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A FAST HEURISTIC FOR PLANNING THE PRODUCTION OF SEVERAL PRODUCTS ON A SINGLE MACHINE SYSTEM

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Abstract: *This paper presents a heuristic that could be used to plan the activity of a system in which resources are allocated to tasks at hand for a number of periods. The proposed method uses a backtracking approach to minimize the inventory levels. To achieve this goal, for the "one machine - several products" case, the heuristic combines the backtracking approach with a procedure that allocates the time available in one period to all products that must be produced in that period. The paper presents the results obtained with two allocation methods. The first one, is of a greedy type while the second one is based on a solution for the well-studied knapsack problem.*

Key words: *production planning, heuristic, knapsack problem, one machine – many products.*

1. INTRODUCTION

Planning has been an important instrument in managing the activity of a system. It provided managers with the possibility to specify how much should they produce, of each product, and what are the inventory levels they need to keep to satisfy the demand of the customers.

Today the planning problem can be solved fairly easy with the help of mathematical models that are implemented in dedicated computer programs or in trivial Excel applications. For this reason, today research focuses on solving the planning problem together with some other related issues. For example, scheduling. As a result, a new research topic has been formulated under the name of integrated planning and scheduling. Finding the solution to such a problem is not a trivial task because scheduling is a notoriously hard problem to solve.

This paper describes a heuristic that could be used to solve the so-called planning problem. This is true because the system under study has been simplified to the point it could be considered as a single machine. Should this assumption be not considered then the simple planning problem would become a difficult one. At the begging of this research initiative, it was thought that if the proposed heuristic could successfully solve the easier problem, the planning problem, it could be later applied to the more difficult one, the integrated planning and scheduling problem.

The quality of the solution provided by the proposed heuristic was not estimated analytically. A "brute force" approach has been used to assess how good the solution provided by the heuristic was. Therefore, a mathematical model has been used to find the optimal solution and thus provide the reference for the assessment process.

2. LITERATURE REVIEW

The word planning has been included in several names of models and procedures dedicated to determining how much it should be produced in each period of the planning horizon so that customer requirements were met with a minimum cost. The most known are aggregate production planning and manufacturing resource planning. Most contributions related to these topics have been made in the seventh and eighth decade of the 20th century. Interestingly enough most of the contributions focused on developing mathematical models and thus find the optimum solution at a time when there was no software that would effectively solve such models, especially large ones.

Later when planning has been integrated with other activities, due to the need to find good solutions quickly, researchers have proposed both mathematical models and algorithms or heuristics to solve them.

In the domain of integrated planning and scheduling (hereafter referred to as IPS) the number of contributions rose quickly so that one of the first literature overviews has been performed as early as 2009 [7]. Maravelias and Sung have proposed a classification scheme based on modelling approaches and solution strategies [7]. Using the first criterion the two authors have divided contributions into: detailed scheduling, relaxation and aggregations of scheduling and finally surrogate models [7]. The last criterion used by Maravelias and Sung generated three more classes namely: hierarchical, iterative and full space [7].

The integration of planning and scheduling processes and, consequently of the model variables related to the two processes, leads to an important decision related to the representation of time. Depending on the decision made by researchers IPS models could have one or two time-grids. In the later case, the planning periods are divided into smaller intervals that are specific and more appropriate from a scheduling point of view.

For example, Erdirik-Dogan and Grossmann have divided the planning intervals into slots so that each interval comprised the same number of slots although of variable length [2].

If the scheduling process required a continuous-time representation then the approach proposed by Gimenez, Henning and Maravelias could also be used. They have considered that the slot could "float" that is the beginning and the end of the task did not have to coincide with a time point [3].

Another modeling feature that could help classify IPS models is the way in which the material balance equation has been implemented. In most cases the equation has the traditional form:

$$Inv_t = Inv_{t-1} + Production_t - Demand_t \quad \forall t \qquad (1)$$

where the inventory at the end of the current period ("Inv_t") is given by the inventory at the end of the previous period, the production and the demand of period "t".

However, Joly, Moro and Pinto have used a slightly different formulation in developing a model for planning and scheduling a petroleum refinery's activity [4]. Given the continuous nature of the processes at hand, the authors have determined the levels in storage tanks, at a particular moment, as the sum of the initial level and the amount that entered the tank minus the amount transferred from the tank up the current moment. This is a new formulation, but it is in fact quite similar to the one in (1).

Planning models have been integrated with distribution models as well. Meinecke and Scholz-Reiter have developed a heuristic using a multi-step decomposition approach as opposed to the more common sequential heuristics that solved subproblems sequentially [8]. Pochet and Vyve have developed a general heuristic for production problems motivated by the fact that most heuristics and Branch&Cut approaches have problems even for small problems. Their heuristic is designed to be used within a Branch&Cut approach [13]. Kreipl and Pinedo have raised the IPS problem to the supply chain level. In [5] they provide a comprehensive overview of the practice of planning and scheduling in supply chains. They also show how these management tools could be used to develop decision support systems.

Velez, Merchan and Maravelias have developed several methods to increase the effectiveness of "tradidional" approaches in solving large scale problems [10]. One of those uses multiple discrete time grids while the other considered tightening constraints on total production, inventory and resource availability. Shah and Ierapetritou have also addressed the issue of intractability of large IPS models [9]. Their solution was to relax the inventory constraints for adjoining periods of the planning/ scheduling horizon using a Lagrangian decomposition method.

Other researchers have tried to combine the mathematical models with heuristics to get better solutions. For example, Chu and his colleagues have proposed a hibrid method that uses a mathematical model for the planning problem and an agent-based method to solve the scheduling problem [1].

Lately, metaheuristics have been given a lot of attention due to their capability to provide a good solution in a very short period of time, an important trait of the solutions required by the industry. In particular, ant colony optimisation (ACO) algorithms have been developed for their ability to dinamically react to changes in the production system. In [6] the authors report on a project that used "ants" as software agents to integrate process planning and shopfloor scheduling while minimizing the makespan. They concluded that ACO algorithms can effectively solve IPS models. This statement is supported also by the work of Chang, Li and Chiang [12]. They have developed a model to integrate production and distribution schedule. Using an ACO algorithm they have tried to minimize the delivery times and have reported near-optimum results.

3. PROBLEM DEFINITION

This paper is the result of a quest to find a different form of the material balance equation (1). It was thought it was possible to find a new formulation that would eliminate a set of decision variables and thus ease the process of solving the mathematical model. Another factor that shaped the results presented here was determined by the requirements that had to be met by the planning/scheduling process in the real production system.

The heuristic presented in this paper has been tested on the data coming from a plant assembling wiring systems for the automotive industry. The system and the input data have been mentioned first in [11]. Briefly, the assembly facility consisted of 24 workstations each of which produced complete wiring systems. This mode of operation led to a modeling decision according to which stations were considered to be working in parallel, independently. In [11] allocation rules have been used to assign products to workstation. This time though such rules have been eliminated to increase the computational effort despite the fact that in practice such rules were used indeed.

When a workstation had to change the product, it was stopped to allow the maintenance team to change its configuration. To reduce the number of such stops workstations produced the same product for a period of approximately four hours. According to the work procedures operators were allowed to have a 20-minute break in between the two four-hour periods that formed the normal shift. In line with this characteristic of the work activity the model developed in [11] considered that the planning/scheduling horizon was formed of slots of 230 minutes. A number of 84 slots have been considered making the length of the planning horizon equal to 14 working days.

With respect to the customer demand data, it should be noted that the system comprising of all 24 workstations was supposed to deliver a number of products per slot of each product type.

Five test data sets have been used to test the proposed heuristic. Test case A was the one that reflected the best the practice in the real system. The other four have modified the profile of customer demand to test the performance of the heuristic. Therefore:

- test case B – reflected orders with higher volumes equivalent to an overall system utilization of 85%;
- test case C – was designed to have a jigsaw type of customer demand profile; it has been formulated by "moving" the demand of one slot to the next one;
- test case D – had a customer demand profile similar to the one of test case C with one modification; in this case there were several consecutive periods of low and high demand (still a jigsaw but with teeth farther apart);
- test case E – was designed to be completely different from test case A: the entire demand for every product type has been concentrated in the last period of the planning/scheduling horizon.

4. MATHEMATICAL FORMULATION

To find the optimum solution for the planning problem, and the reference for the heuristic, a mathematical model has been formulated in line with the traditional aggregate production planning. The formulation of the proposed model has been developed considering a single time grid that had its time points equally distributed at four-hour intervals.

Two indices have been used, namely: "k" for product types and "t" for slots – periods of the planning/scheduling horizon.

In line with these considerations the model tried to minimize the inventory over the entire planning horizon:

$$\sum_{k=1}^{K}\sum_{t=1}^{T} SF_{k,t} \qquad (2)$$

The set of constraints was formed of the following:

$$SF_{k,t} = SF_{k,t-1} + Y_{k,t} - d_{k,t} \qquad \forall t, \forall k \qquad (3)$$

$$Y_{k,t} \leq wCap_k \qquad \forall k, \forall t \qquad (4)$$

$$\sum_{k=1}^{K} asmbTime_k \cdot Y_{k,t} \leq slot \cdot W_t \qquad \forall t \qquad (5)$$

$$\sum_{k=1}^{K} asmbTime_k \cdot Y_{k,t} \geq ulb \cdot slot \cdot W_t \quad \forall t \qquad (6)$$

All test data sets considered allowed feasible solutions to be found. The following parameters have been included:

- W_t - number of workstations available in slot "*t*".
- *K* - number of product types.
- *T* - number of slots in the planning horizon.
- $asmbTime_k$ - duration of the assembly of a wiring system of product type "*k*".
- $d_{k,t}$ - demand for product type "*k*" in slot "*t*";
- $wCap_k$ – the maximum number of products of type "*k*" that could be produced by a workstation.
- *ulb* - utilization lower bound; it was set to 85%.
- *slot* – duration of a period of the planning/scheduling horizon; duration of a slot was set to 230 minutes.

The following decision variables have been defined:

- $Y_{k,t}$ – integer variable that indicates the number of wiring systems of product type "*k*" to be produced in slot "*t*";
- $SF_{k,t}$ – level of inventory of product type "k" at the end of the slot "*t*".

With the help of these parameters and variables a simple mathematical model has been formulated to plan the activity of a single machine system.

The objective function contains a single item in line with the requirement that the optimal solution should minimize the inventory levels.

The first constraint (3) is the well-known material balance constraint. In fact it relates the values of both decision variables to the demand. This constraint is actually the one that determines the duration of the optimization process. In [11] additional constraints have been considered hoping that they would reduce the time needed to find the optimum. In this paper the real goal focuses still on this issue but this time a heuristic has been developed.

Constraint number 4 sets the upper limit for the values of decision variable $Y_{k,t}$. The following constraint plays a similar role but this time the values of $Y_{k,t}$ should be determined as such that the production time needed to produce all products should not exceed the total time available. The last constraint (6) is similar in form with constraint number 5, but it has been considered to ensure that the model provides a solution that will load the system, in every period, to at least a given percentage of its capacity.

The model was written based on the assumption that initial inventory levels are zero for every product. This assumption may simplify the model but, in some cases, it could prevent finding a feasible solution if the demand is higher the system capacity in the first periods of the planning horizon.

5. THE PROPOSED HEURISTIC

The main goal of the proposed heuristic is to determine quantities to be produced so that the inventory levels are as low as possible and customer orders are met in time. The heuristic (hereafter referred to as HPP – the abbreviation for heuristic for production planning) assumes that the inventory levels are zero at the end of the planning horizon. With respect to the initial inventory levels it is worth mentioning that HPP treats them as output data. It determines their values to ensure that feasible solutions are found.

HPP uses a backtracking approach to determine production and inventory levels in slot "t" with the help of the material balance formula (1). In order to distribute the time available to each product HPP could employ several algorithms.

So far only two have been implemented. The first one is a greedy method while the second one uses the algorithm proposed by Rosetta Code organization for the knapsack problem [14].

6. RESULTS

The mathematical model has been solved with CPLEX 12.7 on an LenovoY700 machine with an Intel(R) Core(TM) i7-6700HQ CPU @ 2.60 GHz processor. A time limit of 200 seconds has been imposed on the duration of the optimization process.

The HPP has been implemented in Visual Basic with the help of Microsoft Visual Studio.

Both the mathematical model and the heuristic have been tested on the five data sets mentioned earlier.

Table 1 presents the results obtained with the mathematical model. It indicates the value of the objective function, the best reference value reported by the CPLEX engine, and the time needed to solve the model.

Table 1

Results – mathematical model

	Objective function value	Reference value	Time (milliseconds)
Case A	991	---	507
Case B	5049	4969	200167
Case C	7880	7833	200187
Case D	20266	20194	200162
Case E	58091	57970	200176

Of the five data sets used only for Case A data set CPLEX has found the optimal solution in less than a second. For the other four sets CPLEX could not find the optimum even in 200 seconds.

Table 2 shows the results of HPP when the greedy algorithm was used to allocate the time available in period "t" to all products that were supposed to be produced. It can be noticed that the values for the objective function are slightly higher than the values obtained with the mathematical model. However, the times needed to get those values are much smaller in comparison with the times obtained through "brute force" method.

Table 2

Results – HPP with greedy algorithm

	Objective function value	Time (milliseconds)
Case A	1029	7
Case B	5977	7
Case C	9007	4
Case D	21459	8
Case E	59667	16

The use of the Rosetta Code algorithm has improved the results in terms of the objective function values but the times of finding the solution have increased in reference to the times of the greedy algorithm. Still even with respect to time HPP with Rosetta Code has found good solutions in times far smaller than the times CPLEX needed to solve the mathematical model.

Table 3

Results – HPP with Rosetta Code algorithm

	Objective function value	Time (milliseconds)
Case A	992	306
Case B	5065	1040
Case C	7917	1263
Case D	20276	1961
Case E	58838	4394

7. CONCLUSIONS

This paper presents a heuristic that could be used to produce a plan for a system consisting of one machine and manufacturing several products. The system operates on a slot-based time frame. Experiments conducted with the help of five data sets have shown that the heuristic is producing results much faster than solving the mathematical model in the traditional way. The quality of the solutions is also encouraging especially for the case when the Rosetta Code algorithm is used.

Based on the above observations it can be concluded that the heuristic could be extended to the general case, that of a system consisting of several machines working in parallel, independently.

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O EURISTICĂ RAPIDĂ PENTRU PLANIFICAREA PRODUCȚIEI MAI MULTOR PRODUSE PE UN SISTEM FORMAT DINTR-O SINGURĂ MAȘINĂ

Rezumat: Această lucrare prezintă o euristică care ar putea fi utilizată pentru a planifica activitatea unui sistem în care resursele sunt alocate sarcinilor la îndemână pentru un număr de perioade. Metoda propusă utilizează o abordare de backtracking pentru a minimiza nivelurile stocurilor. Pentru a atinge acest obiectiv, pentru cazul „o singură mașină - mai multe produse”, euristica combină abordarea backtracking cu o procedură care alocă timpul disponibil într-o perioadă tuturor produselor care trebuie produse în acea perioadă. Lucrarea prezintă rezultatele obținute cu două metode de alocare. Prima, este de tip greedy, în timp ce a doua se bazează pe o soluție pentru problema rucsacului.

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