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MODELLING THE VIBRATORY EFFECTS OF RAIL TRAFFIC ON THE ENVIRONMENT

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Abstract: *Despite numerous technological advances, noise and the various forms it takes is perceived as a multiple source of environmental problems that are increasingly borne by humans. Sensitivity to rail traffic vibrations is the most important issue in the opinion of researchers. In this paper is determined the dynamic vibration response of the vehicle-railway system. For this purpose, the most modern integration methods used for such purposes at present are used. For the two-degree-of-freedom, unloaded model for the railway, it is used the one-sided Laplace transform with respect to time that led to the algebraization of the problem, which simplified the integration of the differential equation system. With the Mathematica program, based on the numerical data presented in the paper, we inverted the Laplace transforms, resulting in the displacements as time functions, and then, with the same program, we obtained the graphical representations of the rail displacements caused by its geometrical imperfections. Next, using the same method, we obtained graphical representations of the bogie and wheel displacements caused by forced and damped vibrations induced in the system by the action of a force. For the determination of the deflection of the flexible rail track under a moving load the mathematical model is a partial derivative equation. We integrated this mathematical model by first applying the one-sided Laplace transform with respect to time, resulting in a Laplace image equation, to which we applied the finite sine Fourier transform. Solving the resulting algebraic system, we obtained the dynamic response of the mechanical system in Laplace and Fourier images. Applying first the inverse of the Laplace transform and then the inverse of the Fourier transformation in sine to the algebraic system, we obtained the solution of the partial derivative equation mentioned above in the form of a time and displacement function, which we plotted with the Mathematica program.*

Key words: *modelling, vibrations, Mathematica program, response, rail track*

1. INTRODUCTION

In the paper [1], the author refers to the environmental effects of vibrations produced in railway traffic. This paper aims to develop a reliable methodology to predict the design stages of a railway vehicle or to implement a new railway, as well as the dynamic stresses that the vehicle is likely to transmit to the ground and to estimate the environmental impact.

His results show the important influence of vehicle-railway (track) interaction and the need for a comprehensive model in rail traffic vibration issues. We were interested in determining the dynamic vibration response of the vehicle-railway system. For this purpose, we used the most modern integration methods used for such purposes at present.

The paper [5] specifies that the intensity of mechanical vibrations generated by a moving train increases with increasing train speed. The idea presented in this research is to develop mechanisms with negative stiffness, with utility in the suspension of wagon halls.

In [6] it is shown that locomotive operators in the USA are exposed to multi-axis whole body vibration. In this study the ergonomic design of operator chairs is questioned. In this research vibration exposure was measured according to international regulations (ISO 2631-1, 1997). The aim is to detect musculoskeletal problems of locomotive engineers caused by vibration exposure.

Study [7] aims to investigate and compare vibrations and shocks of railway vehicles used in railway maintenance and construction. It is mentioned in the research that there are no studies available in the literature on the risk and health of machine operators with regard to whole-body vibration and shock exposure among workers.

In paper [8], a collection of vibration measurement reports on railway tracks is analyzed. The vibrations that occur on the railway are generated by the interaction between the wheel and the track, i.e. the rail, and then propagate into the ground. In [9], solutions are proposed to eliminate these vibrations, one of which is to use recycled tires.

Vibrations recorded during truck transport are shown in [10], and mechanical stresses induced by the high temperatures themselves are shown in [11].

In [12], a state of the art in the prediction and control of ground borne noise and vibration is presented. Similar studies are presented in [13] and [14].

The paper [15] presents a human biomechanical model with 4 degrees of freedom of a human body in a car seat with backrest exposed to vertical vibrations.

In [16] it is shown that prolonged exposure of the body vibrations of a tractor operator. An experimental investigation is carried out to determine the vibrations transmitted from the seat to the operator in order to determine the degree of operator comfort in different terrains.

In the quantification of vibrations occurring in mechanical systems, dynamic analysis software for multibody systems, such as ADAMS, can be used. This is used in paper [17] to simulate bearing and shaft vibrations (shaft centre of mass translational deformations).

2. DETERMINATION OF DYNAMIC RESPONSE

Consider the two degree of freedom mechanical model in Fig. 1, where the rail mass is modelled as a concentrated mass of size:

$$m_1 = \rho_s A_s L, \qquad (1)$$

where: ρ_s - the specific rail mass; A_s - area of rail section; L - length of rail.

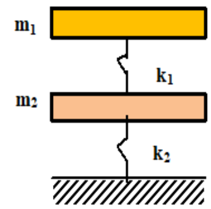

Fig. 1. The two-degree-of-freedom, no-load model for railways.

The mechanical model in Fig.1 generates the mathematical model below:

$$[M]\{\ddot{q}\} + [K]\{q\} = \{0\}, \qquad (2)$$

where: $[M]$ - matrix of masses; $[K]$ - stiffness matrix; $\{q\}$ - matrix of displacements; m_2 - crossbeam mass; k_1 - the rail stiffness coefficient; k_2- the coefficient of stiffness of the cross-beam;

$$[M] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}; \{\ddot{q}\} = \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix}; \{q\} = \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}; [K] = \begin{bmatrix} k_1 + k_2 & -k_1 \\ -k_1 & k_1 \end{bmatrix}; \{0\} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}.$$

Applying the Laplace transform to system (2), initial conditions being:

$$q_1(0) = q_{10}; q_2(0) = q_{20}; \dot{q}_1(0) = 0; \dot{q}_2(0) = 0,$$

so that the vibrations are considered to be generated by the geometrical imperfections of the rail, the algebraic system is obtained:

$$\begin{cases} (m_1 s^2 + k_1 + k_2)\tilde{q}_1(s) - k_1 \tilde{q}_2(s) = m_1 s q_{10} \\ -k_1 \tilde{q}_1(s) + (m_2 s^2 + k_1)\tilde{q}_2(s) = m_2 s q_{20} \end{cases} \qquad (3)$$

Solving the algebraic system elementary (3), results in the Laplace images $\tilde{q}_1(s)$ and $\tilde{q}_2(s)$ of desired motions $q_1(t)$ and $q_2(t)$, as below:

$$\tilde{q}_1(s) = \frac{s[m_1 q_{10}(m_2 s^2 + k_1) + k_1 m_2 q_{20}]}{(m_1 s^2 + k_1 + k_2)(m_2 s^2 + k_1) - k_1^2},$$

$$\tilde{q}_2(s) = \frac{s[m_2 q_{20}(m_1 s^2 + k_1 + k_2) + k_1 m_1 q_{10}]}{(m_1 s^2 + k_1 + k_2)(m_2 s^2 + k_1) - k_1^2} \qquad (4)$$

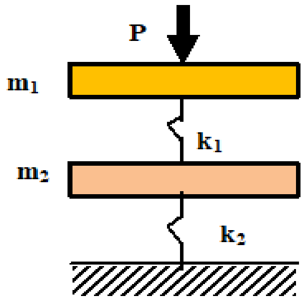

Fig. 2. The two-degree-of-freedom model with loading for railways.

In Fig. 2 a train axle load of size P is considered. This time we have worked under homogeneous initial conditions, i.e.

$$q_1(0) = 0; q_2(0) = 0; \dot{q}_1(0) = 0; \dot{q}_2(0) = 0$$

System (2) becomes:

$$[M]\{\ddot{q}\} + [K]\{q\} = \{Q\}, \tag{5}$$

where:

$$\{Q\} = \begin{Bmatrix} P \\ 0 \end{Bmatrix}$$

Applying, under these conditions, the one-sided Laplace transform with respect to the system time (5), results in the algebraic system in the unknowns $\tilde{q}_1(s)$ and $\tilde{q}_2(s)$:

$$\begin{cases} s(m_1 s^2 + k_1 + k_2)\tilde{q}_1(s) - sk_1\tilde{q}_2(s) = P \\ -k_1\tilde{q}_1(s) + (m_2 s^2 + k_1)\tilde{q}_2(s) = 0 \end{cases} \tag{6}$$

Solving the elementary algebraic system (6), results in Laplace images $\tilde{q}_1(s)$ and $\tilde{q}_2(s)$ of motions searched $q_1(t)$ and $q_2(t)$, as below:

$$\tilde{q}_1(s) = \frac{P(m_2 s^2 + k_1)}{s(m_1 s^2 + k_1 + k_2)(m_2 s^2 + k_1) - sk_1^2},$$

$$\tilde{q}_2(s) = \frac{Pk_1}{s(m_1 s^2 + k_1 + k_2)(m_2 s^2 + k_1) - sk_1^2} \tag{7}$$

Using Mathematica, based on the data in Table 1, we inverted the Laplace transforms (4), after which, with the same program, we obtained the graphical representations from Fig. 3 and Fig. 4.

With the program Mathematica we inverted the Laplace transforms (7), based on the data in Table 1, then with the same program we obtained the graphical representations in Fig. 5 and Fig. 6

It is easy to see from these representations that the displacements caused to the rail by the vibrations induced by the P load are larger than those caused by a geometrical non-uniformity of the rail. Introducing damping into the system leads, in the case of vibrations generated only by geometrical imperfections of the rail, to the mathematical model below.

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{0\}, \tag{8}$$

where:

$$[K] = \begin{bmatrix} k_1 + k_2 & -k_1 \\ -k_1 & k_1 \end{bmatrix}; [C] = \begin{bmatrix} c_1 + c_2 & -c_1 \\ -c_1 & c_1 \end{bmatrix}; \{\ddot{q}\} = \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix}; \{\dot{q}\} = \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix}; \{q\} = \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix};$$

$$\{Q\} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Applying, under initial conditions $q_1(0) = q_{10}; q_2(0) = q_{20}; \dot{q}_1(0) = 0; \dot{q}_2(0) = 0,$

Laplace transform to system (8), results in the algebraic system:

$$\begin{cases} [m_1 s^2 + (c_1 + c_2)s + k_1 + k_2]\tilde{q}_1(s) - (c_1 + k_1) \\ \quad \tilde{q}_2(s) = m_1 q_{10} s + (c_1 + c_2)q_{10} - c_1 q_{20} \\ -(c_1 + k_1)\tilde{q}_1(s) + (m_2 s^2 + c_1 s + k_1)\tilde{q}_2(s) = \\ \quad m_2 q_{20} s - c_1 q_{10} + c_1 q_{20} \end{cases} \tag{9}$$

Solving elementary the algebraic system (9), results in Laplace images $\tilde{q}_1(s)$ and $\tilde{q}_2(s)$ of movements searched $q_1(t)$ and $q_2(t)$, as below:

$$\begin{cases} \tilde{q}_1(s) = \dfrac{[m_1 q_{10} s + (c_1 + c_2)q_{10} - c_1 q_{20}](m_2 s^2 + c_1 s + k_1) + (c_1 + k_1)(m_2 q_{20} s - c_1 q_{10} + c_1 q_{20})}{[m_1 s^2 + (c_1 + c_2)s + k_1 + k_2](m_2 s^2 + c_1 s + k_1) - (c_1 + k_1)^2} \\ \tilde{q}_2(s) = \dfrac{[m_1 s^2 + (c_1 + c_2)s + k_1 + k_2](m_2 q_{20} s - c_1 q_{10} + c_1 q_{20}) + (c_1 + k_1)[m_1 q_{10} s + (c_1 + c_2)q_{10} - c_1 q_{20}]}{[m_1 s^2 + (c_1 + c_2)s + k_1 + k_2](m_2 s^2 + c_1 s + k_1) - (c_1 + k_1)^2} \end{cases} \tag{10}$$

Introducing damping into the system, in the case of train loading, on the axle, with load P, leads to the mathematical model:

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{Q\} \tag{11}$$

Applying, under homogeneous initial conditions, the Laplace transform to the system (11), the following system is obtained:

$$\begin{cases} s[m_1 s^2 + (c_1 + c_2)s + k_1 + k_2]\tilde{q}_1(s) \\ \qquad\qquad -s(c_1 s + k_1)\tilde{q}_2(s) = P \\ -(c_1 s + k_1)\tilde{q}_1(s) + (m_2 s^2 + c_1 s + k_1)\tilde{q}_2(s) = 0, \end{cases} \tag{12}$$

with the solution, in Laplace images $\tilde{q}_1(s)$ and $\tilde{q}_2(s)$ of motions searched $q_1(t)$ and $q_2(t)$,

$$\begin{cases} \tilde{q}_1(s) = \dfrac{P(m_2 s^2 + c_1 s + k_1)}{s[m_1 s^2 + (c_1+c_2)s + k_1 + k_2](m_2 s^2 + c_1 s + k_1) - s(c_1 s + k_1)^2} \\ \tilde{q}_2(s) = \dfrac{P(c_1 s + k_1)}{s[m_1 s^2 + (c_1+c_2)s + k_1 + k_2](m_2 s^2 + c_1 s + k_1) - s(c_1 s + k_1)^2} \end{cases} \tag{13}$$

With the Mathematica program we inverted the Laplace transforms (13), based on the data in Table 1, then with the same program we obtained the graphical representations in Fig. 9 and Fig. 10.

With the Mathematica program we inverted the Laplace transforms (10), then with the same program, based on the data in Table 1, we obtained the graphical representations in Fig.7 and Fig.8.

In [18], Krylov proposes an analytical method to determine the deflection *w* (*x, t*) of a flexible railway track under moving load and the effects on the ground.

He thus takes up a quasi-static approach based on the equation describing the vertical behaviour of a beam on a Winkler foundation:

$$E_1 I_1 \frac{\partial^4 w(x,t)}{\partial x^4} + K_f w(x,t) = P(x - v_o t), \tag{14}$$

where the parameter K_f is the stiffness (per unit length) of the foundation, which is approximated by:

$$K_f = \left(\frac{L}{k_1} + \frac{L}{k_2}\right)^{-1},$$

and v_0 is the speed of the train set, which we further take to be 100 $\left[\frac{Km}{h}\right]$.

Applying the unilateral Laplace transform with respect to time to equation (14), we obtain the equation:

$$E_1 I_1 \frac{\partial^4 \tilde{w}(x,s)}{\partial x^4} + K_f \tilde{w}(x,s) = Px\frac{1}{s} - Pv_o\frac{1}{s^2}, \tag{15}$$

to which applying the Fourier transform in the sine (the resemblance conditions allowing this) leads to an algebraic equation whose solution is:

$$\tilde{w}_s(n,s) = \frac{P}{\alpha_n\left(E_1 I_1 \alpha_n^4 + K_f\right)}\frac{1}{s}\left\{(-1)^{n+1}L - v_0\left[1 + (-1)^{n+1}\right]\frac{1}{s}\right\} \tag{16}$$

Applying the inverse of the Laplace transform in (16), we obtain the solution of the problem in the Fourier transform of the sought solution as:

$$w_s(n,t) = \frac{P}{\alpha_n\left(E_1 I_1 \alpha_n^4 + K_f\right)}\begin{Bmatrix}(-1)^{n+1} LH(t) - \\ v_0\left[1 + (-1)^{n+1}\right]t\end{Bmatrix} \tag{17}$$

Applying the inverse of the sine Fourier transform to relation (17), the solution of equation (14) is given as:

$$w(x,t) = \frac{2}{L}\sum_{n=1}^{n=\infty}\frac{P}{\alpha_n\left(E_1 I_1 \alpha_n^4 + K_f\right)}\begin{Bmatrix}(-1)^{n+1} LH(t) - \\ v_0\left[1 + (-1)^{n+1}\right]t\end{Bmatrix}\sin(\alpha_n x), \tag{18}$$

where $\alpha_n = \frac{n\pi}{L}$, and H(t) is the function of Heaviside.

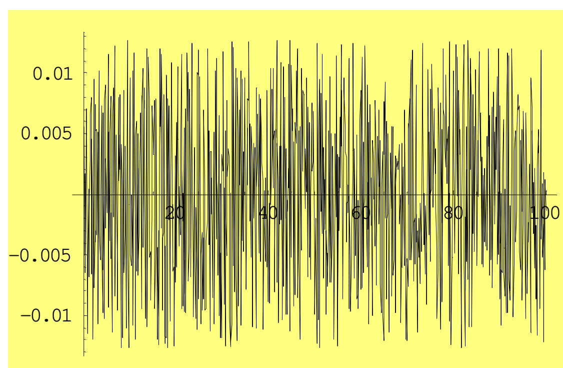

Fig. 3. Mathematica computed displacement $q_1 = q_1(t)$

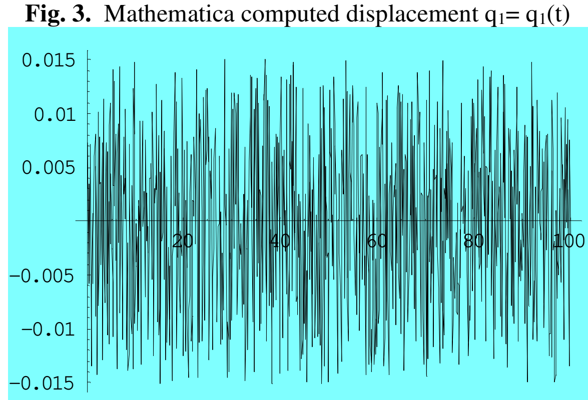

Fig. 4. Mathematica computed displacement $q_2 = q_2(t)$

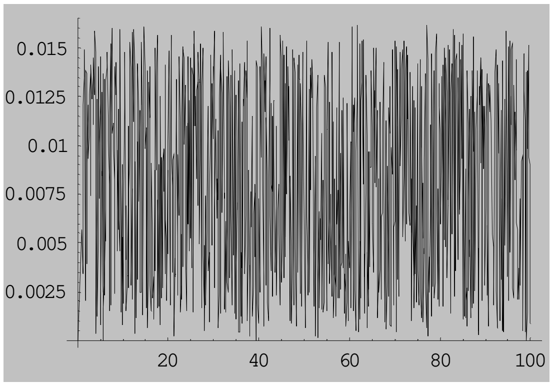

Fig. 5. Mathematica computed displacement $q_1 = q_1(t)$

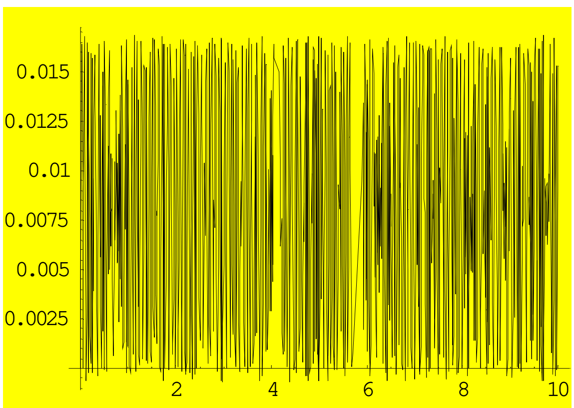

Fig. 6. Mathematica computed displacement $q_2 = q_2(t)$

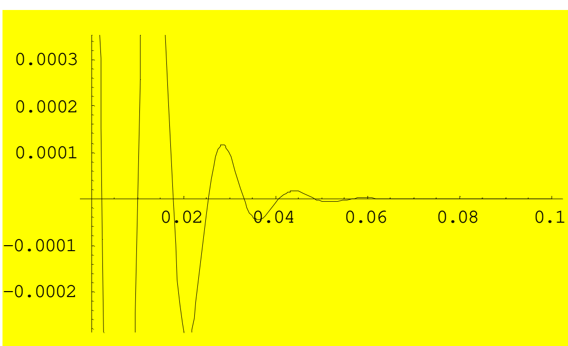

Fig.7. Mathematica computed displacement $q_1 = q_1(t)$

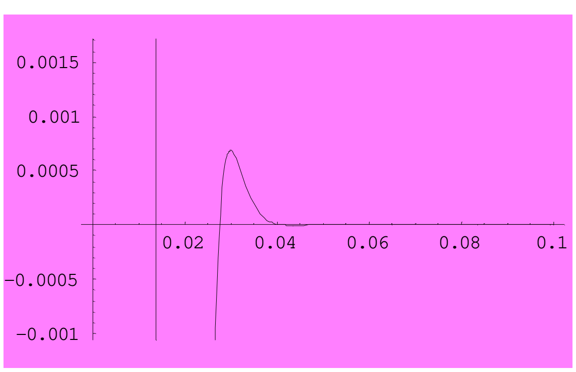

Fig. 8. Mathematica computed displacement $q_2 = q_2(t)$

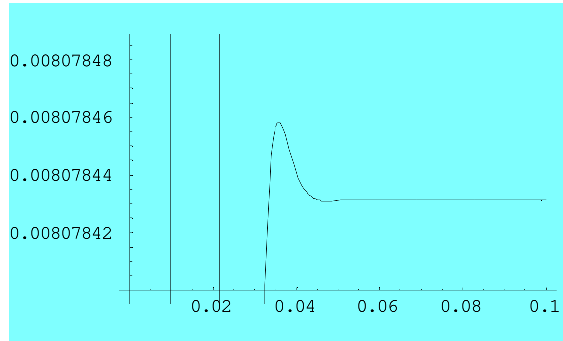

Fig. 9. Mathematica computed displacement $q_1 = q_1(t)$

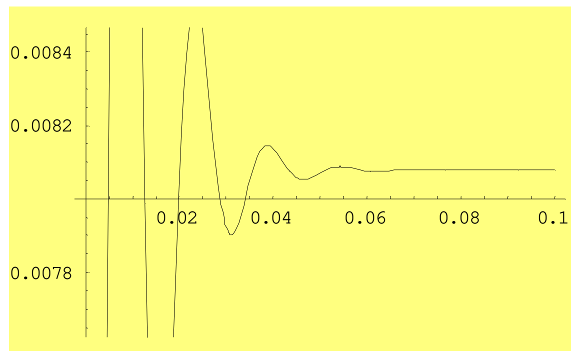

Fig. 10. Mathematica computed displacement $q_2 = q_2(t)$

Comparing the latest graphs, where amortization has occurred, with the previous ones, which were not subject to amortization, it can be seen, as expected, that the latter are much lower. With the data in Table 1, using Mathematica, the 3D graphical representation of the displacement is obtained, given by (18) induced to the rail by the vibrations of the system, as below

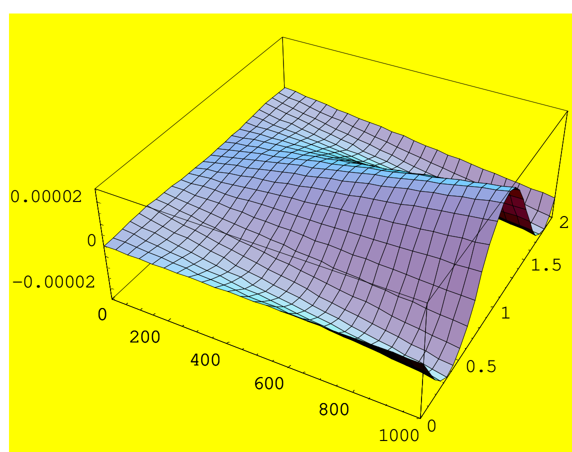

Fig. 11. Graphical representation of the temporal deflection behaviour of a flexible railway track.

In the following we have shown how the vibrations of a multibody wagon-bogie-wheel system could be modelled. For the vehicle in Fig.12 each axial axle load is modelled by a system with three degrees of freedom.

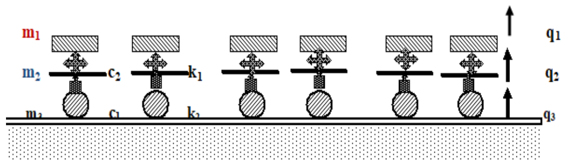

Fig. 12. The multibody model, with three degrees of freedom, of the railway vehicle.

2.1. Modelling of forced and damped vibrations

The mathematical model of the forced and damped vibrations of the railway vehicle in Fig.12 is

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{Q\}, \qquad (19)$$

where:

$[M]$ - matrix of masses; $[C]$ - damping matrix; $[K]$ - stiffness matrix; $\{q\}$ - matrix of displacements; $\{Q\}$ - generalized force matrix;

where:

$$[M] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_1 & 0 \\ 0 & 0 & m_2 \end{bmatrix}; [C] = \begin{bmatrix} c_2 & -c_2 & 0 \\ -c_2 & c_1 + c_2 & -c_1 \\ 0 & -c_1 & c_1 \end{bmatrix}; [K] = \begin{bmatrix} k_2 & -k_2 & 0 \\ -k_2 & k_1 + k_2 & -k_1 \\ 0 & -k_1 & k_1 \end{bmatrix};$$

$$\{q\} = \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix}; \{\dot{q}\} = \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{Bmatrix}; \{\ddot{q}\} = \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{Bmatrix}; \{Q\} = \begin{Bmatrix} -m_1 g \\ -m_2 g \\ F - m_3 g \end{Bmatrix}$$

m_1 - mass of the wagon; m_2 - bogie mass; m_3 - wheel mass; k_1 - the stiffness coefficient related to the wagon; k_2 - the stiffness coefficient related to the bogie; c_1 - the depreciation coefficient related to the wagon; c_2 - the corresponding depreciation coefficient related to the bogie.

Applying the unilateral Laplace transform, with respect to time, the system of differential equations with constant coefficients (10), results in an elementary algebraic system, which has as unknowns the Laplace transforms $\tilde{q}_1(s), \tilde{q}_2(s), \tilde{q}_3(s)$ of displacements $q_1(t)$, $q_2(t)$, $q_3(t)$, in the form:

$$\begin{cases} s(m_1 s^2 + c_2 s + k_2)\tilde{q}_1(s) - s(c_2 s + k_2)\tilde{q}_2(s) = -m_1 g \\ -s(c_2 s + k_2)\tilde{q}_1(s) + s[m_2 s^2 + (c_1 + c_2)s + k_1 + k_2]\tilde{q}_2(s) \\ \qquad -s(c_1 s + k_1)\tilde{q}_3(s) = -m_2 g \\ -s(c_1 s + k_1)\tilde{q}_2(s) + s(m_3 s^2 + c_1 s + k_1)\tilde{q}_3(s) = F - m_3 g \end{cases} \qquad (20)$$

Solving the algebraic system elementary (20) results in Laplace images $\tilde{q}_1(s), \tilde{q}_2(s), \tilde{q}_3(s)$ of displacements $q_1(t)$, $q_2(t)$, $q_3(t)$, in the form:

$$\tilde{q}_i(s) = \frac{P_i(s)}{P(s)}, i = \overline{1,3} \qquad (21)$$

where:

$$P_1(s) = -m_1 g s^2\{[m_2 s^2 + (c_1 + c_2)s + k_1 + k_2](m_3 s^2 + c_1 s + k_1) - (c_1 s + k_1)^2\} + \\ + s^2(c_2 s + k_2)[-m_2 g(m_3 s^2 + c_1 s + k_1) + (c_1 s + k_1)(F - m_3 g)]$$

$$P_2(s) = s^2(m_1 s^2 + c_2 s + k_2)[-m_2 g(m_3 s^2 + c_1 s + k_1) + (c_1 s + k_1)(F - m_3 g)] - \\ - m_1 g s^2(c_2 s + k_2)(m_3 s^2 + c_1 s + k_1)$$

$$P_3(s) = s^2(m_1 s^2 + c_2 s + k_2)\{[m_2 s^2 + (c_1 + c_2)s + k_1 + k_2](F - m_3 g) - m_2 g(c_1 s + k_1)\} - \\ - s^2(c_2 s + k_2)^2(F - m_3 g)$$

$$P(s) = s^3(m_1 s^2 + c_2 s + k_2)\{[m_2 s^2 + (c_1 + c_2)s + k_1 + k_2](m_3 s^2 + c_1 s + k_1) - (c_1 s + k_1)^2\} - \\ - s^2(c_2 s + k_2)(m_3 s^2 + c_1 s + k_1)$$

2.2. Modelling of forced and non-damped vibrations

The mathematical model of the forced and non-damped vibrations of the railway vehicle in Fig.12 is:

$$[M]\left\{\ddot{q}\right\} + [K]\{q\} = \{Q\} \tag{22}$$

Applying the unilateral Laplace transform, with respect to time, the system of differential equations with constant coefficients (10), results in an elementary algebraic system, which has as unknowns the Laplace transforms $\tilde{q}_1(s), \tilde{q}_2(s), \tilde{q}_3(s)$ of displacements $q_1(t)$, $q_2(t)$, $q_3(t)$, in the form:

$$\tilde{q}_1(s) = \frac{P_1(s)}{P(s)}, \tilde{q}_2(s) = \frac{P_2(s)}{P(s)}, \tilde{q}_3(s) = \frac{P_3(s)}{P(s)}, \tag{23}$$

where:

$$\begin{aligned} P_1(s) = -m_1 g s^2[(m_2 s^2 + k_1 + k_2)(m_3 s^2 &\\ + k_1) - k_1^2] &\\ + s^2 k_2[-m_2 g(m_3 s^2 + k_1) &\\ + k_1(F - m_3 g)] & \end{aligned}$$

$$\begin{aligned} P_2(s) = s^2(m_1 s^2 + k_2)[-m_2 g(m_3 s^2 + k_1) &\\ + k_1(F - m_3 g)] &\\ - m_1 g k_2 s^2(m_3 s^2 + k_1) & \end{aligned}$$

$$\begin{aligned} P_3(s) = s^2(m_1 s^2 &\\ + k_2)[(m_2 s^2 + k_1 + k_2)(F &\\ - m_3 g) - m_2 g k_1] &\\ - k_2^2 s^2(F - m_3 g) & \end{aligned}$$

$$\begin{aligned} P(s) = s^3(m_1 s^2 &\\ + k_2)[(m_2 s^2 + k_1 + k_2)(m_3 s^2 &\\ + k_1) - k_1^2] &\\ - k_2 s^2(m_3 s^2 + k_1) & \end{aligned} \tag{24}$$

Inverting the Laplace transforms, with data from Table 1 of application, with the Mathematica program, results the graphical representations from Fig.13 and Fig.14 of the movements of the bogie and wheels caused by forced and damped vibrations induced by the force action system of 118 *k*N. Also, in Fig.15 and Fig.16 are given graphical representations of the movements of the bogie and the wheels caused by the forced and non-damped vibrations induced by the system by the action of a force of 118 *k*N. Comparing the displacements in the graphs in Fig.13 and Fig.14, with those in the graphs in Fig.15 and Fig.16, it is clear that the former is much smaller than the latter. This shows the importance of introducing damping devices into vibrating mechanical systems.

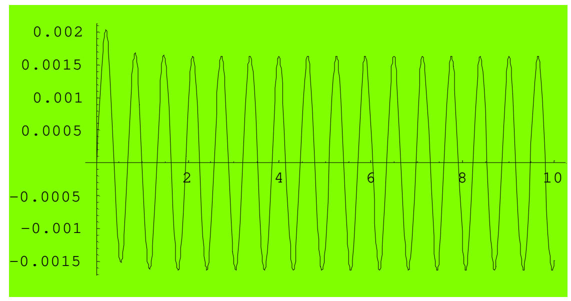

Fig. 13. Mathematica computed displacement $q_2 = q_2(t)$

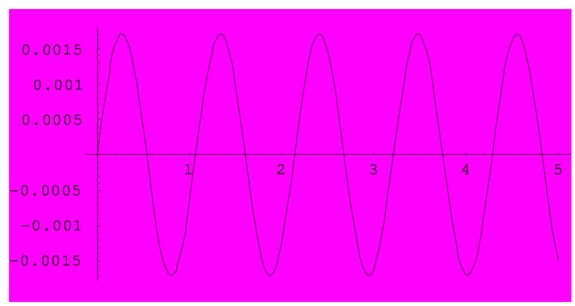

Fig. 14. Mathematica computed displacement $q_3 = q_3(t)$

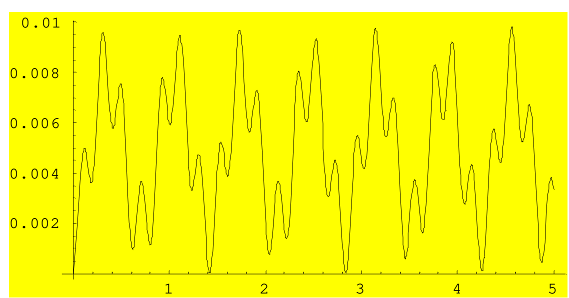

Fig. 15. Mathematica computed displacement $q_2 = q_2(t)$

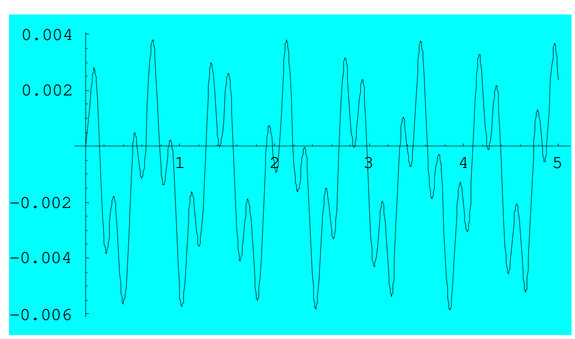

Fig. 16. Mathematica computed displacement $q_3 = q_3(t)$

Table 1

Parameter	Unit of measure	Symbol	Size
Longitudinal modulus of elasticity of the rail	$[\mathrm{GPa}]$	E1	210
Moment of inertia of the cross sections of the rail	$[\mathrm{cm}^4]$	I_1	1987
Density of the material	$\left[\frac{\mathrm{Kg}}{\mathrm{m}^3}\right]$	ρ_1	7850
Stiffness coefficient related to the rail	$\left[\frac{\mathrm{MN}}{\mathrm{m}}\right]$	k_1	90
The cross-sectional area of the rail	$\left[\frac{\mathrm{MN}}{\mathrm{m}}\right]$	k_2	25,5
The cross-sectional area of the rail	$[\mathrm{cm}^2]$	A_1	63,8
Rail length	$[\mathrm{m}]$	L_1	0,72
Cross mass	$[\mathrm{Kg}]$	m_2	90,84
Damping coefficient for the rail	$\left[\frac{\mathrm{kNs}}{\mathrm{m}}\right]$	c_1	30
Damping coefficient related to the crossbar	$\left[\frac{\mathrm{kNs}}{\mathrm{m}}\right]$	c_2	40
Loading	$[\mathrm{kN}]$	P	206
Geometric irregularity of the rail	$[\mathrm{m}]$	q_{10}	0,005
Geometric irregularity of the crossbar	$[\mathrm{m}]$	q_{20}	0,003

3. CONCLUSIONS

In this paper, an analysis was performed to determine the deformation *w* (*x, t*) of the flexible railway subjected to a mobile load, as well as the effects on the ground.

The determination of the dynamic behaviour of the structures subjected to vibrations was done with the help of integral transformations. These, leading to the so-called algebraization of the problem, facilitate the determination of the dynamic responses of vibrating systems.

The graphical representation of the temporal behaviour of the flexible rail deflection was obtained with the Mathematica program. For the model with two degrees of freedom, without load, for the railway, the unilateral Laplace transformation in relation to time was also resorted to.

With the Mathematica program, based on the numerical data presented in the paper, we reversed the Laplace transformations, resulting in displacements in the form of time functions, after which, with the same program, we have graphical representations of displacements and cause imperfections and geometries. .

Next, using the same presentations, we obtained graphical representations of the bogie and wheel movements caused by the forced and damped vibrations induced by the system by the action of a force.

Comparing the movements of the bogie and the wheels caused by the forced and damped vibrations, induced by the action of a force, with those caused by the forced and non-damped vibrations, it is found that the former is smaller, which does not show the importance of introducing damping means in mechanical vibration systems.

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MODELAREA EFECTELOR VIBRATORIILORLOR ALE TRAFICULUI FEROVIAR ASUPRA MEDIULUI

Rezumat: În ciuda numeroaselor progrese tehnologice, zgomotul şi diversele forme sub care el se manifestă este perceput ca o sursă multiplă de probleme asupra mediului înconjurător,

care sunt din ce în ce mai mult suportate de oameni. Sensibilitatea față de vibrațiile traficului feroviar este cea mai importantă în opinia cercetătorilor. În această lucrare ne-am propus să determinăm răspunsul dinamic la vibrații al sistemului vehicul-cale ferată. În acest scop am folosit cele mai moderne metode de integrare folosite în astfel de scopuri la ora actuală. Pentru modelul cu două grade de libertate, fără încărcare, pentru calea ferată, am apelat la transformata Laplace unilaterală în raport cu timpul care a dus la algebrizarea problemei, ceea ce a simplificat integrarea sistemului de ecuații diferențiale. Cu programul Mathematica, în baza datelor numerice prezentate în lucrare, am inversat transformatele Laplace, rezultând deplasările sub forma unor funcții de timp, după care, cu același program, am obținut reprezentările grafice ale deplasărilor șinei provocate de imperfecțiunile ei geometrice. În continuare, apelând la aceeși metodă, am obținut reprezentările grafice ale deplasărilor boghiului şi roţilor provocate de vibraţiile forţate şi amortizate induse sistemului de acţiunea unei forţe. Pentru determinarea deflexiei căii feroviare flexibile supuse unei încărcări mobile modelul matematic îl reprezintă o ecuație cu derivate parțiale. Acest model matematic l-am integrat aplicându-i, mai întâi, transformata Laplace unilaterală în raport cu timpul, rezultând o ecuație în imagini Laplace, căreia i-am aplicat transformata Fourier finită în sinus. Rezolvând sistemul algebric astfel rezultat, am obținut răspunsul dinamic al sistemului mecanic în imagini Laplace și Fourier. Aplicând, mai întâi, inversa transformatei Laplace și apoi inversa transformatei Fourier în sinus soluției sistemului algebric, am obținut soluția ecuației cu derivate parțiale, amintită mai sus, sub forma unei funcții de timp și deplasare, pe care am reprezentat-o grafic cu programul Matematica.

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