decreases and the vehicle speed is inversely proportional to the increasing of the ramp.

From the intersection of the power curve to the wheel (Fig. 6) $P2_{R_{j,s}}$ with those of external resistance powers $P2_{ext_{vD,p}}$, the maximum travel speeds obtained by the electric-powered passenger car at each longitudinal inclination of the road shall be determined and noted as follows: $v2_{max0}$, $v2_{max5}$ and $v2_{max10}$.

From the results captured in Figure 7, the wheel power curves $P3_{R_{j,s}}$ can be identified for the hybrid-powered vehicle and the three curves of the external resistance power $P3_{ext_{vD,p}}$, corresponding to each longitudinal inclination of the road. Above each curve of the exterior resistance powers (Fig. 7) $P3_{ext_{vD,p}}$, of the three situations of longitudinal inclination of the road, the availability of the gears in each case can be noted. Through the intersections of the wheel powers with the external resistance powers due to the three longitudinal inclinations of the road, the maximum travel speeds of the hybrid vehicle are obtained, denoted as follows: $v3_{max0}$, $v3_{max5}$ and $v3_{max10}$ (Fig. 7), corresponding to each inclination of the road.

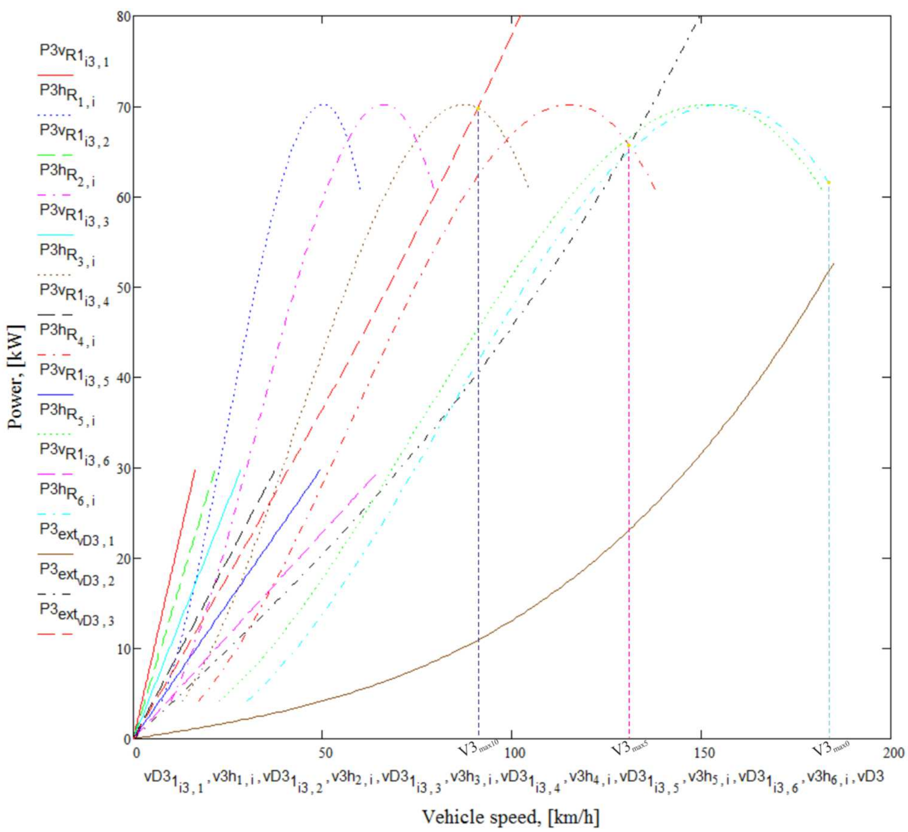

Fig. 7. The power characteristic of the hybrid vehicle for the situation of the different longitudinal inclinations of the road and the maximum travel speed corresponding to each inclination.

4. CONCLUSIONS

Depending on the propulsion system of each vehicle, in this case three vehicles with internal combustion, electric and hybrid propulsion systems are captured, and the numerical calculation model developed provides data to capitalize the vehicles performance on roads with different longitudinal inclinations.

Based on the obtained results, a comparative study can be carried out to show the maximum travel speeds that can be achieved by the vehicles at each of the three longitudinal bends of the road, as well as the gears available to achieve these speeds.

The design parameters of each studied vehicle have a significant influence on the forward resistance forces, but also on the external resistance powers. Peculiarities captured in this study shows that the maximum design speed of vehicles is obtained only when they are driven on a horizontal road, and at the maximum longitudinal inclination of the road, each vehicle is influenced by the following parameters: dynamic radius of the wheels, engine performance, propulsion system, type of transmission, number of gears, weight of the vehicle, etc.

The numerical calculation model developed can be adapted to any type of vehicle, considering any operating situation, thus allowing comparative studies to be carried out between different vehicles. In this numerical calculation model, different natures of the road can also be captured, as well as different loading situations of the vehicles, thus identifying the influence of the vehicle's weight on their performance, respectively on the propulsion capacity in different operating conditions. The numerical calculation model can also be developed in such a way as to allow the identification of the vehicle's performance parameters and by means of the force or excess power and their characteristic, or by means of the dynamic characteristic.

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POSIBILITĂȚI DE EVALUARE COMPARATIVĂ A PERFORMANȚELOR DINAMICE ALE AUTOVEHICULELOR CU PROPULSIE CLASICĂ, ELECTRICĂ ȘI HIBRIDĂ, PRIN MODELARE NUMERICĂ

Rezumat: *În lucrare se evaluează din punct de vedere matematic performanțele autovehiculelor prin identificarea posibilităților de înaintare ale acestora în diferite situații de exploatare luate în considerare. Forțele la roți, forțele de rezistență la înaintare și puterile necesare învigerii lor, pentru diferite condiții de deplasare, se valorifică prin dezvoltarea unui model de calcul numeric în programul MathCad, cu ajutorul căruia se obțin rezultate comparative, cu interpretare grafică, referitoare la caracteristicile de tracțiune și de putere ale autovehiculelor luate în studiu, în care se ține cont de sistemul de propulsie, parametrii constructivi, treptele de viteză utilizate, diferite înclinări longitudinale ale drumului etc. Astfel, se poate identifica influența parametrilor constructivi ai autovehiculelor și a condițiilor de deplasare, asupra performanțelor dinamice, cu referire la posibilitățile de tracțiune și accelerare ale autovehiculelor considerate, dar și identificarea treptelor de viteză care pot fi utilizate în diferitele condiții de exploatare.*

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