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MULTI-OBJECTIVE DYNAMIC OPTIMIZATION OF A NOVEL SUSPENSION SYSTEM FOR RACE CARS

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Abstract: The work approaches the dynamic optimization of an innovative suspension system (with triadbased wheel guidance mechanism) used in Formula Student race cars. The work actually continues the study from a previous paper by the authors, in which the kinematic optimization of the proposed suspension was approached. While in the case of the kinematic model, where the chassis is fixed, the optimization aimed at minimizing wheel movement-specific variations, the dynamic optimization targets the angular movements of the chassis, namely roll, pitch and yaw. The dynamic optimization process is conducted on the basis of regression models and DOE (Design of Experiments) investigation strategies, by using specific modules from the ADAMS software package (namely Insight and View). **Key words:** race car, suspension system, dynamics, optimization.

1. INTRODUCTION

The optimal design of the wheel suspension mechanism is an essential element for the safety and stability of racing cars (in this case, Formula Student), considering the special dynamic regimes in which these cars run. It should be noted that before being able to compete in the track, such a car must pass a series of static tests, including the behavior of the suspension system, the regulations in this regard being very strict [1-8].

The traditional suspension system (for both front and rear wheels) used for race cars is the one based on 4-bar mechanism, often entitled double-wishbone (Figure For 1). such mechanisms used in the suspension of passenger cars, the spring & damper assembly is arranged in a vertical plane, (Figure 1.a), commonly between the upper control arm of the 4-bar mechanism and the car body, which is not possible for race cars due to the limited available vertical space. Under these conditions, the race cars commonly use the solution with the arrangement of the spring & damper assembly in a horizontal plane, the forces being transmitted to the chassis through a push-rocker group (Figure 1.b).



Fig. 1. Suspension setups based on 4-bar mechanism.

From structural and kinematic point of view, the two variants of suspension based on 4-bar mechanism each have one degree of mobility (DOM), namely the wheel vertical displacement (Y_K) due to the profile of the running track. Although it is a simple solution, the 4-bar suspension mechanism has the well known issue of contradictory changes of wheel track and camber (by decreasing one of these variations the other will increase, this resulting in a nonlinear dynamic behavior) [9].

The decoupling of the two contradictory variations can be achieved by using a 2-DOM mechanism, based on a 5-bar setup (see Figure 2), which implies a supplementary degree of mobility (for example at the level of the upper arm movement) by reference to the classical 4-bar mechanism.



Fig. 2. Suspension based on 5-bar mechanism.

The second mobility in the 5-bar mechanism can be controlled both electronically (so active suspension) and mechanically (passive suspension). In the active suspension, the control of the second degree of mobility can be realized by adding a linear actuator that pulls/pushes the upper rod of the 5-bar mechanism (Figure 3), thus cancelling, as the case may be, the variation of the wheel track or of the camber angle. This solution was discussed in a previous work [10], focusing on the optimal design of the control system so that to minimize the wheel track variation.



Fig. 3. Active suspension derived from the 5-bar setup.

In the passive suspension (Figure 4), the movement of the upper rod is geometrically controlled through the rocker MM_0 , whose arrangement is made so that the trajectory of the M point will ensure the cancellation (or at least the minimization) of the wheel track or camber angle variation, as the case may be.



Fig. 4. Passive suspension derived from the 5-bar setup.

The passive suspension system derived from the 5-bar setup (which is in fact a triad-based mechanism) was discussed in [11], but only from a kinematic point of view, through a multiobjective optimization process aimed at minimizing or canceling (as appropriate) the unwanted linear and angular displacements of the wheel. Through the present work, the authors aim to continue the study on the innovative passive suspension mechanism by addressing specific issues for optimizing the dynamic behavior, so that the approach of the proposed design to be a complete one.

2. INNOVATIVE SUSPENSION SYSTEM CONCEPT

The design of the suspension system with triad-based wheel guiding mechanism was performed in the steps mentioned below [11]:

(1) canceling the variation of the camber angle (whose significance is schematically shown in Figure 5) in the 5-bar mechanism through the kinematic restriction $\Delta \gamma = \gamma - \gamma_0 = 0$;



Fig. 5. The camber angle of the wheel.

(2) carrying out the kinematic simulation of the suspension mechanism under the conditions in which the wheel vertical displacement is imposed by an in-time variation law, $Y_K=Y_K(t)$;

(3) obtaining the trajectory of a convenient point M on the upper rod, whose trajectory is as close as possible to a semicircle or hemisphere;

(4) replacing the kinematic restriction from step (1) with the guiding rocker MM_0 , the global coordinates of the focal point on chassis (M_0) resulting from a numerical algorithm based on the least squares method [11].

The passive suspension obtained by going through the previously presented design algorithm corresponds to a mono-mobile mechanism in its planar version. In the following, the study is developed based on the spatial configuration of the suspension mechanism (which is more realistic for practical implementation). Compared to the planar version of the mechanism, the corresponding spatial configuration has one supplementary degree of mobility, corresponding to the angular displacement from the vertical axis of the wheel, briefly named caster (Figure 6).



Fig. 6. The caster angle of the wheel.

For the spatial configuration of the mechanism, two kinematic restrictions are used in the first step from the previously mentioned algorithm with the purpose to cancel the variations of the camber ($\Delta \gamma = \gamma - \gamma_0 = 0$) and caster ($\Delta \beta = \beta - \beta_0 = 0$) angles. Subsequently, at step (4), these restrictions will be substituted through the guiding rocker MM₀. The kinematic optimization is further performed with the purpose to minimize the variations of wheel track, wheelbase and bump steer [11].

3. DEFINING THE DYNAMIC MODEL

As stated above, the dynamic model of the proposed suspension mechanism is based on the spatial configuration schematically rendered in Figure 7. In addition to the bodies that appear on the planar version of the mechanism (that is, the one in Figure 4), the model also contains the toe link (denoted by 6), which is spherically connected to wheel carrier and chassis (for the non-steered rear wheels), respectively to wheel carrier and steering rack (for the steered front wheels).

The dynamic optimization will take place in the spring & damper layout chain ("DC" in Figure 7), which from a kinematic point of view has an imposed movement based on the behavior of the kinematic chain ("KC") that was previously optimized [11].



Fig. 7. The spatial scheme of the suspension system.

Unlike the kinematics, in which it is sufficient to use quarter car models (the chassis being fixed), the dynamic study (where the chassis is a moving part) is going to be based on half-car models, considering the front or rear suspension at a time.

The half-car car model is obtained by removing from the full-car model of one of the axles, front or rear by case, as shown in Figure 8.



Fig. 8. Half-car models: front axle (a), rear axle (b).

To ensure the equilibrium of the chassis, in the given conditions to the half-car models, the missing axle (as the case may be, front or rear) must be replaced with a fictitious connection between the chassis and the ground. In this regard, previous research by the authors [12-14] has shown that the use of a ball joint is best suited, as it allows the chassis to perform all the angular oscillations induced by the car's running on the track, namely roll, pitch and yaw. The ball joint locations (denoted by P_r and P_f in Figure 8) were determined by the double-conjugate point's method [15].

The half-car models are simulated/tested in the regime of passing over bumps, by generating the dynamic forces on wheels with a specific designed rig. The wheels will be linked to the actuators of the rig with contact joints, and the actuators will recreate the rolling surface vertical oscillations. The rolling surface profile for the left and right wheel was created by imposing motions to the rig actuators. An asymmetrical movement of the left to right oscillation was considered, as follows: while the right wheel is not oscillating (there is no irregularities on that side of the rolling surface), the left hand side actuator will impose a sinus wave of +/- 25mm.

4. DYNAMIC OPTIMIZATION PROCESS

The dynamic model of the proposed suspension mechanism was developed with the help of ADAMS/View, while the optimization study was addressed through a multi-objective parametric design process in ADAMS/Insight, within the ADAMS software package existing conduits between these modules.

The dynamic optimization is required for determining the optimal arrangement of the spring & damper assembly so as to ensure the minimization of the roll, pitch, and yaw movements of the chassis (in terms of root mean square during simulation), which play the role of design objectives (i.e. responses). In ADAMS/Insight, each of the three responses is to be transposed as a regression function.

The design variables (called factors in ADAMS/Insight) for the dynamic optimization are the global coordinates (X, Y, Z) of the spring and damper attachment hard points to the chassis, I and J, and to the rockers, K and L (see Figure 9). As points I/K and J/L are symmetrical to the longitudinal plane of the chassis/car, there will result only six factors (see Table I), whose initial/standard values for the front axle

suspension mechanism are as follows: $f_01 - f_06 = \{-204.6, 406.35, 69.069, -94.434, 349.69, 111.36\}$ mm. For each factor, the variation range is ±20mm from the initial value.



Fig. 9. Spring & damper assembly layout.

Table 1

The factors for the front axle suspension.

Point	Х	Y	Z	
Ι	f_01	f_02	f_03	
J	f_04	f_05	f_06	
K	(-1* f_01)	f_02	f_03	
L	(-1* f_04)	f_05	f_06	

For determining the regression functions attached to the three responses, there must be specified/defined the investigation strategy (e.g. Screening, Response Surface, Variation-Monte Carlo, Study-Perimeter), the type of regression model (e.g. Linear, Linear with Interactions, Quadratic, Cubic) and the DOE (Design of Experiments) technique (e.g. Plackett Burman, Fractional Factorial, Full Factorial, D-Optimal) [16], according to which the so-called work space matrix is subsequently generated. The work space will be later populated with the combinations of values of the factors and the resulting responses, which are determined by running in ADAMS/View the half-car suspension model for each trial specified in this matrix.

combinations of Several investigation strategies. regression models and DOE techniques were tested in order to identify the one that is best suited (as accuracy of the regression models), namely DOE Screening -Linear with Interactions - D-Optimal, whose working matrix. The matrix of the work space corresponding to the picked investigation strategy is presented in Figure 10, where r_0i are the notations for the three responses, in the following order: yaw (r_01), pitch (r_02), and roll (r 03).

W	Work Space										
	Trial		f_01	f_02	f_03	f_04	f_05	f_06	r_01	r_02	r_03
1	Trial	1	-184.6	426.35	49.069	-114.434	329.69	91.366	0.00411246	2.21337	0.000496108
2	Trial	2	-184.6	386.35	89.069	-114.434	329.69	91.366	0.00274581	4.20051	0.000229862
3	Trial	3	-184.6	386.35	49.069	-74.434	329.69	91.366	0.0050141	2.05103	0.000366194
4	Trial	4	-184.6	386.35	49.069	-114.434	369.69	91.366	0.00239115	4.66949	0.000269176
5	Trial	5	-184.6	386.35	49.069	-114.434	329.69	131.366	0.00285151	2.98112	0.000265971
6	Trial	6	-224.6	426.35	89.069	-114.434	329.69	91.366	0.00544574	1.16177	0.000255985
7	Trial	7	-224.6	426.35	49.069	-74.434	329.69	91.366	0.00563925	1.10892	5.43163e-005
8	Trial	8	-224.6	426.35	49.069	-114.434	369.69	91.366	0.00433765	1.90813	0.000577285
9	Trial	9	-224.6	426.35	49.069	-114.434	329.69	131.366	0.00494522	0.608296	5.71125e-005
10	Trial	10	-224.6	386.35	89.069	-74.434	329.69	91.366	0.00359739	0.523158	0.000115194
11	Trial	11	-224.6	386.35	89.069	-114.434	369.69	91.366	0.000977019	3.23939	0.000151966
12	Trial	12	-224.6	386.35	89.069	-114.434	329.69	131.366	0.0068272	2.25113	0.000468945
13	Trial	13	-224.6	386.35	49.069	-74.434	369.69	91.366	0.00455718	0.646745	0.000167007
14	Trial	14	-224.6	386.35	49.069	-74.434	329.69	131.366	0.0141701	1.59442	0.000234857
15	Trial	15	-224.6	386.35	49.069	-114.434	369.69	131.366	0.00917694	1.84539	0.000571314
16	Trial	16	-184.6	386.35	89.069	-74.434	369.69	131.366	0.00229693	2.74018	0.000247354
17	Trial	17	-184.6	426.35	89.069	-74.434	369.69	91.366	0.00271921	2.00419	0.000215656
18	Trial	18	-184.6	426.35	49.069	-74.434	369.69	131.366	0.00432609	1.03978	0.000200226
19	Trial	19	-224.6	426.35	89.069	-74.434	369.69	131.366	0.00227832	0.487011	6.70109e-005
20	Trial	20	-184.6	426.35	89.069	-114.434	369.69	131.366	0.000622336	3.31443	0.00079584
21	Trial	21	-184.6	426.35	89.069	-74.434	329.69	131.366	0.00346689	0.846705	0.000142182
22	Trial	22	-224.6	386.35	49.069	-114.434	329.69	91.366	0.00671689	2.2338	0.000460636

Fig. 10. Work space matrix.



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	r_01		r_02		r_03			r 01	r 02	r 03
R2	1	0	1	0	1	•	E 14			
R2adj	1	0	1	0	1	(ГІІ			0
Р	0	0	0	0	0	0	Term		•	
R/V	1e+20	0	1e+20	0	1e+20	0	Residuals		•	C

Fig. 11. Evaluating the regression models.

In ADAMS/Insight, the accuracy of the regression models is reported by means of graphical indicators (red - value should be investigated, yellow - value is not wrong but should be considered, green - value is appropriate), by using statistics evaluation methods, such as Cook's statistics, studentized residuals, term significances, goodness-of-fit, and rules-of-thumb summary [16, 17].

For the strategy identified as the best suited (DOE Screening - Linear with Interactions - D-Optimal), all evaluation parameters have appropriate values (so, green indicators), as shown in Figure 11. Based on the values in the work space matrix, the regression functions for the three dynamic responses are subsequently generated. The specific coefficients of the regression models are the ones presented in Figure 12. The products between factors (f_01*f_02 , and so on) are specific to the linear regression model with interactions.

The effective optimization of the half-car suspension system model was carried out by using the OptDes-GRG algorithm (provided with ADAMS/Insight), with the MinTo operator, which constraints the response to be as close as possible to the target.

rer	т соепі	cients for	model "I	vioaei_0
	r_01	r_02	r_03	Term
1	8.3883	-175.49	-148.61	1 (constant)
2	0.0083909	-0.43013	-1.3677	f_01
3	-0.026545	-0.10474	0.89781	f_02
4	0.0095285	0.29984	2.179	f_03
5	0.0007221	-1.1708	-0.21064	f_04
6	-0.0092888	0.60065	-0.74541	f_05
7	-0.0088591	-0.12147	-2.4712	f_06
8	-4.1686e-05	-0.0014517	0.0042055	f_01 * f_02
9	2.2202e-05	0.0015318	9.2141e-05	f_01 * f_03
10	-3.0721e-05	-0.0056148	-0.001762	f_01 * f_04
11	4.6638e-06	0.0019012	-0.00058851	f_01 * f_05
12	1.316e-05	-0.001446	-0.001574	f_01 * f_06
13	5.2745e-05	-0.00055487	0.0032413	f_02 * f_03
14	-1.402e-05	0.0010298	0.0036553	f_02 * f_04
15	3.1766e-05	-0.00061113	-0.00033284	f_02 * f_05
16	1.2762e-05	0.0008064	0.0010673	f_02 * f_06
17	2.0133e-05	-0.001417	0.009013	f_03 * f_04
18	-6.3916e-05	0.00074406	-0.0060845	f_03 * f_05
19	-1.697e-05	-0.00070156	-0.0054864	f_03 * f_06
20	-8.3552e-06	-0.0017288	-0.0063516	f_04 * f_05
21	-4.6e-06	0.0015819	-0.0022383	f_04 * f_06
22	2.4229e-05	-0.0010106	0.0061979	f_05 * f_06

Fig. 12. Regression function coefficients.

As a result of the effective optimization, the optimal values of the factors are found, as follows: $f_01 - f_06 = \{-224.6, 386.35, 89.069, -74.434, 329.69, 131.36\}$ mm. With these values, through the dynamic analysis of the half-car model, there are obtained the in-time variations of the roll, pitch and yaw angles, as shown in Figures 13-15 (blue solid curves). The initial variations (before optimization) of these parameters are also presented (red dash curves), noting a significant improvement of all the monitored dynamic parameters.









Fig. 15. Yaw angle variation for the front axle model.

The rear axle optimization was carried out in a similar way to the front axle, considering the following initial values of the factors: f_01 $f_06 = \{306.2, 366.15, -1753.0, 142.04, 244.85, -1750.6\}$ mm. The correlation between factors and hard points coordinates are similar with the ones in Figure 9 and Table I.

By running the optimization process (with the same settings as for the front half-car model), the following optimal values of the design variables were obtained: $f_01 - f_06 = \{286.2, 346.15, -1733, 162.04, 224.85, -1770.6\}$ mm. The resulting responses are shown in Figures 16-18, noting again a significant improvement in the dynamic behavior.



Fig. 16. Roll angle variation for the rear axle model.



Fig. 17. Pitch angle variation for the rear axle model.



Fig. 18. Yaw angle variation for the rear axle model.

5. CONCLUDING REMARKS

Regarding the elements of originality of this work, it should be mentioned that not only the suspension system with triad-based wheel guiding mechanism is an innovative one, but also the way it is designed through the algorithm briefly presented in the 2^{nd} chapter of the work.

The results of the kinematic study [11] along with those corresponding to the dynamic behavior approached in this work confirm that the proposed suspension solution fulfills the basic function for which it was designed, namely the decoupling of contradictory variations of the wheel track and camber angle from the classical (frequently used) 4-bar suspension mechanism. Moreover, the proposed suspension system ensures the maintenance of the other kinematic and dynamic parameters within the limits allowed by the rules of the Formula Student competition.

The dynamic analysis and optimization based on half-car models precedes the achievement of the full-car model (shown in Figure 19), which is obtained by coupling/linking the two above optimized front and rear suspension systems, thus allowing the chassis to be detached from any other connection except for those with a physical equivalent from the suspension system, so that it can perform all the six degrees of freedom that really exist.



Fig. 19. Full-vehicle virtual prototype.

The full-vehicle dynamic model will allow a more accurate (close to reality) assessment of the dynamic behavior of the race car, given the mutual influences between the front and rear axles through the chassis, this remaining to be studied in a further work, along with the development, implementation and testing of the physical prototype (which is based on the specifications of the virtual prototype in Figure 19).

On the other hand, the optimal design algorithm used in this work (based on regression models and DOE investigation strategies) is characterized by general and unitary character, since it can be used/adapted for all types of suspension systems (front and rear), for both kinematic and dynamic optimization.

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Optimizarea dinamică multi-obiectiv a unui sistem inovativ pentru suspensia automobilelor de cursă

Rezumat: În această lucrare se prezintă optimizarea dinamică multi-obiectiv a unui sistem de suspensie inovativ (cu mecanism de ghidare a roții bazat pe triadă) utilizat la automobile de cursă tip Formula Student. Lucrarea este, de fapt, o continuare a studiului prezentat într-un articol anterior al autorilor, în care optimizarea mecanismului de suspensie a fost abordată strict din punct de vedere cinematic. Dacă în cazul modelului cinematic, la care șasiul monopostului este considerat fix (deci, baza mecanismului), optimizarea a urmărit minimizarea variațiilor unor parametri de mișcare specifici roții, în cazul modelului dinamic, optimizarea vizează parametri de mișcare ai șasiului (care este mobil), și anume oscilațiile de ruliu, tangaj și girație. Procesul de optimizarea dinamică este efectuat pe bază de modele de regresie și strategii de investigație DOE (Design of Experiments), prin utilizarea de module specifice din pachetul software de prototipare virtuală ADAMS (și anume ADAMS Insight și View).

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