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CONNECTING ROD CURVES GENERATED BY THE R-PPR-PPR-PRP MECHANISM

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Abstract: *This paper presents the study of a novel linkage designed to generate interesting connecting rod curves. Starting from an imagined structural diagram with three dyads the geometric-kinematic model has been done, resulting in an original mechanism consisting of a R-type driving link and three PPR, PPR and PRP dyads. The trajectories of some points of interest of the mechanism are determined, resulting in the curves: Pascal snails, oval, ellipse. The geometry of the curves for different kinematic parameters of the mechanism was explored. Certain kinematic parameters of the mechanism were successively modified and a wide range of curves of the same type was obtained, but with different dimensions and oriented differently with respect to the axes of the reference system*

Key words: *mechanism, curves of the slider-crank mechanism, three dyads, Pascal snails*

1. INTRODUCTION

The connecting rod curves have always been studied over the years, starting with the invention of the steam engine, in order to find the trajectory necessary to pass from the linear movement of the piston to the rotational movement of the wheel. Many mathematicians have studied the related theory, i.e. the deduction of the sixth-degree equation to the four-bar linkage and the fourth degree to the crank-connecting rod Mechanism and of other degrees to other curves. The lack of efficient computing means did not allow the analytical solution of the synthesis of the mechanisms generating certain curves, so graphical methods were used. A reference work was elaborated by J.A. Hrones and G.L. Nelson, respectively an atlas of the connecting rod curves, generated by the four-bar linkage [1]. Starting with 1951, Artobolevskii's books were published, that presented many mechanisms for which, in most cases, the drawn curves were indicated, such as [2].

Starting with the last decades of the twentieth century, computer science developed a lot so that the studies of the mechanisms were made mainly by analytical methods that allowed the computerized design of the mechanisms in different program. A matrix method for determining the parametric equations of the connecting rod curves is presented in [3]. Synthesis of plane and spatial linkages by an analytical method is shown in [4]. Interesting studies on the generation of the connecting rod curves are also found in [5], [6] and [7]. In-depth research shows that the literature offers many articles and books that address the mechanisms generating the connecting rod curves. Also, some of authors were concerned to the generation of curves, studies on the mechanisms generating the connecting rod curves [8-12].

This paper presents the study of a novel linkage designed to generate interesting connecting rod curves. A structural diagram with three dyads and a rotating driving link was developed and the geometric-kinematic model of the mechanism was configured. A series of geometric parameters of the mechanism were successively modified and the trajectories generated by the proposed mechanism were determined. The similarities and differences of the obtained trajectories were analyzed. Thus, a wide range of curves was determined: Pascal snails, ovals, ellipses. The results obtained from the study show that the newly created mechanism generates useful connecting rod curves. Original curves generated by this mechanism may be of interest for applications in various fields such as light industry, automated lines, technological equipment.

2. METHODS

The structural synthesis of the mechanism was performed, consisted in the design of structural diagrams, based on the following: the principles of structural functionality of the mechanisms, the structural components of the mechanisms (kinematic elements and joints) that have to assure desmodrome kinematic chains. Assur's principle was applied to the design of the mechanism . The succession of dyads was made so that they transfer complicated movements, preferring to take the movements from elements in planar motion.

In order to establish the kinematic functions of the studied plane mechanism, the closed-loop method is used, so that the requirements and configuration of the mechanism are expressed through mathematical relations. The closed-loop method is based on the projection of the vectorial contours on the axes of an Cartesian system.

A comparative analysis of the curves described by certain points of interest of the mechanism was also performed.

2.1 The kinematic synthesis of the mechanism

In order to create the mechanism, a structural diagram was conceived with a driving link (1) with rotational movement and three dyads, given in Fig. 1.

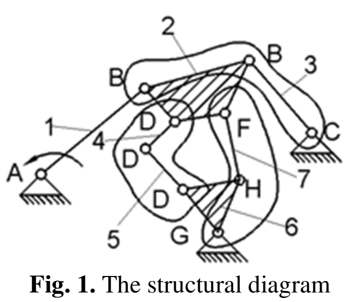


Fig. 1. The structural diagram

Based on Fig 1, the geometric-kinematic model of the mechanism was configured and presented in Fig. 2.

The kinematic joints are: A (1,0), B (1,2), B (3,2), C (3,0), D (2,4), D (4,5), D (5.6), F(6.7), F (7.2), G (6.0).

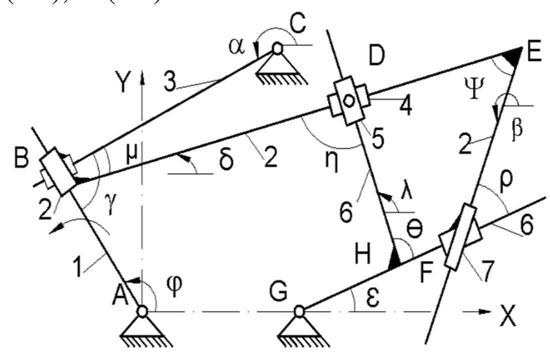


Fig. 2. The kinematic diagram

The mechanism's degree of mobility is calculated with the equation for plane mechanisms: $M = 3n-2C_5 – C_4$. In the equation is introduced the numerical values for the number n of the kinematic elements, C_5 - the number of V^{th} class joints (rotation and translation), C_4 - the number of IV^{th} class joints. For the mechanism proposed in Fig. 2, the degree of mobility results: $M = 3x\ 7- 2x\ 10 = 1$, that means, the mechanism needs a single driving element.

The structural analysis of the proposed mechanism is presented in the diagrams in Fig. 3. The mechanism is composed of a rotating driving link (R) and three dyades of PPR, PPR and PRP type. Thus, it is considered the mechanism to be of the R-PPR-PPR-PRP type.

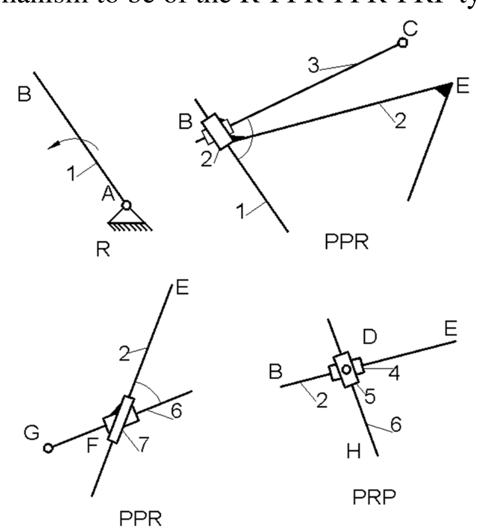


Fig. 3. The kinematic groups

The length of the driving link, i.e. the crank (1), is variable. A PPR dyad consists of two translation joints provided in point B and the rotation joint in C. The element connecting the two sliders in B is zero length. The second PPR dyad consists of the rotation joint in G and two translation joints provided at point F and connected by a zero length kinematic element. The third dyad is of PRP type and consists of a rotation joint and two translation joints, all three joints are provided in point D and the elements 4 and 5 have lengths equal to zero. Based on Fig. 4. the correlation between the angular geometric parameters is established.

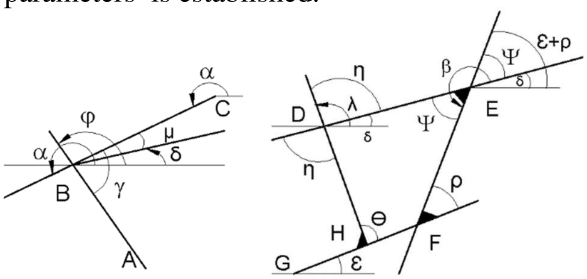


Fig. 4. Correlations between angular geometric parameters

2.2. The parametric equations

Based on Fig. 2, the following equations are written based on the closed-loop method:

$$x_B = AB \cdot cos\varphi = x_C + CB \cdot cos\alpha \quad (1)$$

$$y_B = AB \cdot sin\varphi = y_c + CB \cdot sin\alpha \quad (2)$$

$$\alpha = \varphi + \gamma\,;\, \delta + \mu = \alpha - \pi \quad (3)$$

$$tg\alpha = \frac{AB{\cdot}sin\varphi - y_c}{AB{\cdot}cos\varphi - x_c} \quad (4)$$

$$(AB \cdot cos\varphi - x_C)tg\alpha = AB \cdot sin\varphi - y_C \quad (5)$$

$$AB = \frac{-y_c + x_c\, tg\alpha}{cos\varphi{\cdot}\, tg\alpha - sin\varphi} \quad (6)$$

$$x_D = x_B + BD \cdot cos\delta = x_G + GH \cdot cos\varepsilon + HD \cdot cos\lambda \quad (7)$$

$$y_D = y_B + BD \cdot sin\delta = y_G + GH \cdot sin\varepsilon + HD \cdot sin\lambda \quad (8)$$

$$tg\delta = \frac{y_G - y_B + GH{\cdot}sin\varepsilon + HDsin\lambda}{x_G - x_B + GH{\cdot}cos\varepsilon + HDcos\lambda} = \frac{b_1 + HD{\cdot}sin\lambda}{b_2 + HD{\cdot}cos\lambda} \quad (9)$$

$$(b_2 + HD \cdot cos\lambda)tg\delta = b_1 + HD \cdot sin\lambda \quad (10)$$

$$HD = \frac{b_1 - b_2 tg\delta}{cos\lambda\, tg\delta - sin\lambda} \quad (11)$$

From Eqs. (1) to (6), AB, CB, α, XB and YB are determined. From Eq. (3) results the correlations between the angles. BD, HD, XD, YDresuts from Eqs. (7), (8), (9), (10) and (11). The following equations are also written:

$$x_E = x_B + BEcos\delta \quad (12)$$

$$y_E = y_B + BEsin\delta \quad (13)$$

$$\lambda = \varepsilon + \theta \quad (14)$$

$$\varepsilon + \rho + \pi = \beta \quad (15)$$

$$\eta = \lambda - \delta \quad (16)$$

$$\beta = \delta + \psi + \pi \quad (17)$$

$$\varepsilon = \delta + \psi - \rho \quad (18)$$

$$x_F = x_G + \mathrm{GF}cos\varepsilon = x_E + \mathrm{EF}cos\beta \quad (19)$$

$$y_F = y_G + \mathrm{GF}sin\varepsilon = y_E + \mathrm{EF}sin\beta \quad (20)$$

$$tg\beta = \frac{y_B + GFsin\varepsilon - y_E}{x_G + GFcos\varepsilon - x_E} \quad (21)$$

$$GF = \frac{y_G - x_G tg\beta + x_E tg\beta - y_E}{cos\varepsilon\, tg\beta - sin\varepsilon} \quad (22)$$

$$EF = \frac{x_G + GFcos\varepsilon - x_E}{cos\beta} \quad (23)$$

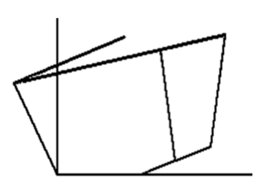
From Eqs. (12) and (13) are determined XE, YE. From Eqs. (14),(15), (16), (17) and (18) are determined the relationships between the angles and from Eqs. (19) to (23) GF, EF, XF, YF, β are resulting.

Thus, the parametric equations of the points of interest for generating the curves were obtained. Points of interest are points that are located on elements in planar motion, whose trajectories differ from arcs to straights lines.

3. RESULTS AND DISCUSSION

The following input data were considered (based on a scale representation of the mechanism): XG = 40; GH = 18; BE = 106; γ= 88; η = 82; θ = 76; ρ= 62; XG = 41; XC = 33; YC = 68; Ψ = 70; μ = 10 (lengths in millimeters and angles in degrees).

Fig. 5 shows the mechanism for the position of φ=115 degrees, and Fig. 6 shows the mechanism in a number of successive overlapped positions. With the chosen input data and based on the relationships given above, the trajectories of the points were found : E (Fig. 7) - a deformed oval , D (Fig. 8) - Pascal snail. The trajectory of B is an ellipse, as seen in Fig. 9.



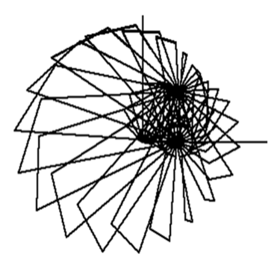
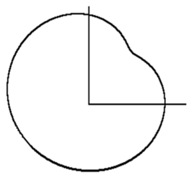


Fig.5. Mechanism for φ=115; **Fig.6.** Successive positions



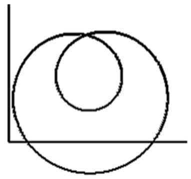


Fig. 7. The trajectory of E; **Fig. 8.** The trajectory of D

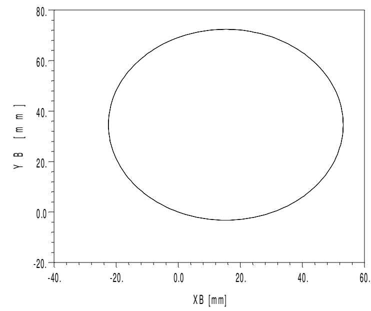


Fig. 9. The trajectory of point B

Further, some dimensions of the mechanism changed, resulting in the same types of curves as above, but variable in position with respect to the axes of the system and oriented with the loop or flattening at other angles. The paper presents some of the results obtained regarding the curves generated by points of interest D, E and F.

The following figures indicate the value of the modified linear and angular kinematic parameters.

For point D the following curves were obtained:

- The change of the coordinates (XC, YC) and then (XG, YG) gives as result the Pascal snails in different positions, but also some deformed ovals (Fig. 10. - Fig.13);

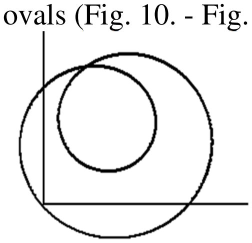


Fig. 10. *XC=10, YC=75*

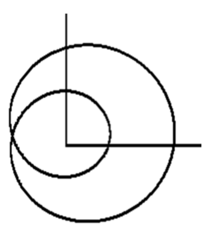


Fig. 11. *XC= - 30, YC=0*

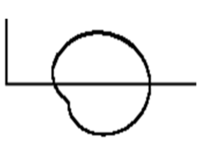


Fig. 12. *XC=30, YC=0*

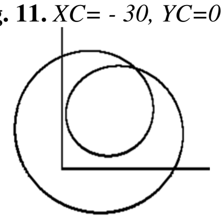


Fig. 13. *XG=0; YG=0*

- The change of the length BE of the kinematic element 2 also led to Pascal snails (Fig. 14, Fig. 15);

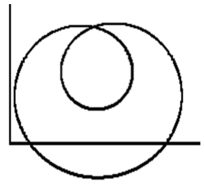


Fig. 14. *BE=20*

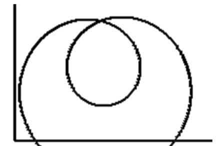


Fig. 15. *BE= - 20*

- The modification of the GH length on the kinematic element 6 gives as result: a circle (Fig. 16) and Pascal snails (Fig. 17, Fig. 18);

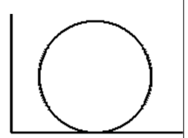


Fig. 16. *GH=0*

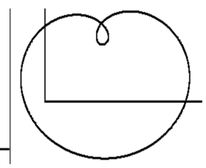


Fig. 17. *GH=45*

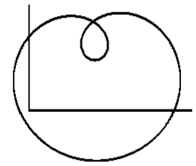


Fig. 18. *GH= - 35*

- The change of the angular parameter γ , leds also to Pascal snails but in different positions (Fig. 19 – Fig. 21);

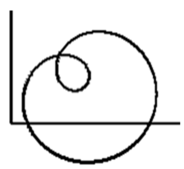


Fig. 19. *γ=20*

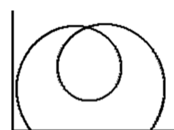


Fig. 20. *γ =90*

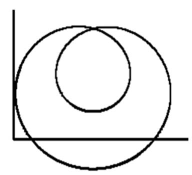


Fig. 21. *γ =125*

- The change of the the angle η , gives also as result a Pascal snail (Fig. 22);
- The change of the angle θ , leds to the Pascal snail (Fig. 23), and this kind of curve was obtained at several values of θ;
- The change of the angle ψ , gives as result a curve (Fig. 24).

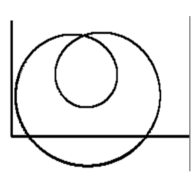


Fig. 22. η=*15*

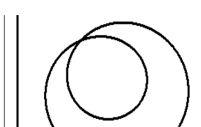


Fig. 23. *θ=45*

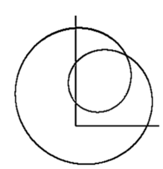


Fig. 24. *ψ=120*

For point E the following curves were obtained:

- Modifying the angle ψ leds to the curves from Fig. 25 and Fig.26; it's noticed that ψ angle does not influence the trajectory of point E.

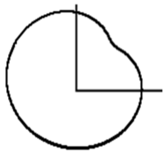
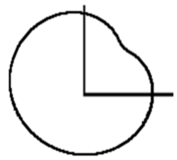



Fig. 25. *ψ=0* **Fig. 26.** *ψ=240*

- Changing the coordinates of point C (XC, YC) , results curves of the oval type (Fig. 27, Fig. 28) which are positioned at different angles to the axes system;

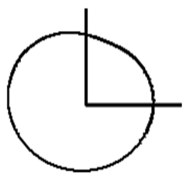
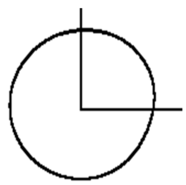



Fig. 27. *XC=0; YC=50* **Fig. 28**. *XC=40; YC=0*

- Changing the angle γ , interesting curves, both Pascal snails and ovals result (Fig. 29 - Fig.32) ;

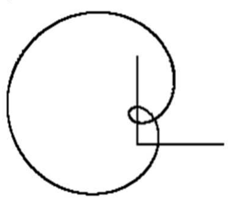
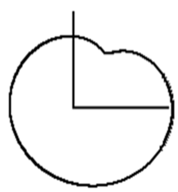



Fig. 29. *γ=30* **Fig. 30**. *γ=120*

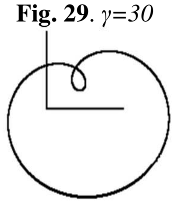
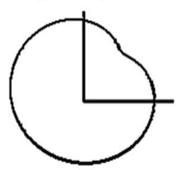



Fig. 31. *γ=330* **Fig. 32.** *γ=270*

The following curves were obtained for point F:

- By changing the angle ψ, also Pascal snails and ovals results (Fig. 33, Fig. 34);

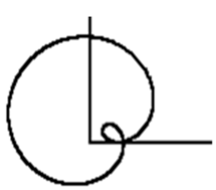
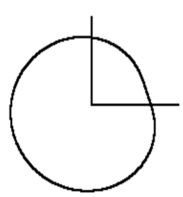



Fig.33. *ψ =30* **Fig. 34**. *ψ =90*

Several trajectories of the different points of interest of the mechanism were obtained through variations of the angular and linear kinematic parameters. Based on the obtained results, a comparative analysis of the generated curves can be made. For newly created mechanism is noticed as choosing different values for certain linear and angular kinematic parameters resulted in getting several types of curves: Pascal snails, ovals, ellipses. It is observed that a series of Pascal snails with certain characteristics resulted. All images are symmetrical in geometry, but differ in positioning relative to the xOy axes system. It can also be seen that Pascal snails with different dimensions and geometric shapes of the loop were obtained. Oval curves can be said to be more or less deformed, but they all admit an axis of symmetry. Also, the ovals are positioned differently in relation to the xOy coordinate axes. In only one case, when determining the trajectory of point B, an ellipse curve resulted. The exploration of the curves generated by the mechanism for different values of the kinematic parameters, leds to obtaining a variety of curves with interesting characteristics.

From the obtained images can be selected those one capable to fulfil the requirements of certain practical applications.

4. CONCLUSION

The study started from a structural diagram of the mechanism, designed so as to have three dyads, the third dyad picking up its input movements from the movements of the first and second dyad. The kinematic diagram was then designed and the R-PPR-PPR-PRP mechanism resulted. Based on the equations resulting from the application of the contour method, the mechanism was represented in a position, to check the created computer program. The trajectories of the points of interest were drawn, resulting in variants of Pascal snails, ovals, ellipses. Further, different values were adopted for some of the linear and angular kinematic parameters that define the mechanism. The result is a wide range of curves, of the same type as the initial ones, but with different characteristics, such as: different positioning in relation to the xOy axes system, more or less deformed ovals, Pascal snails with loops of different geometric shapes and sizes and positioned at different angles to the axes of the reference system. The results obtained from the study and presented in the paper show that the newly created mechanism generates useful connecting rod curves. Original curves generated by this mechanism may be of interest for applications in various fields such as light industry, automated lines, technological equipment.

This paper contributes to the development of an important field in Theory of Mechanisms, namely Mechanisms for generating plane curves. Further, the studies can be extended to the design of the mechanisms that are based in operation on the kinematic diagram presented in this paper.

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CURBE DE BIELĂ GENERATE DE MECANISMUL R-PPR-PPR-PRP

Rezumat: Această lucrare prezintă studiul unui mechanism nou creat, conceput pentru a genera curbe interesante de bielă. S-a pornit de la o diagramă structurală imaginată cu trei diade și s-a construit modelul geometric-cinematic, rezultând un mecanism original format din elementul conducător de tip R și trei diade PPR, PPR și PRP. Se determină traiectoriile unor puncte de interes ale mecanismului, rezultând curbele de tip: melcul lui Pascal, oval, elipsă. A fost analizată geometria curbelor pentru diferiți parametri cinematici ai mecanismului. Anumiți parametri cinematici ai mecanismului au fost modificați succesiv și s-a obținut o gamă largă de curbe de același tip, dar cu dimensiuni diferite și orientate diferit față de axele sistemului de referință.

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